What are the conditions for acheiving Linear Phase?

Generalyzed Linear Phase:

$$H(\omega) = \underbrace{H_m(\omega)}_{\substack{\text{Real} \\ \text{positive or negative.}}} e^{j(\beta - \alpha\omega)}$$

α = group delay.

$\measuredangle H(\omega) = \beta - \alpha\omega$

$-\frac{d}{d\omega} \measuredangle H(\omega) = \alpha \leftarrow$

$$H(\omega) = H_m(\omega) \cos(\beta - \alpha\omega) + j\, H_m(\omega) \sin(\beta - \alpha\omega)$$

$$\tan\left(\measuredangle H(\omega)\right) = \frac{\sin(\beta - \alpha\omega)}{\cos(\beta - \alpha\omega)} = \tan(\beta - \alpha\omega)$$

$\leftarrow$ eqn 1.

How can we deriven $\measuredangle H(\omega)$ in Terms of $h(n)$?

$$H(\omega) = \sum_n h(n)\, e^{-j\omega n}$$

$$H(\omega) = \sum_n h(n) \cos(\omega n) - j \sum_n h(n) \sin(\omega n)$$

$$\tan\left[\measuredangle H(\omega)\right] = \frac{-\sum_n h(n) \sin(\omega n)}{\sum_n h(n) \cos(\omega n)} \leftarrow \text{Eqn 2.}$$

Combine Eqn 1 & 2 :

$$\boxed{\frac{\sin(\beta - \alpha\omega)}{\cos(\beta - \alpha\omega)} = \frac{-\sum_n h(n) \sin(\omega n)}{\sum_n h(n) \cos(\omega n)}}$$

$\rightarrow$ necessary condition for $h(n)$ To be G.L.P.

$$\sin(\beta - \alpha\omega) \sum_n h(n) \cos \omega n + \cos(\beta - \alpha\omega) \sum_n h(n) \sin \omega n = 0$$

$$\Rightarrow \boxed{\sum_{n=0}^{N-1} h(n) \sin\left[\omega(n-\alpha) + \beta\right] = 0}$$

↳ necessary condition for $h(n)$ to be G.L.P.

Consider 2 cases:

Case ① : $\beta = 0$ or π

$$\Rightarrow \sum_n h(n) \sin(\omega(n-\alpha)) = 0$$

Can show: If $\boxed{h(n) = h(2\alpha - n)}$ then This is True.

$\boxed{N = 2\alpha + 1}$

Case ② $\beta = \pi/2$ or $3\pi/2$

$$\boxed{\sum_{n=0}^{N-1} h(n)\cos[(n-\alpha)\omega] = 0}$$

Can show: If $\boxed{h(2\alpha - n) = -h(n)}$ $\boxed{N = 2\alpha + 1}$

Then satisfy

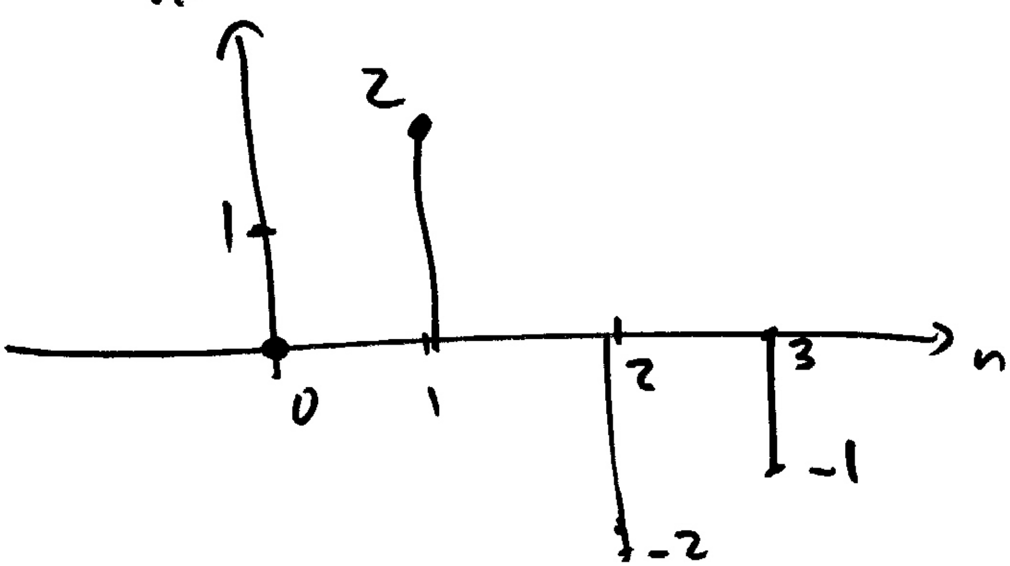
$h(N-1-n) = -h(n)$

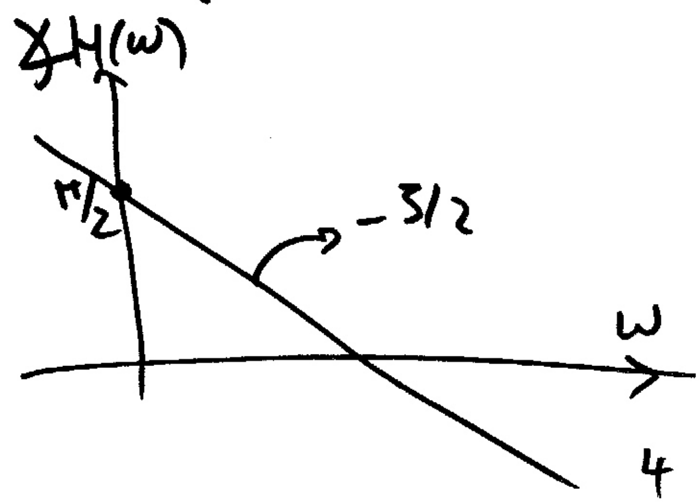
$\underline{EX}$ $N = 4 = \#$ of Taps $\quad N = 2\alpha + 1 = \#$ of Taps.

