Filter Design

LTI

FIR: Determining coeffs of $h(n)$

IIR

Rational Transfer fn
$$H(z) = \frac{P(z)}{Q(z)}$$

Determining coeffs $Q(z)$ and $P(z)$

implement using D.E.

not a Rational Transfer fn.

3 steps of building filters

1. Specification $\longrightarrow$ Application dependent
2. Design $\longrightarrow$ Determine coeff.
3. Realization $\longrightarrow$ Direct form 1, 2, Cascade, Parallel.
4. Implementation
   - C program on PC.
   - matlab prog.
   - Program DSP chip.
   - Xilinx FPGA.
   - Programable ASIC
   - ASIC

FIR

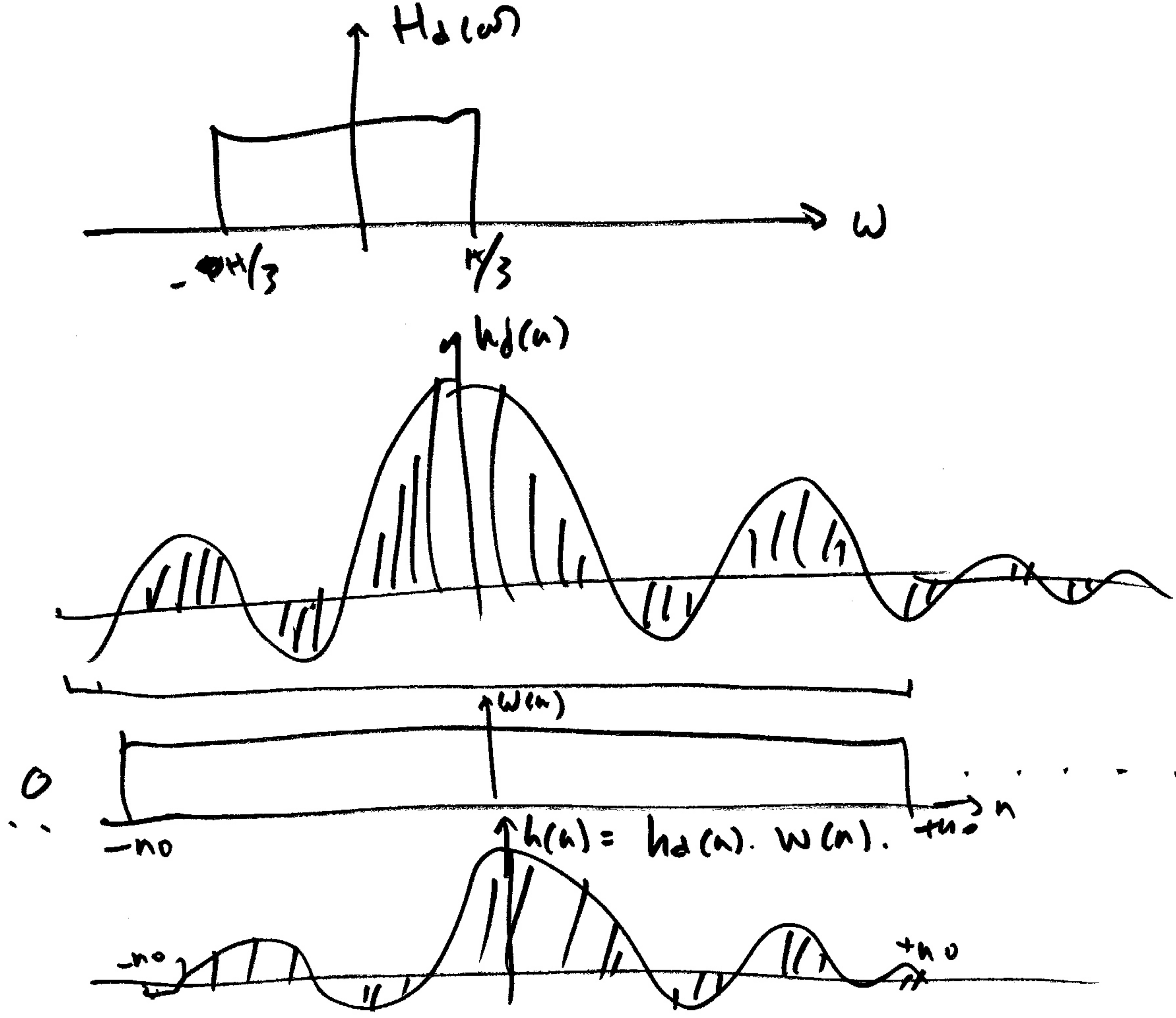
Windows. Optimum.

