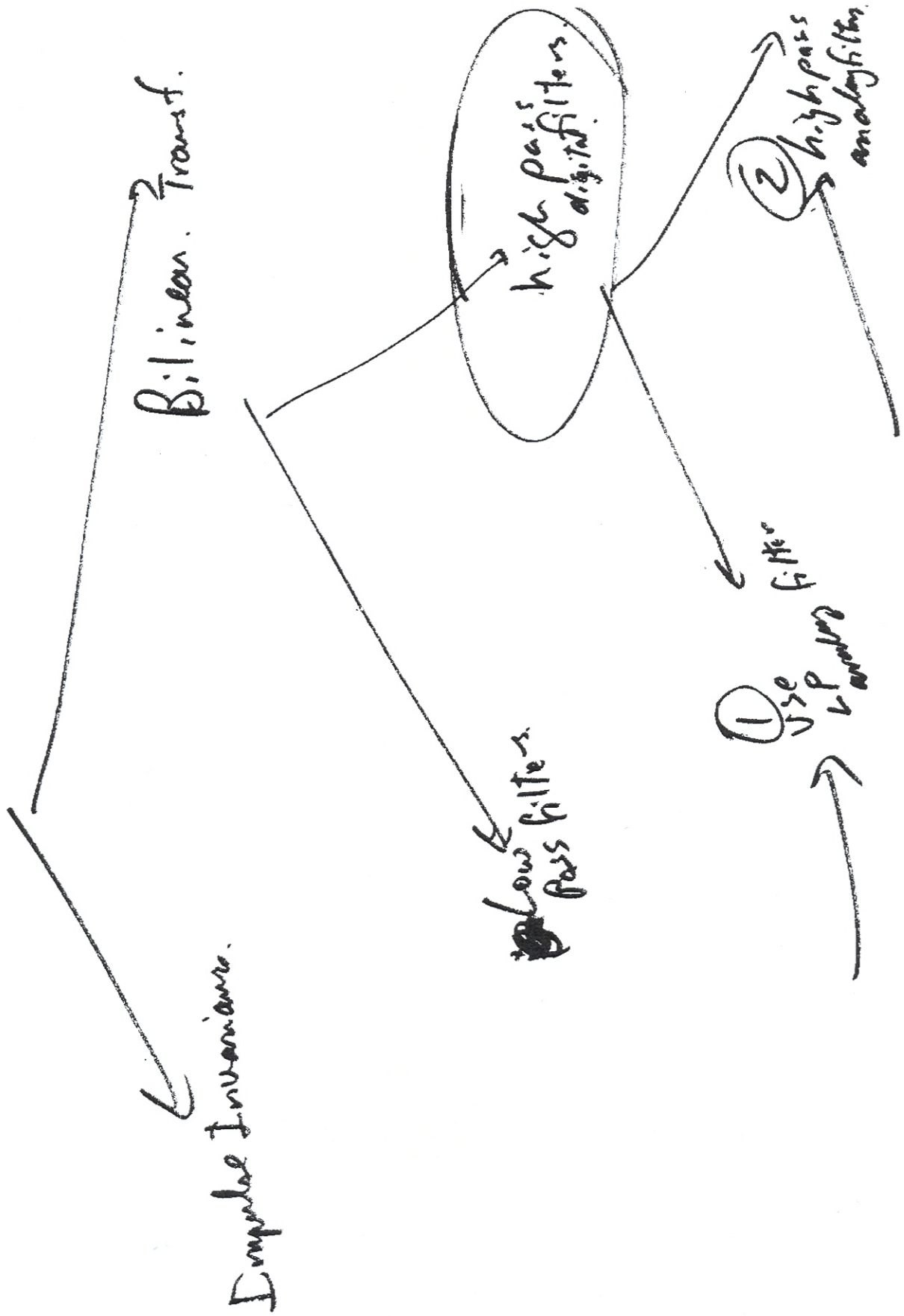


Dec. 5, 2006

IIR Filter Design Transformation



Approach 1 To design Digital high pass IIR filters



C = continuous.