$-h(n) = h(3-n)$ $\quad \beta = \pi/2 \implies \alpha = 3/2$

$$H(\omega) = H_m(\omega)\, e^{+j\left(\pi/2 - \frac{3}{2}\omega\right)}$$

$h(3-n) = -h(n)$

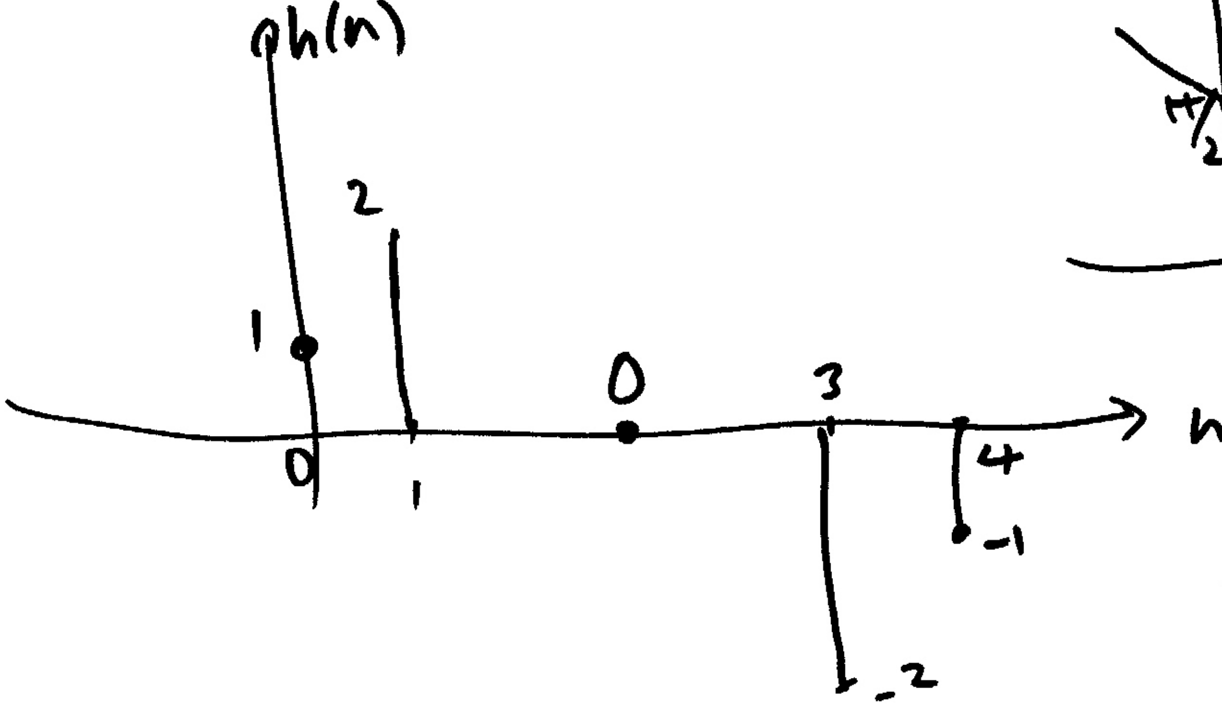




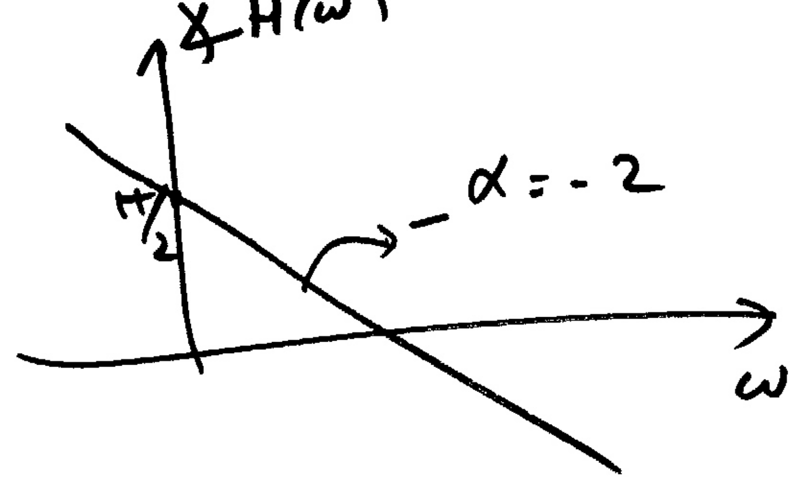
Ex $h(n)$ 5 points

$$N = 2\alpha + 1$$
$$5 = 2\alpha + 1$$
$$\Longrightarrow \alpha = 2$$

$$h(2\alpha - n) = -h(n)$$
$$h(4 - n) = -h(n)$$



$\beta = \pi/2$



$$H(\omega) = H_m(\omega)\, e^{+j(\cancel{\beta} - \alpha\omega)}$$

Can show 2 things.

Case ① $\beta = 0$

$$h(n) = \begin{cases} h(N-1-n) & 0 \leq n < N \\ 0 & \text{otherwise} \end{cases}$$

Then

$$\boxed{H(\omega) = H_m(\omega)\, e^{-j\left(\frac{N-1}{2}\right)\omega}}$$

$$\boxed{\beta = 0}$$
$$\boxed{\alpha = \frac{N-1}{2}}$$

Case 2 $\quad \beta = \pi/2$

$$h(n) = \begin{cases} -h(N-1-n) & 0 \le n < N \\ 0 & \end{cases}$$

$H(\omega) = H_m(\omega)\, e^{j(\beta - \alpha\omega)}$ with $\beta = \pi/2$, $\alpha = \frac{N-1}{2}$

$$\boxed{H(\omega) = H_m(\omega)\, e^{j\pi/2}\, e^{-j\omega\left(\frac{N-1}{2}\right)}}$$

Case ① $\beta = 0$ symmetry

odd # of Taps — even

Case ② $\beta = \pi/2$ anti-symmetry.

odd — even.