FIR Filter Design Using Windows.

① start with desired freq. Response $H_d(\omega)$

② Compute IDTFT $\{H_d(\omega)\} = h_d(n)$ = = Desired impulse response.

③ $h(n) = h_d(n)\, W(n)$ finite length window function.

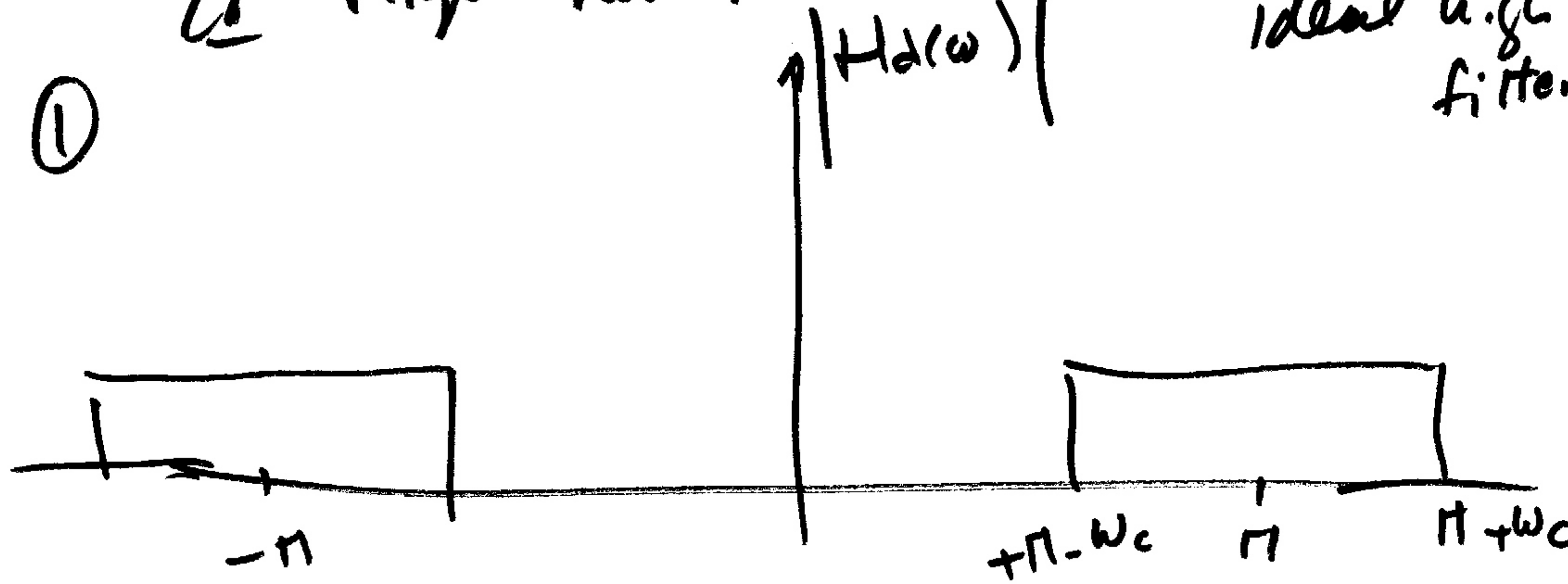


$H_d(\omega)$
ω
$-\pi/3$
$\pi/3$
$h_d(n)$
$w(n)$
n
$-n_0$
$+n_0$
$h(n) = h_d(n) \cdot w(n)$.
$-n_0$
$+n_0$

Ex High Pass Filter

ideal high pass filter.