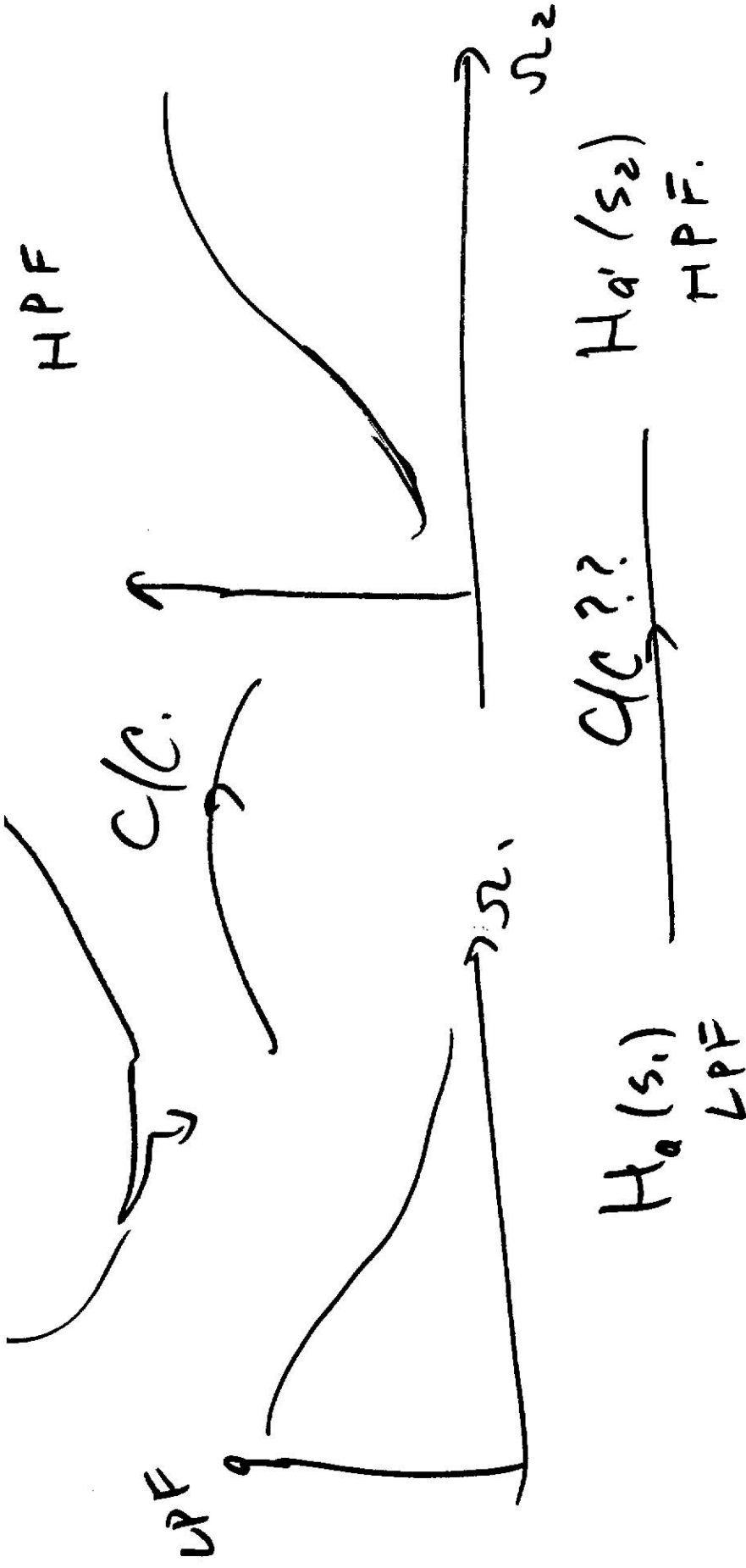
a, a' = analog.