$\Longrightarrow$ 4 Types of GLP FIR

odd ← Type I
even ← Type II

$\Big\}$ → symmetric

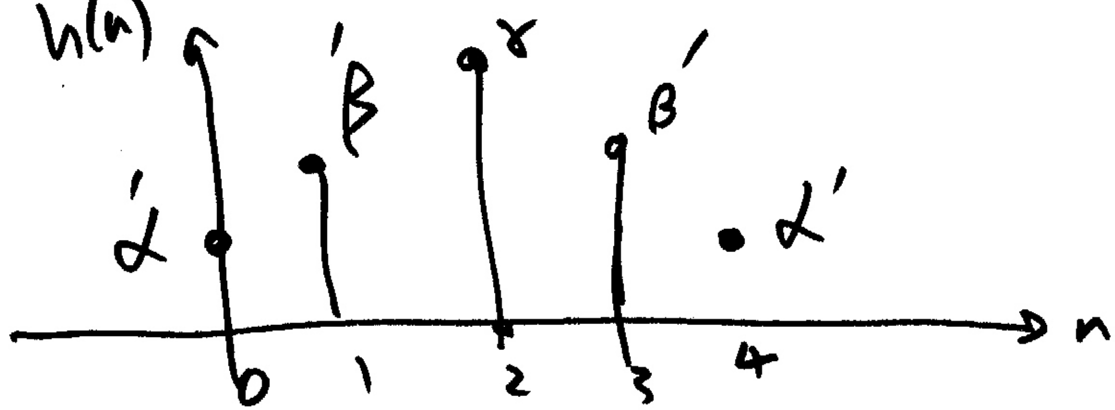
odd ← Type III
even ← Type IV

$\Big\}$ → Anti-symmetric.

Type I : symmetry   # of taps odd.

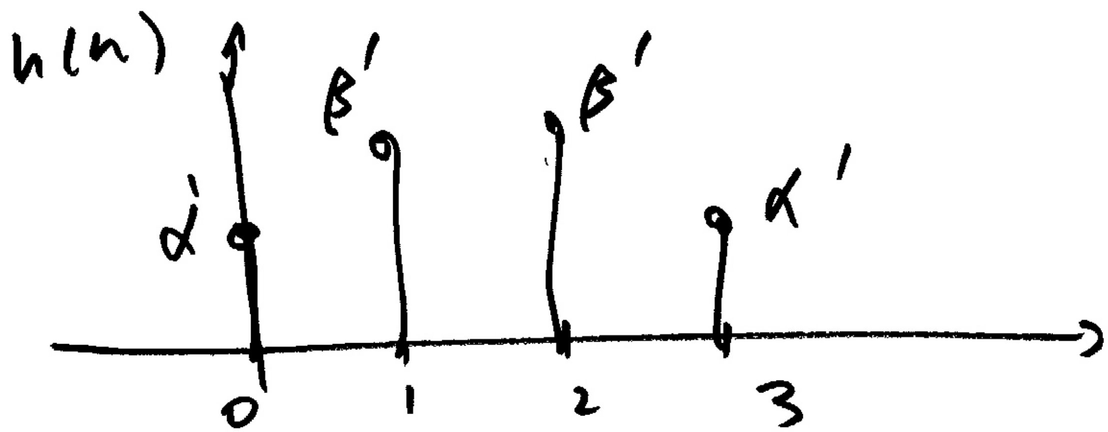
$h(n) = h(N-1-n)$

$N = 5$.



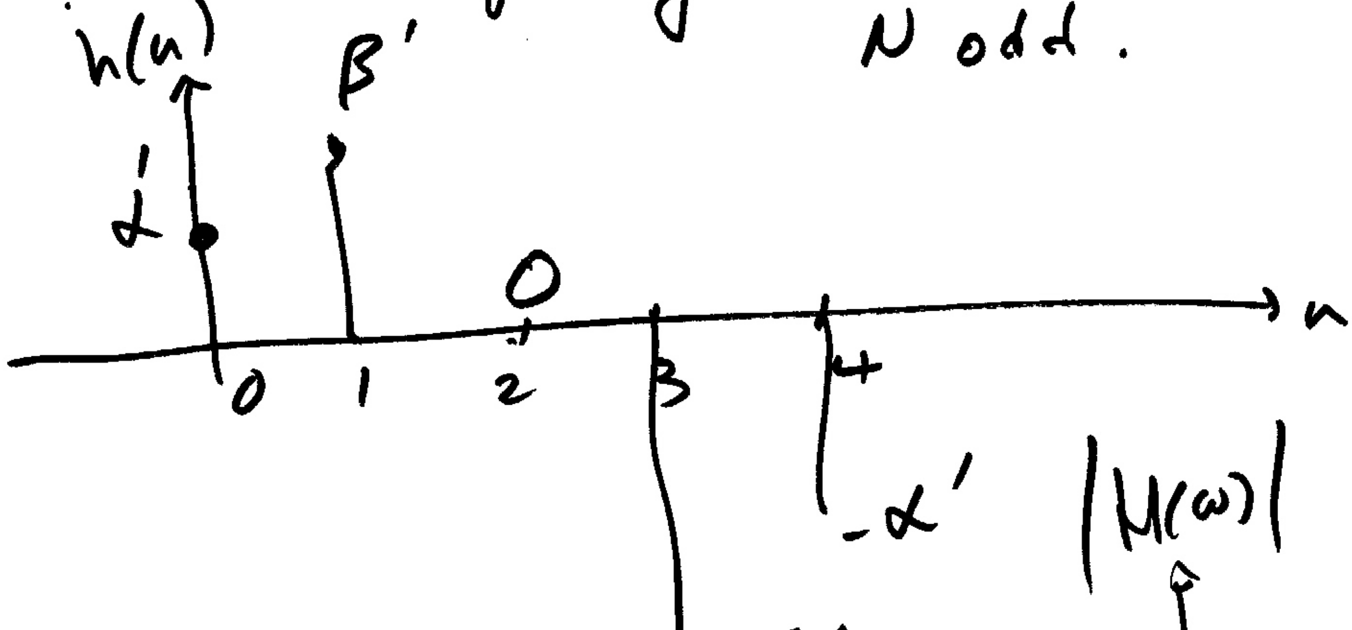
<u>Type II</u>   symmetry   # of taps is even.