(1)



(2) Compute I.D.T.F.T. $\{H_d(\omega)\}$

Assume linear Phase filter. (desired/ideal, final FIR)

Assume Type I.

$$\left.\begin{array}{l}\beta = 0 \\ \alpha = \dfrac{N-1}{2} \\ \#\text{ of taps is odd}\end{array}\right\} \rightarrow H_d(\omega) = \underbrace{H_r(\omega)}_{\text{Real}} e^{-j\alpha\omega}$$

$$|H_d(\omega)| = \begin{cases} 1 & \pi - \omega_c < \omega < \pi + \omega_c \\ 0 & \text{otherwise.} \end{cases}$$

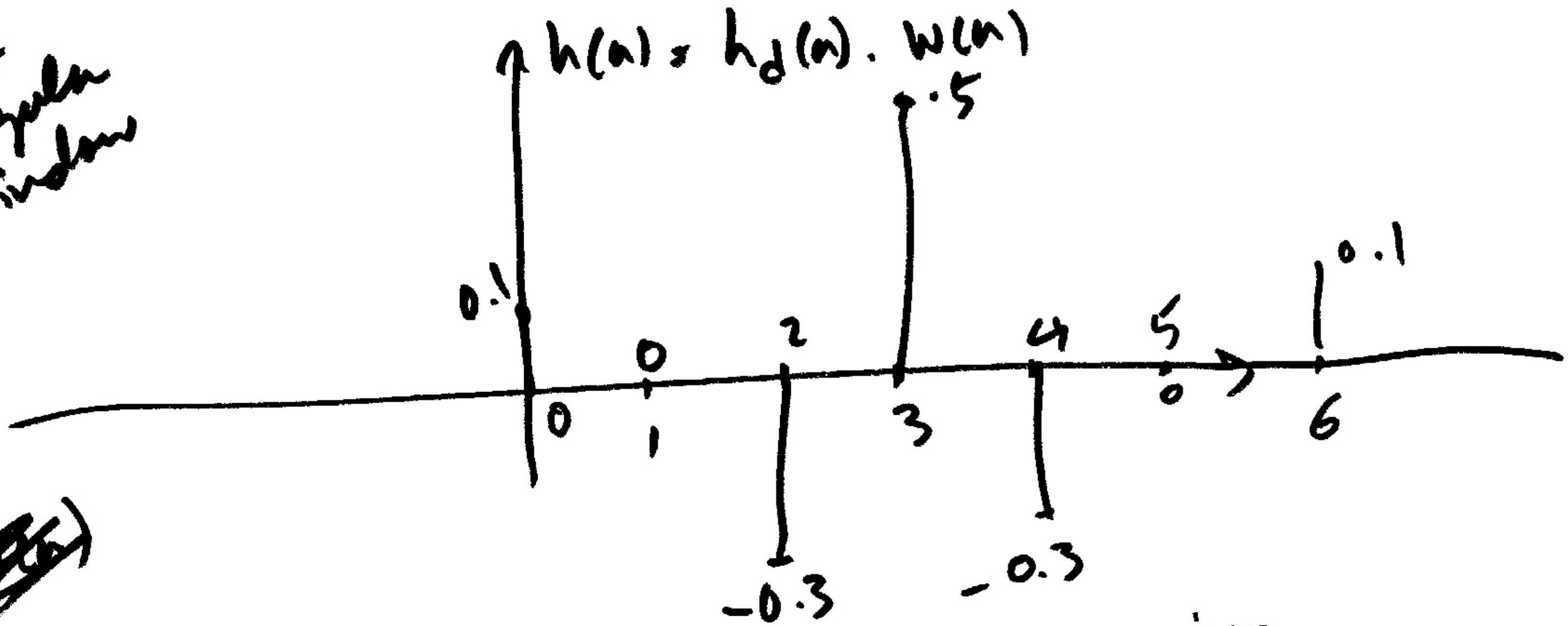
$$H_d(\omega) = \begin{cases} e^{-j\alpha\omega} & \pi - \omega_c \leq \omega < \pi \\ 0 & \text{otherwise.} \end{cases}$$

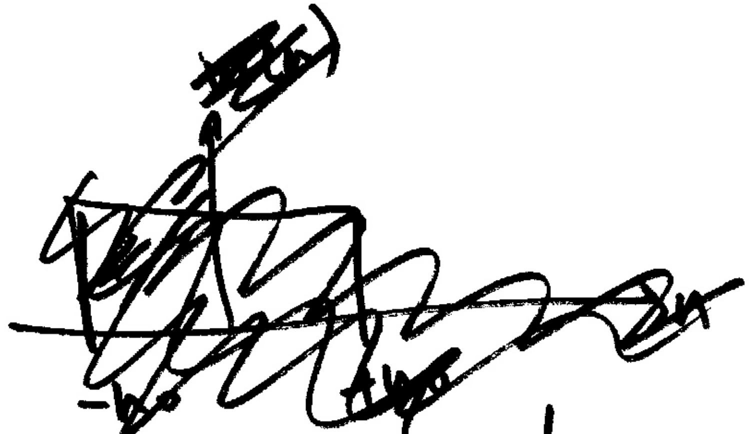
$$\text{I.DTFT}\{H_d(\omega)\} = \frac{1}{2\pi} \int_{\pi-\omega_c}^{\pi+\omega_c} e^{-j\alpha\omega} e^{j\omega n} \, d\omega$$