d. = discrete = d

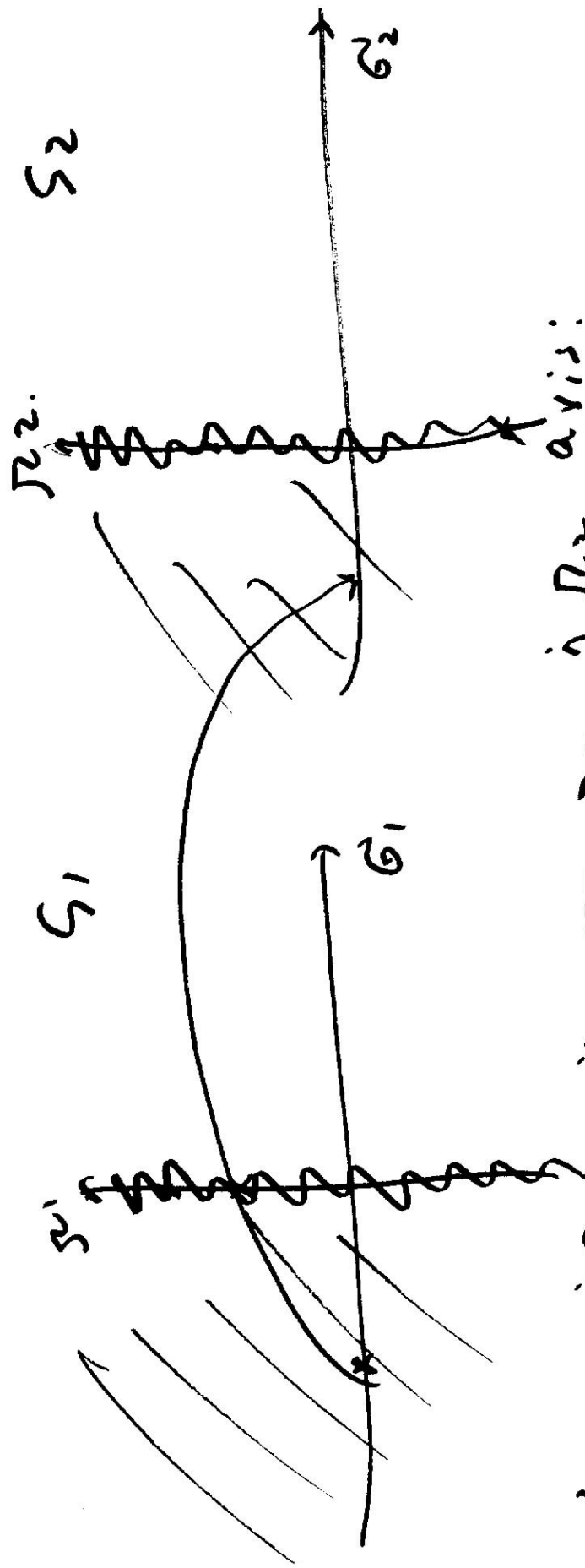
T = Time.

HPT = high pass filter.

LPP = low pass filter.



C/C.: Propose $H_0'(s_2) = [H_0(s_1)]_{s_1 = \frac{\omega_u}{s_2}}$
 $\omega_u = \text{positive constant.}$



Show $j\omega_1$ axis \rightarrow $j\omega_2$ axis:

Let $s_1 = j\omega_1 = \frac{\omega_1}{s_2} \Rightarrow s_2 =$

$\omega_2 = 0, \quad \omega_2 = -\frac{\omega_1}{s_1}$

Yes: $j\omega_1 \rightarrow j\omega_2$.