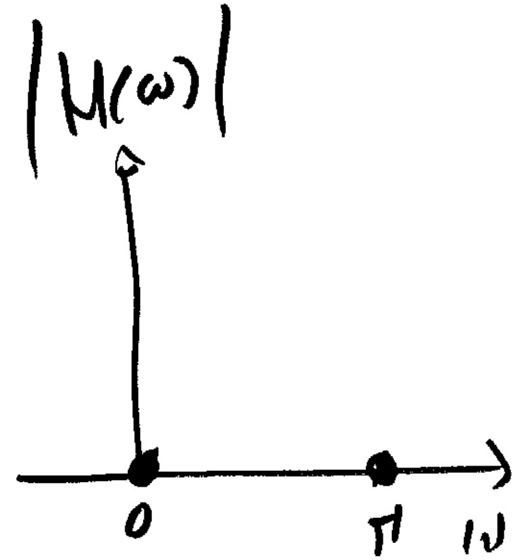
$N = 4$



Type III. Anti-Symmetry $h(n) = -h(N-1-n)$

N odd. $N = 5$





Can show $H(0) = H(\pi) = 0$

Cannot be low pass

Cannot be high pass

Type IV antisymmetry ~~&~~ $N =$ even. $\rightarrow \beta = N/2$

$N = 4$

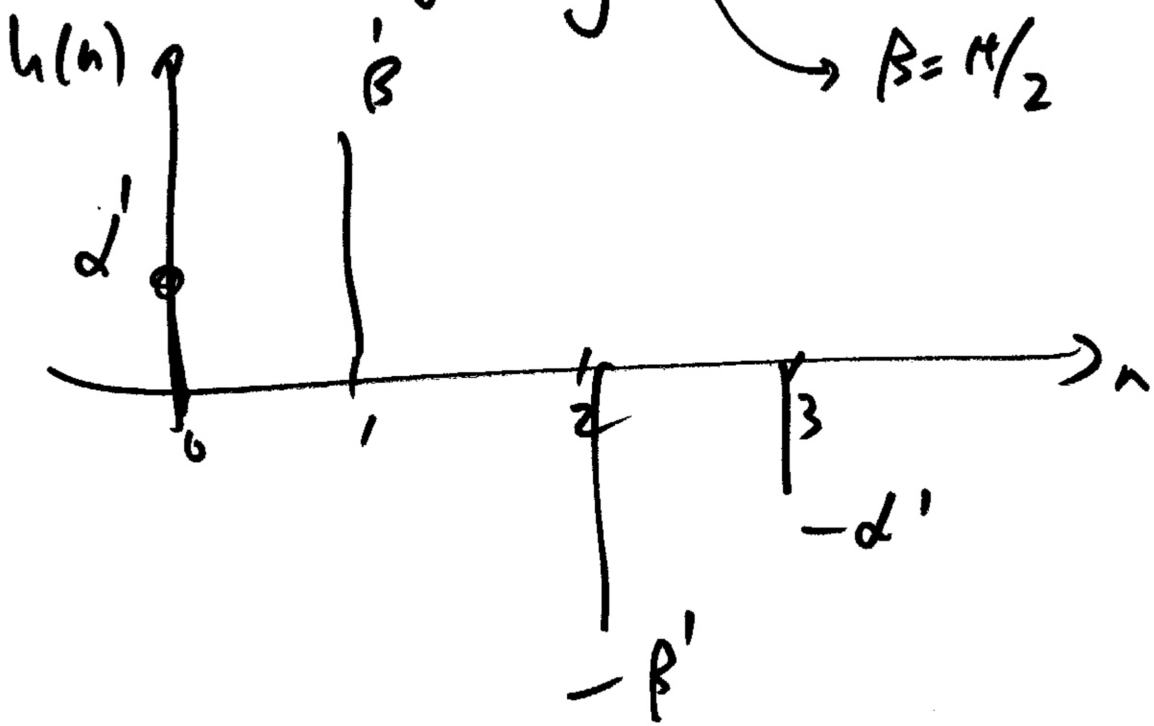
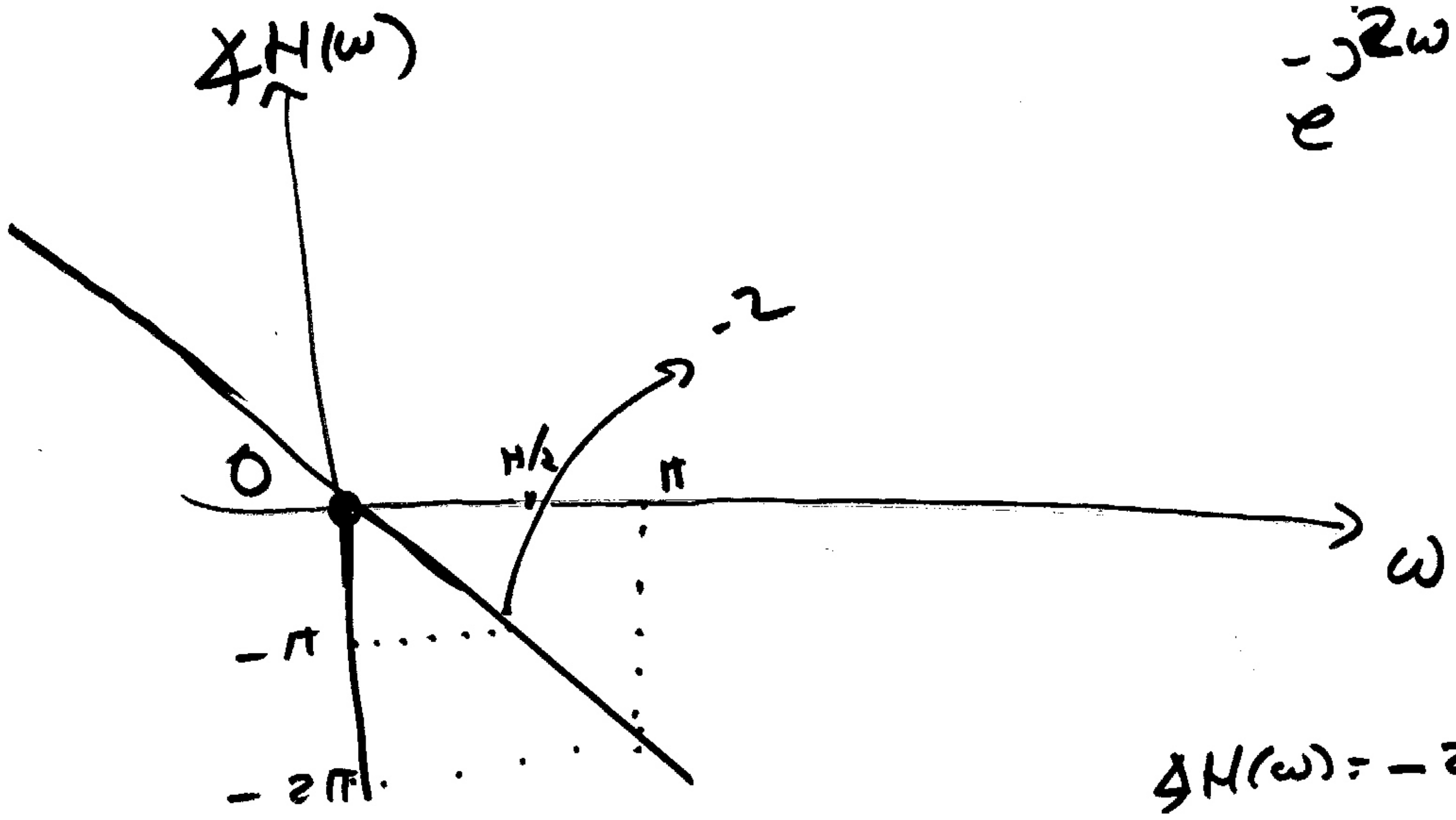