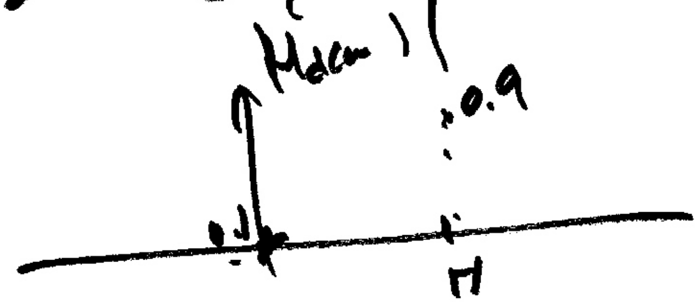
$$h_d(n) = \frac{(-1)^{n-\alpha}}{\pi(n-\alpha)} \sin\left(\omega_c (n-\alpha)\right)$$

(3) Multiply $h_d(n)$ by a finite length window to get FIR filter

$N = 7$
Rectangular Window







$H(w) = \sum h(n) e^{-jwn}$

$[H(w)]_{w=0} = \sum h(n) = 0.1$

$\Rightarrow$ D.C. value of $H(w)$ in $\sum h(n)$

$[H(w)]_{w=\pi} = \sum h(n) e^{-j\pi n}$

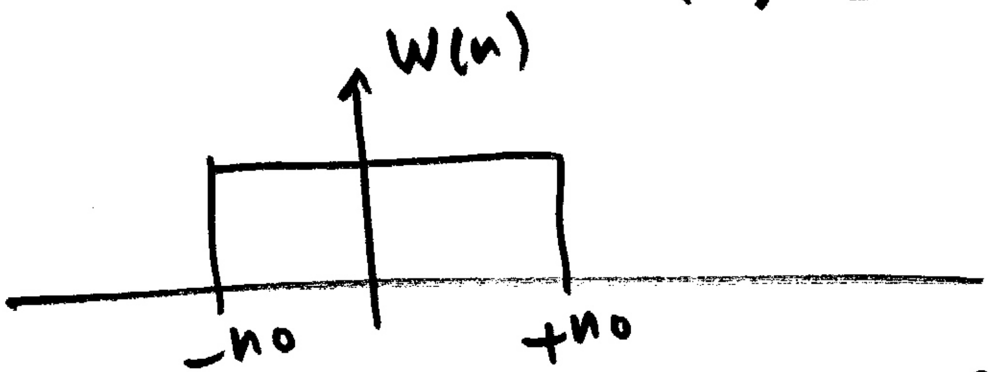
$= \sum_{n=0}^{N-1} h(n)(-1)^n$

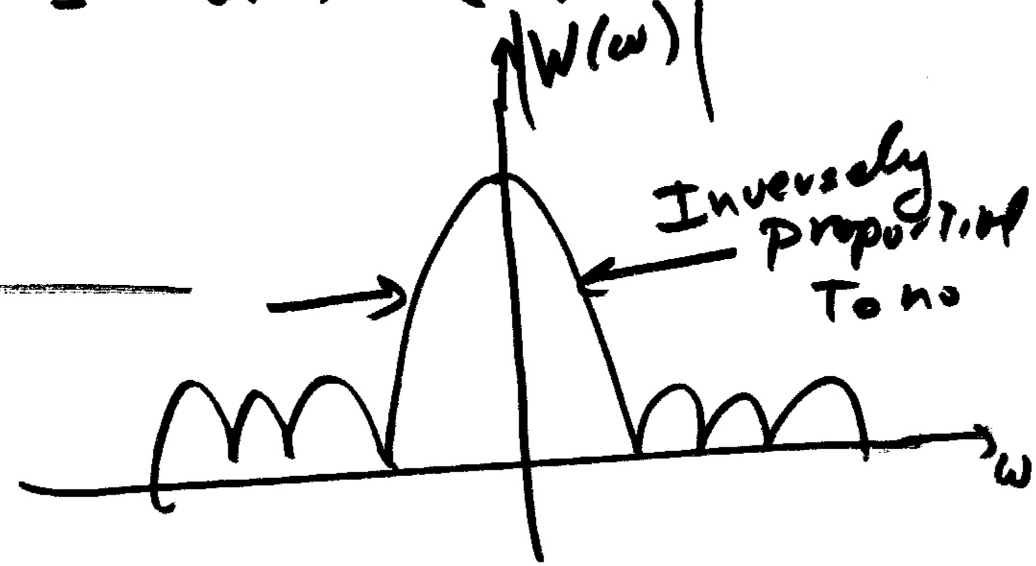
$= 0.1 - 0.3 - 0.5 - 0.3 + 0.1$

$= -0.9$

$$h(n) = h_d(n)\, w(n) \leftarrow \text{window}$$

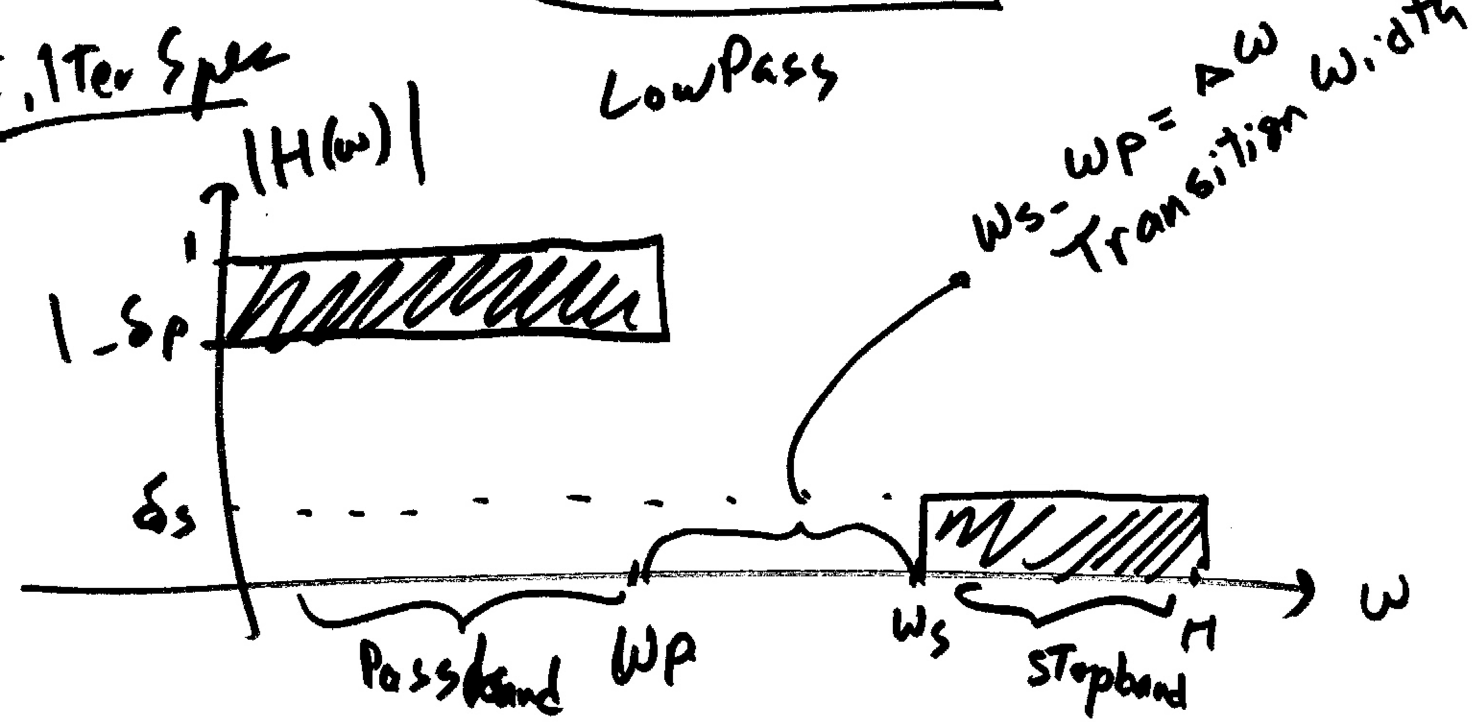
$$H(w) = H_d(\omega) * W(\omega)$$





Terminology.