Show: LHP plane maps onto LHP plane in s_2 ? Yes

Contour + stable in s_1 \rightarrow Contour + stable in s_2

~~transfer~~ LH plane in s_1 \rightarrow LHP in s_2

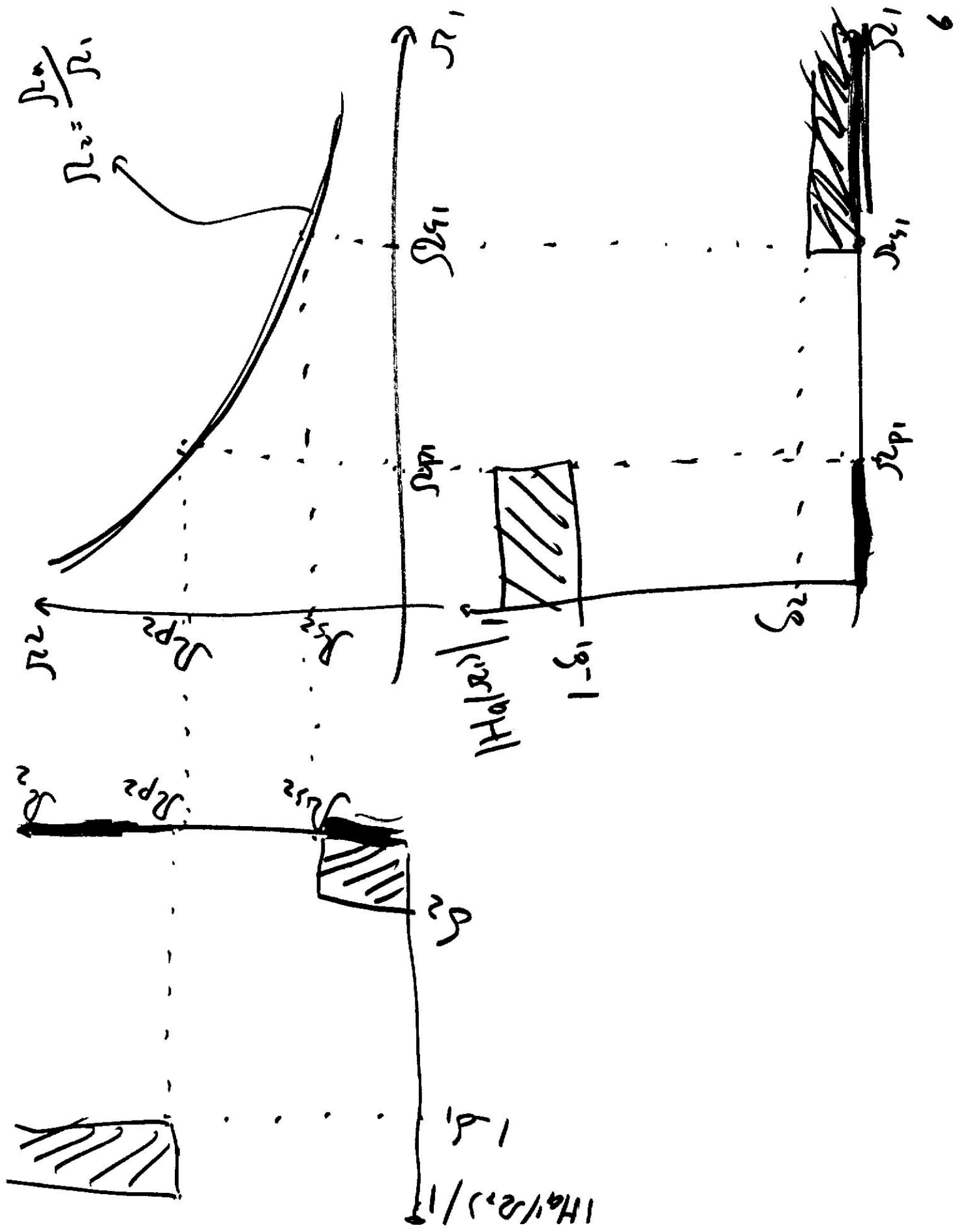
Translation Specif.