Fig 5.36, 5.37, 5.38, 5.39, 5.40

O&S



$e^{-j2\omega}$

$H_m(\omega)$

$\measuredangle H(\omega) = -2\omega$

$\omega = \pi/2$

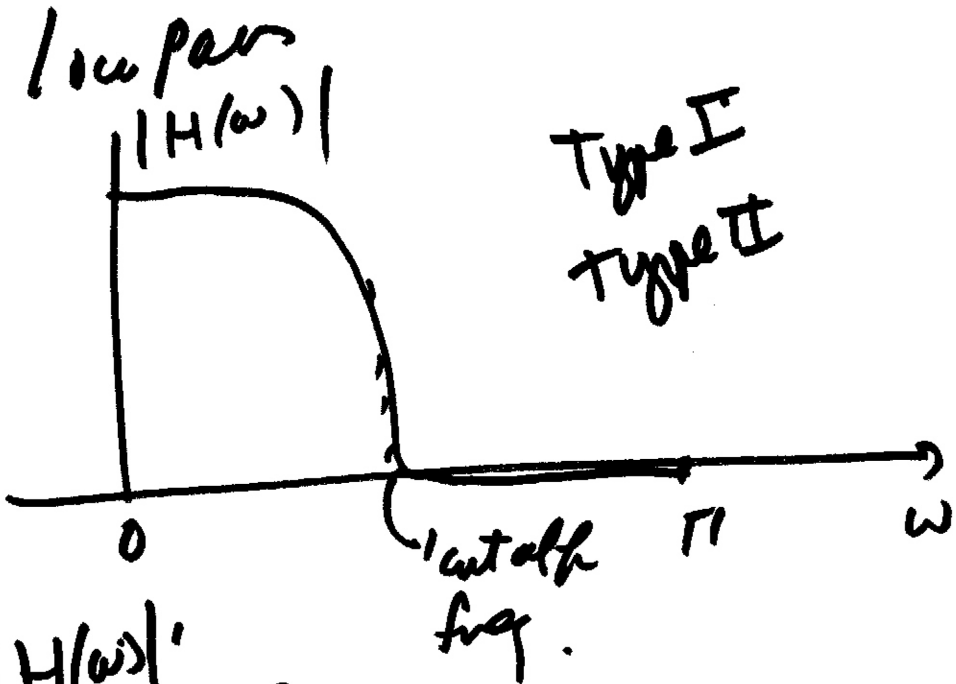
$\measuredangle H(\omega) = -\pi$

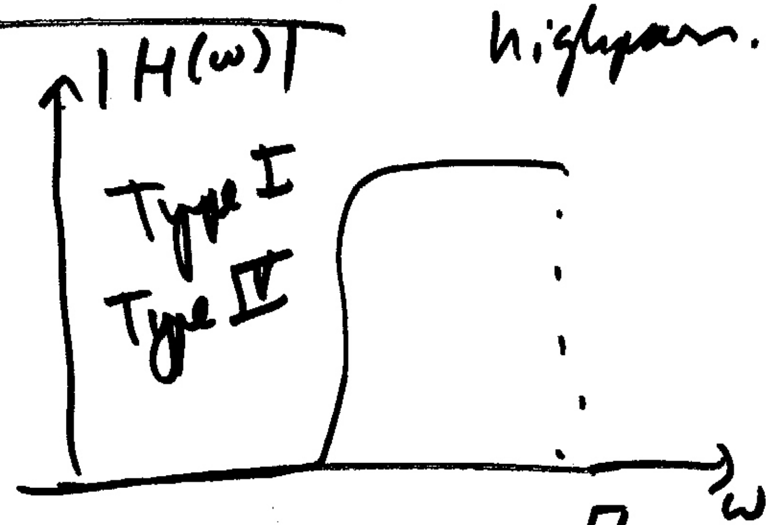
$\omega = \pi$

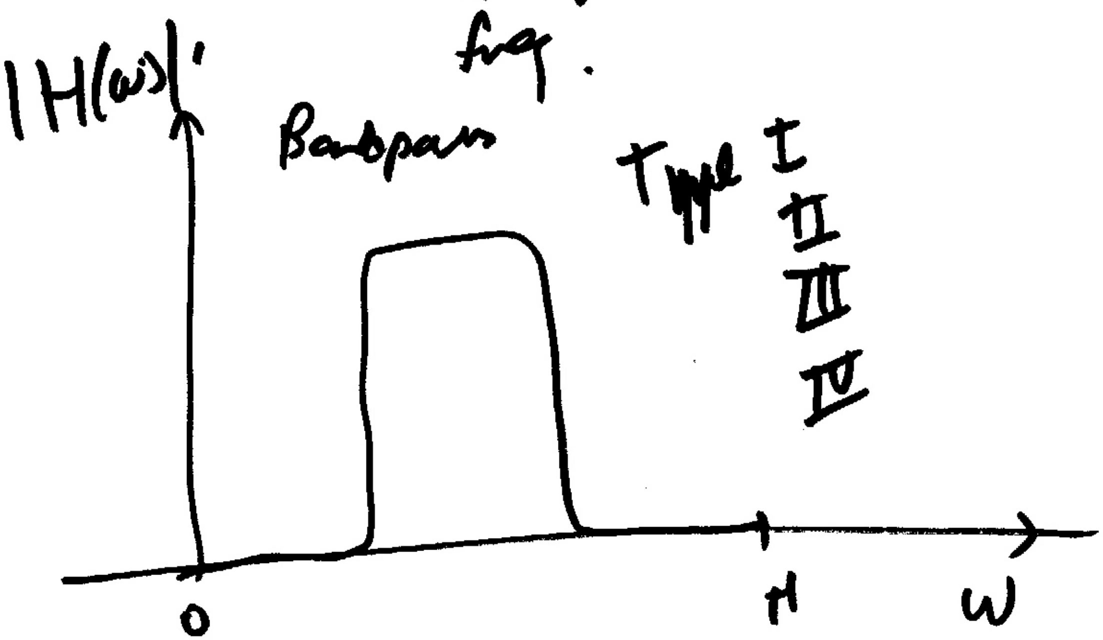
5.38

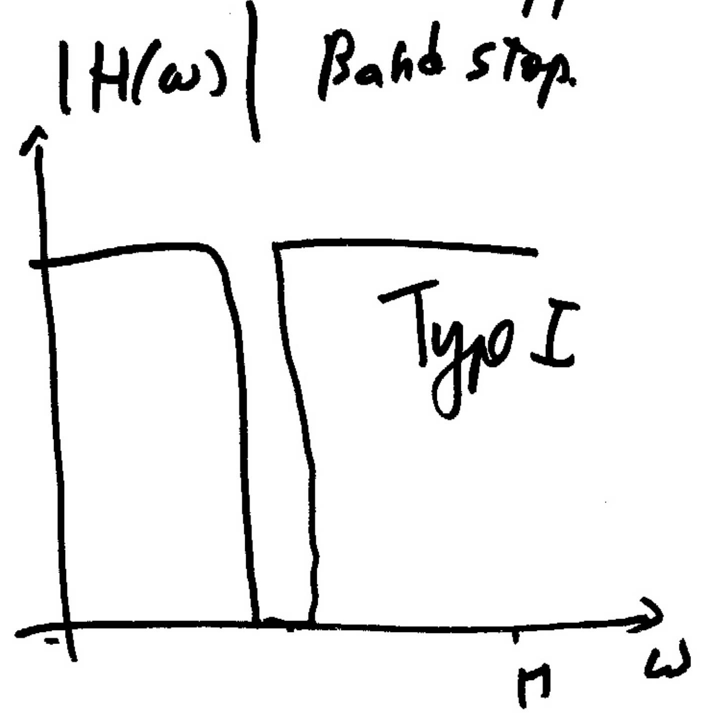
	Symmetry or antisymmetry	N even or odd	α	β	$H_r(\omega)$	Constraints $H_r(\omega)\, e^{j\beta}$
type I	$h(n) = h(N-1-n)$	odd	$\frac{N-1}{2}$	0	$\sum_{0}^{(N-1)/2} a(n) \cos \omega n$	Real
Type II	$h(n) = h(N-1-n)$	even	$\frac{N-1}{2}$	0	$\sum_{n=1}^{N/2} b(n) \cos \omega(n-\frac{1}{2})$	Real $H(\pi)=0$
type III	$h(n) = -h(N-1-n)$	odd	$\frac{N-1}{2}$	$\pi/2$	$\sum_{n=1}^{\frac{N-1}{2}} c(n) \sin \omega n$	purely Imaginary. $H(0)=0$ $H(\pi)=0$
Type IV	$h(n) = -h(N-1-n)$	even	$\frac{N-1}{2}$	$\pi/2$	$\sum_{n=1}^{N/2} d(n) \sin \omega(n-\frac{1}{2})$	purely imaginary. $H(0)=0$

4 Classes of Filters









	Low pass	high pass	Band pass	Band stop.
I	✓	✓	✓	✓
II	✓	×	✓	×
III	×	×	✓	×
IV	×	✓	✓	×

Filter Design

LTI

FIR: Determining coeffs of $h(n)$

IIR

Rational Transfer fn $H(z) = \frac{P(z)}{Q(z)}$

Determining coeffs $Q(z)$ and $P(z)$

implement using D.E.

not a Rational Transfer fn.

3 steps of building filters

1. Specification → Application dependent
2. Design → Determing coeff.
3. Realization → Direct form 1, 2, Cascade, Parallel.
4. Implementation
   - → C program on PC.
   - → matlab prog.
   - → Program DSP chip.
   - → Xilix FPGA.
   - → Programable ASIC
   - → ASIC