Filter Spec LowPass



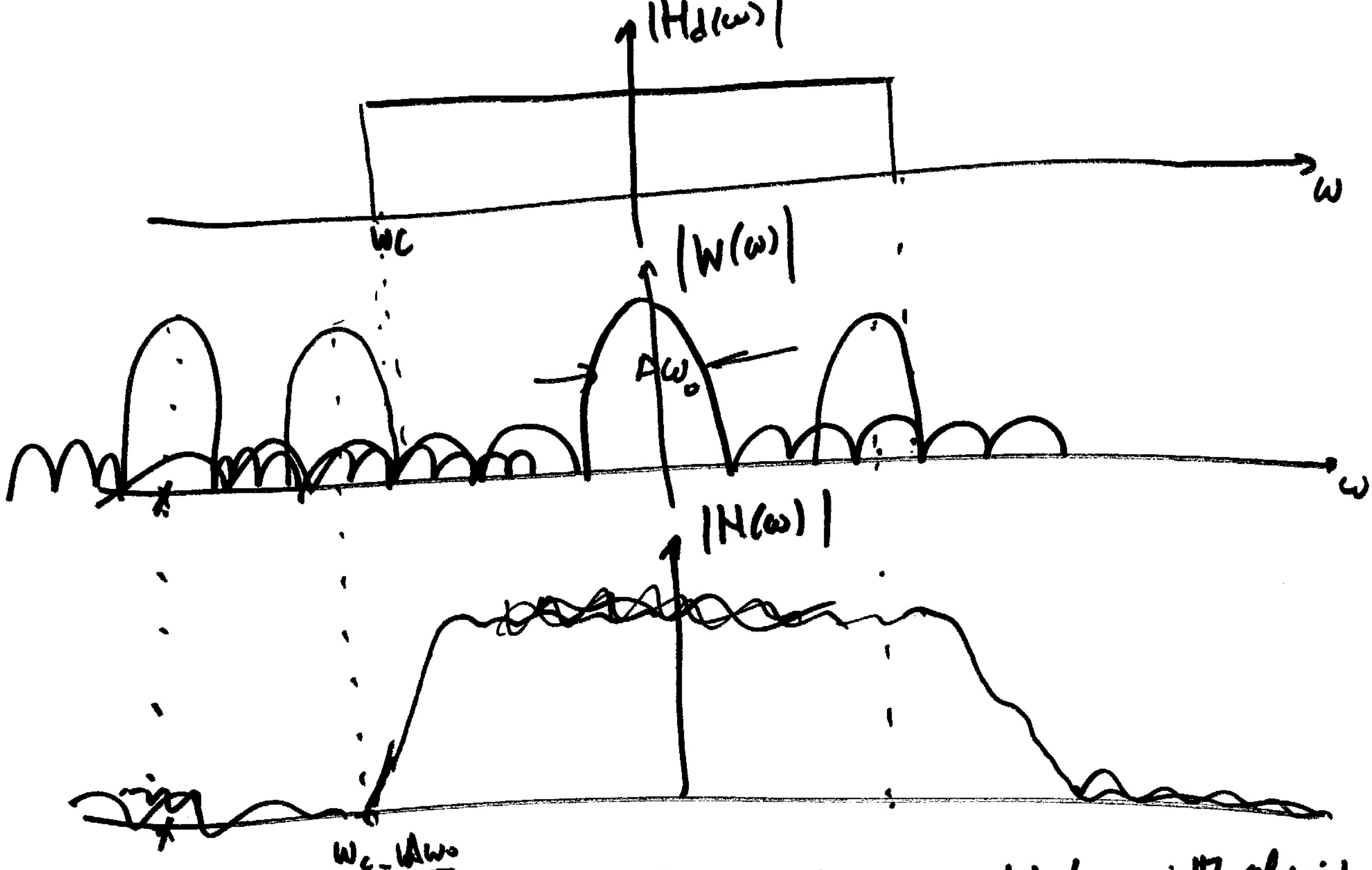
$0 \leq \omega \leq \omega_p$ passband.

$\omega_s \leq \omega \leq \pi$ stopband.