FIR

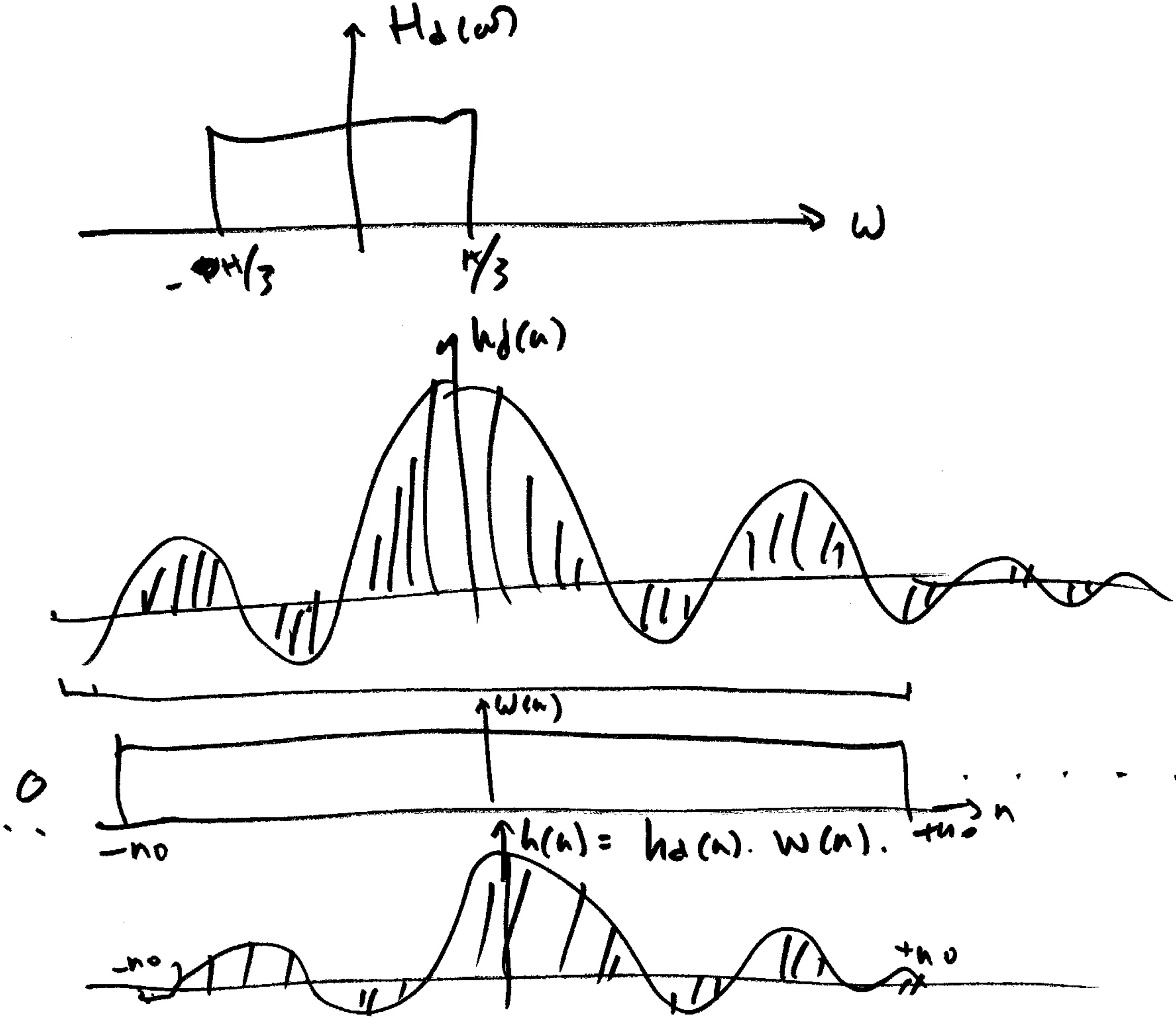
Windows. OPTimum.

FIR Filter Design Using Windows.

(1) start with desired freq. Response $H_d(\omega)$

(2) Compute IDTFT $\{H_d(\omega)\} = h_d(n) =$ = Desired impulse response.

(3) $h(n) = h_d(n)\, W(n)$ ← finite length window function.

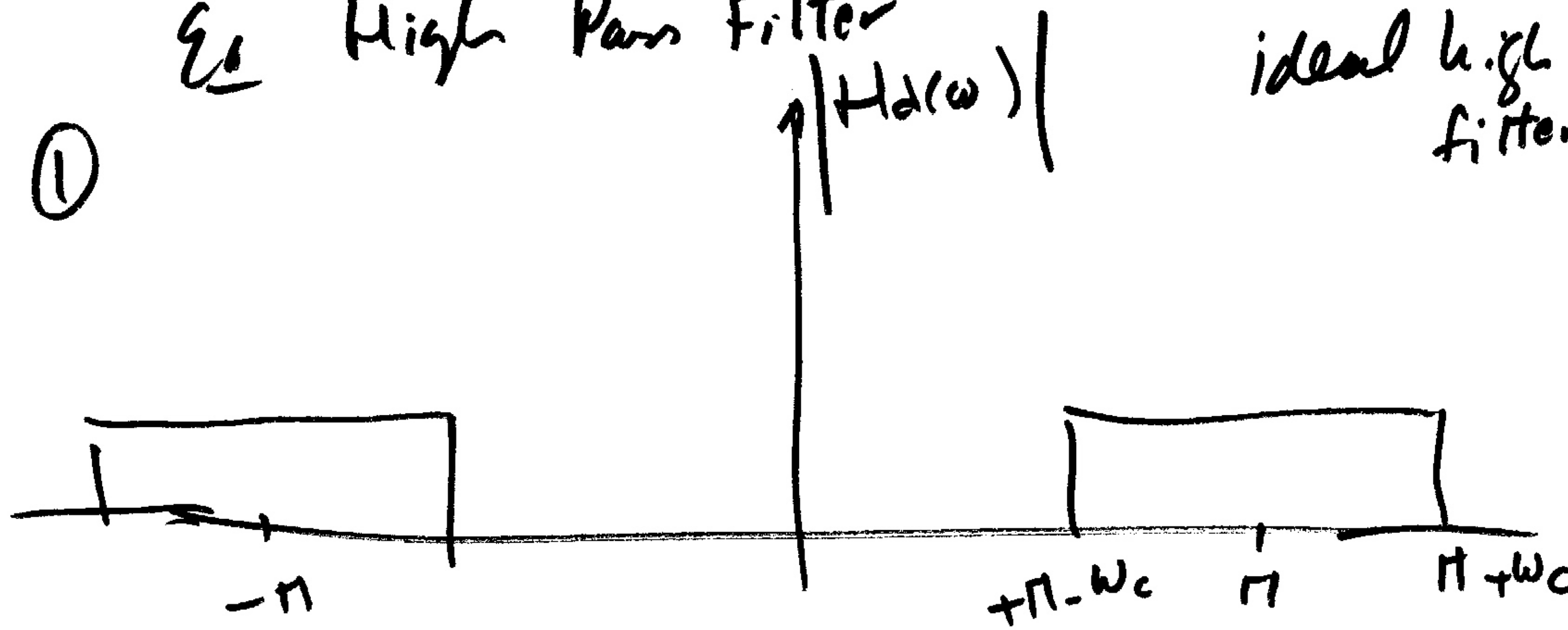


$H_d(\omega)$
ω
$-\pi/3$
$\pi/3$
$h_d(n)$
$w(n)$
n
$-n_0$
$+n_0$
$h(n) = h_d(n) \cdot w(n)$.
$-n_0$
$+n_0$

Ex High Pass Filter

ideal high pass filter.

(1)



(2) Compute I.D.T.F.T. $\{H_d(\omega)\}$ (desired/ideal, final FIR)

Assume linear Phase filter.

Assume Type I. $\left.\begin{array}{l}\beta = 0 \\ \alpha = \frac{N-1}{2} \\ \text{\# of taps is odd}\end{array}\right\} \rightarrow H_d(\omega) = \underbrace{H_n(\omega)}_{\text{Real}} e^{-j\alpha\omega}$

$$|H_d(\omega)| = \begin{cases} 1 & \pi - \omega_c < \omega < \pi + \omega_c \\ 0 & \text{otherwise.} \end{cases}$$

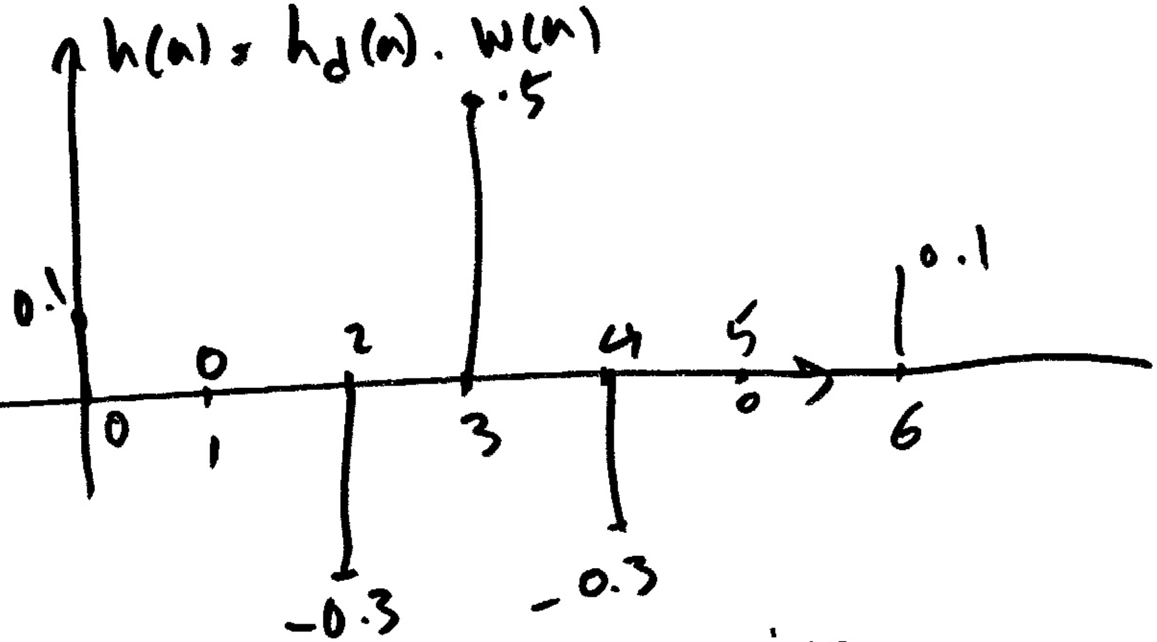
$$H_d(\omega) = \begin{cases} e^{-j\alpha\omega} & \pi - \omega_c \leq \omega < \pi \\ 0 & \text{otherwise.} \end{cases}$$

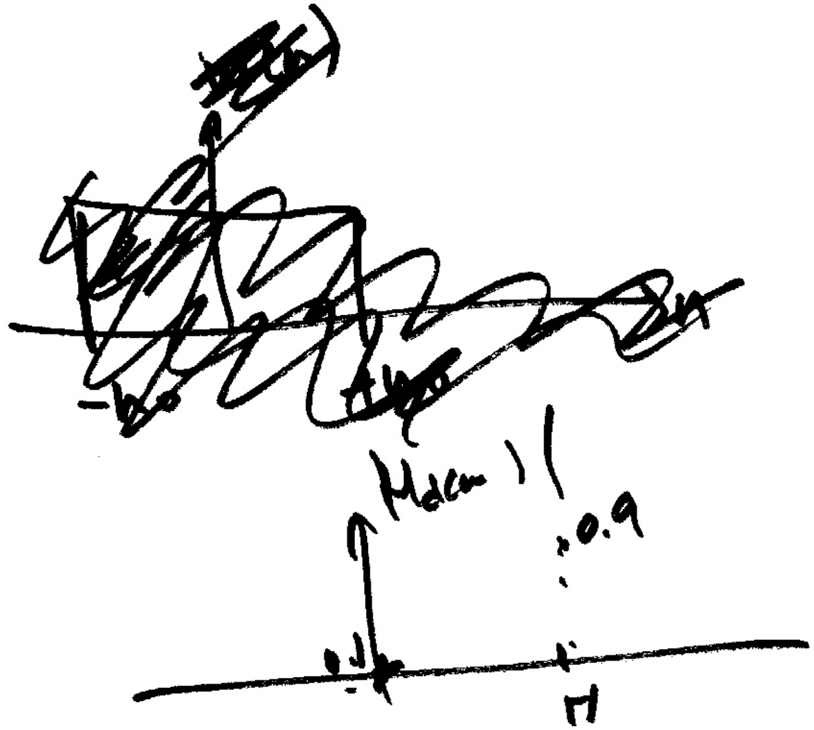
$$\text{I.DT FT}\left\{ H_d(\omega) \right\} = \frac{1}{2\pi} \int_{\pi-\omega_c}^{\pi+\omega_c} e^{-j\alpha\omega} e^{j\omega n} \, d\omega$$

$$h_d(n) = \frac{(-1)^{n-\alpha}}{\pi(n-\alpha)} \sin\left(\omega_c (n-\alpha)\right)$$

(3) Multiply $h_d(n)$ by a finite length window to get FIR filter

N = 7
Rectangular Window





$H(\omega) = \sum h(n) e^{-j\omega n}$

$[H(\omega)]_{\omega=0} = \sum h(n) = 0.1$

$\Rightarrow$ D.C. value of $H(\omega)$ in $\sum h(n)$

$[H(\omega)]_{\omega=\pi} = \sum h(n) e^{-j\pi n}$

$\sum_{n=0}^{N-1} h(n)(-1)^n$

$= 0.1 - 0.3 - 0.5 - 0.3 + 0.1$

$= -0.9$