δ_p = passband ripple

δ_s = stopband ripple.

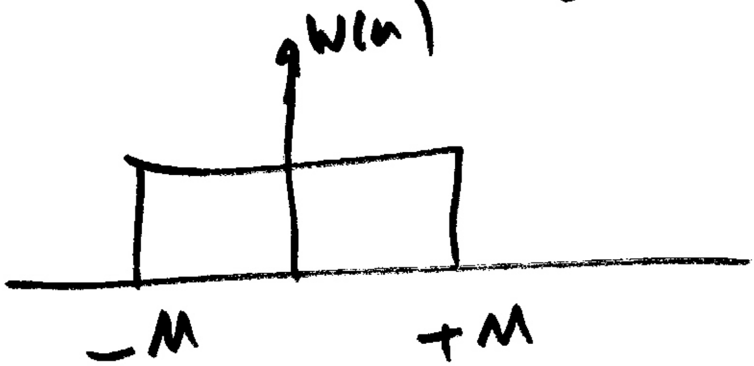
$\Delta\omega = \omega_s - \omega_p$ = Transition width

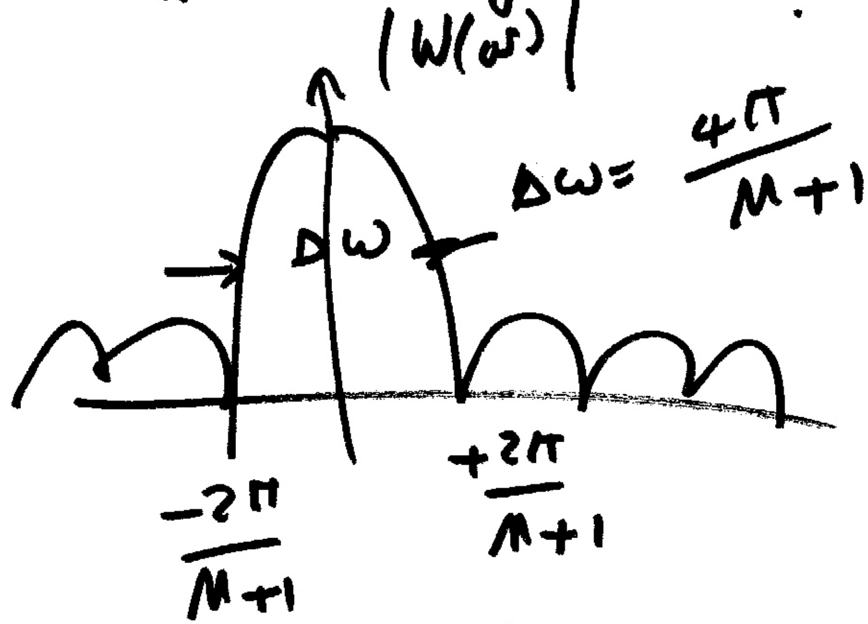


① Transition width of $H(\omega)$ depends on mainlobe width of window.
② Ripple in $H(\omega)$ depends on ripple of window fn.

Q How To design window $w(n)$ To get good mainlobe / sidelobe behavior $H(\omega)$?
δ_p small δ_s small $\omega_s - \omega_p$ small

Q1 How To control Transition Width of $H(\omega)$ i.e. Mainlobe of $W(\omega)$?





(a) * Longer Windows in Time domain have narrower mainlobe width in freq Domain $\Longrightarrow$ small Transition width in final FIR filter.

(b) Shape of Window:

Fixed size (duration) window.
but different shape. have different.
main-lobe width

Fig 7.22 & 25

Rectangular $\longrightarrow$ smallest mainlobe width.

Blackman $\longrightarrow$ highest mainlobe width.

Q2 How to design $w(n)$ to get good side lobe behavior for $w(n)$ i.e.. a good ripple behavior for my final FIR filter.

- Shape of $w(n)$ controls sidelobe behavior.
- But size of $w(n)$ (duration) does not significantly affect sidelobe behavior

shape $\to$ sidelobe and mainlobe $\quad w(n)$.

size $\xrightarrow{\text{only}}$ mainlobe of $w(n)$

Strategy

(1) use shape to control sidelobe behaviour of $W(\omega)$

(2) use size to control mainlobe behaviour

FIR Filter Design using Windows

$N = M+1$
\# of Taps.
$2\alpha + 1 = N$
$\alpha = \frac{N-1}{2}$

Kaiser Window:

$$W(n) = \begin{cases} I_0\left[\underbrace{\beta'\left(1 - \left(\frac{n-\alpha}{\alpha}\right)^2\right)^{\frac{1}{2}}}_{x} \right] & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

I_0 = zeroth order modified Bessel function.

$$I_0(x) = 1 + \frac{x^2}{2^2 (1!)^2} + \frac{x^4}{2^4 (2!)^2} + \frac{x^6}{2^6 (3!)^2} + \ldots$$

Soln to the following diff eqn:

$$\frac{x^2\, dy^2}{dx^2} + x \frac{dy}{dx} - (x^2 + n^2)\, y = 0$$

nth order modified Bessel fn.

β' controls the shape of Kaiser window allowing Trade off between sidelobe and mainlobe.

Design Using Kaiser Window

(1) $\Delta\omega = \omega_s - \omega_p$: Transition width.

(2) ripple $= \delta \longrightarrow A = -20 \log_{10} \delta$

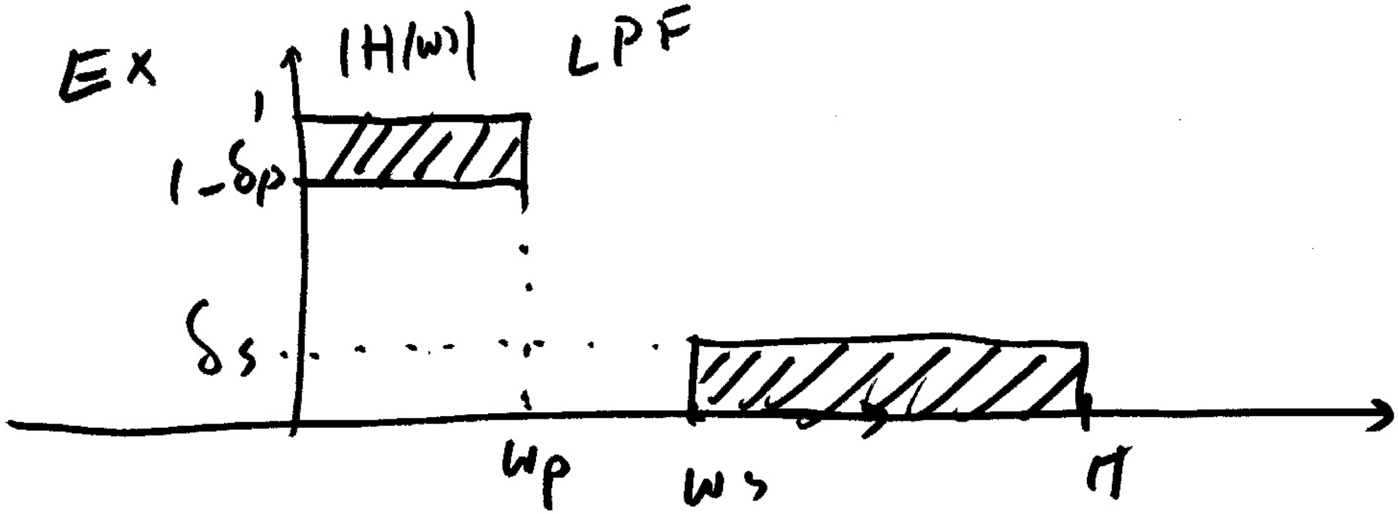
choose $\alpha = \frac{M}{2} = \frac{N-1}{2}$ and β' as follows.

(a) $M = 2\alpha = \dfrac{A - 8}{2.285\,\Delta\omega}$

(b)

$$\beta' = \begin{cases} 0.1102(A - 8.7) & A > 50 \\ 0.5842(A - 21)^{0.4} & 21 \leq A < 50 \\ 0 & A < 21 \end{cases}$$

Ex



linear phase filter.

$H_d(\omega) \longrightarrow h_d(n)$

$H(\omega) = \underbrace{H_m(\omega)}_{\text{Real}} \, e^{j(\alpha\omega - \beta)}$ ← generalized LP filter.

$\beta = 0 \longrightarrow$ Type I or Type II $\longrightarrow$ Both capable of LPF.

<u>Specification</u>:

$\omega_p = 0.4\pi$

$\omega_s = 0.6\pi$