$$|H_a'(s_2)| = |H_a(s_1)| \left. \begin{array}{l} \omega_{s_1} = -\frac{s_2}{s_1} \\ \Rightarrow \end{array} \right\} \Rightarrow$$

assume $h_a(t)$ real \Rightarrow
 \Rightarrow Freq. Res. mag. is symmetric

$$|H_a'(s_2)| = |H_a(-s_1)| \left. \begin{array}{l} \omega_{s_1} = -\frac{s_2}{s_1} \\ \Rightarrow \end{array} \right\} \Rightarrow$$

$$|H_a'(s_2)| = |H_a(s_1)| \left. \begin{array}{l} \omega_{s_1} = \frac{s_2}{s_1} \\ \Rightarrow \end{array} \right\} \Rightarrow$$

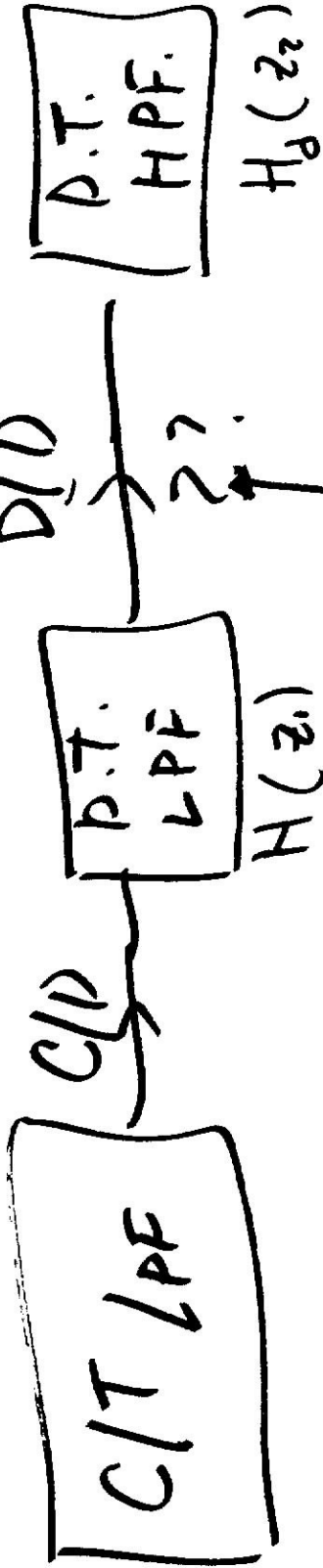


Steps To design FIR Digital filter using Approach 2.

1. Start spec in D.T. ^{domain.} for HPF = high pass filter
2. Translate specs D.T., NPF \rightarrow C.T. HPF.
3. Translate specs C.T. HPF \rightarrow C.T. LPF.
4. Design: C.T. LPF \rightarrow Bottom Chebyshev elliptic.
5. Transform C.T. LPF \rightarrow C.T. HPF.
6. Transform C.T. HPF \rightarrow D.T. HPF.

Approach #2
Design DT. IIR HPF.

Transformation



Proposed:

$$H_d(z_2) = [H(z)]_{z_1 = \frac{\alpha + z_2}{1 + \alpha z_2}}$$

$$\alpha = \text{real}, \quad |\alpha| < 1$$

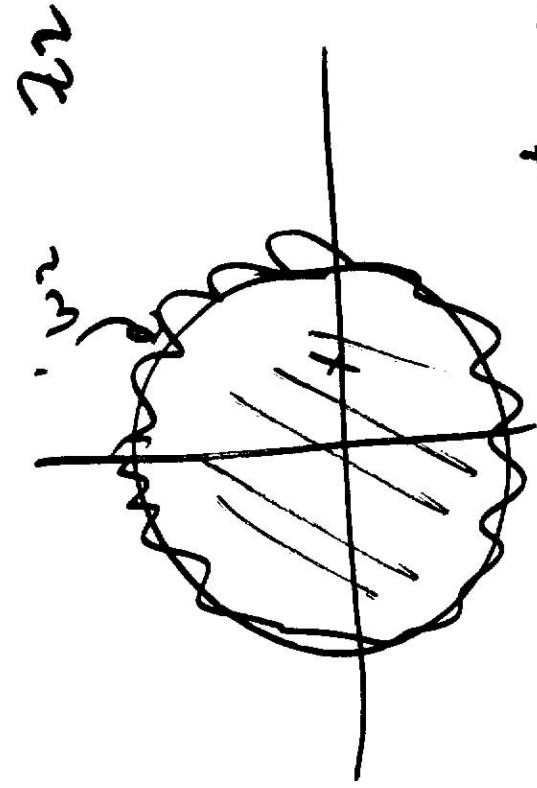
Does it map $e^{j\omega_1}$ in z_1 to $e^{j\omega_2}$ in z_2 ?

Let $z_1 = e^{j\omega_2}$ \Rightarrow $\frac{e^{j\omega_2}}{\alpha + e^{j\omega_2}} = \frac{e^{j\omega_2}(1 + \alpha e^{-j\omega_2})}{(1 + \alpha e^{j\omega_2})}$

$z_1 = \frac{\alpha + e^{j\omega_2}}{1 + \alpha e^{j\omega_2}} = \frac{e^{j\omega_2}(1 + \alpha e^{-j\omega_2})}{1 + \alpha e^{j\omega_2}}$

$|z_1| = \frac{|e^{j\omega_2}| |1 + \alpha e^{-j\omega_2}|}{|1 + \alpha e^{j\omega_2}|} = 1$

\Rightarrow unit circle in z_1 maps to unit circle in z_2 .
 Yes $e^{j\omega_2} \rightarrow e^{j\omega_1}$



inside unit circle

\Rightarrow causal + stable HPF



inside unit circle

\Rightarrow causal stable LPF

$$e^{j\omega_1} = - \frac{d + e^{j\omega_2}}{1 + \alpha e^{j\omega_2}}$$

mapping.

$$w_1 = f(w_2, \alpha)$$

show: if $w_1 = f(w_2, \alpha) \Rightarrow$
 $w_1 = f(-w_2, \alpha)$

$$|H_d(w_2)| = |H(w_1)|_{w_1 = f(w_2, \alpha)}$$

Assume H_d has real impulse response \Rightarrow

$$|H_d(w_2)| = |H_d(-w_2)|$$

$$|H_d(-w_2)| = |H_d(w_2)| = |H(w_1)|_{w_1 = f(-w_2, \alpha)}$$

Suppose $\alpha = 0$

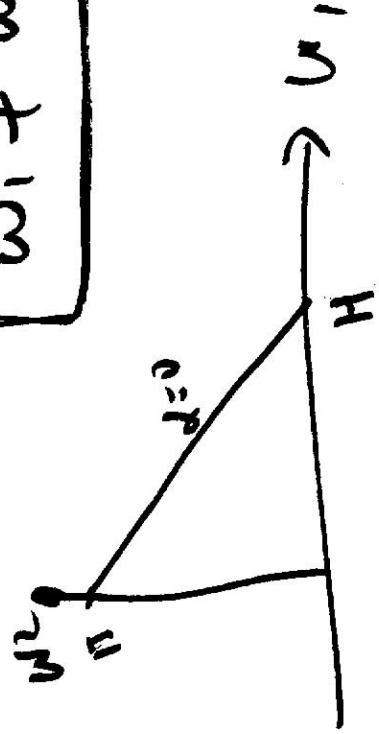
$$e^{j\omega_1} = - \frac{e^{j\omega_2}}{\alpha + e^{j\omega_2}}$$

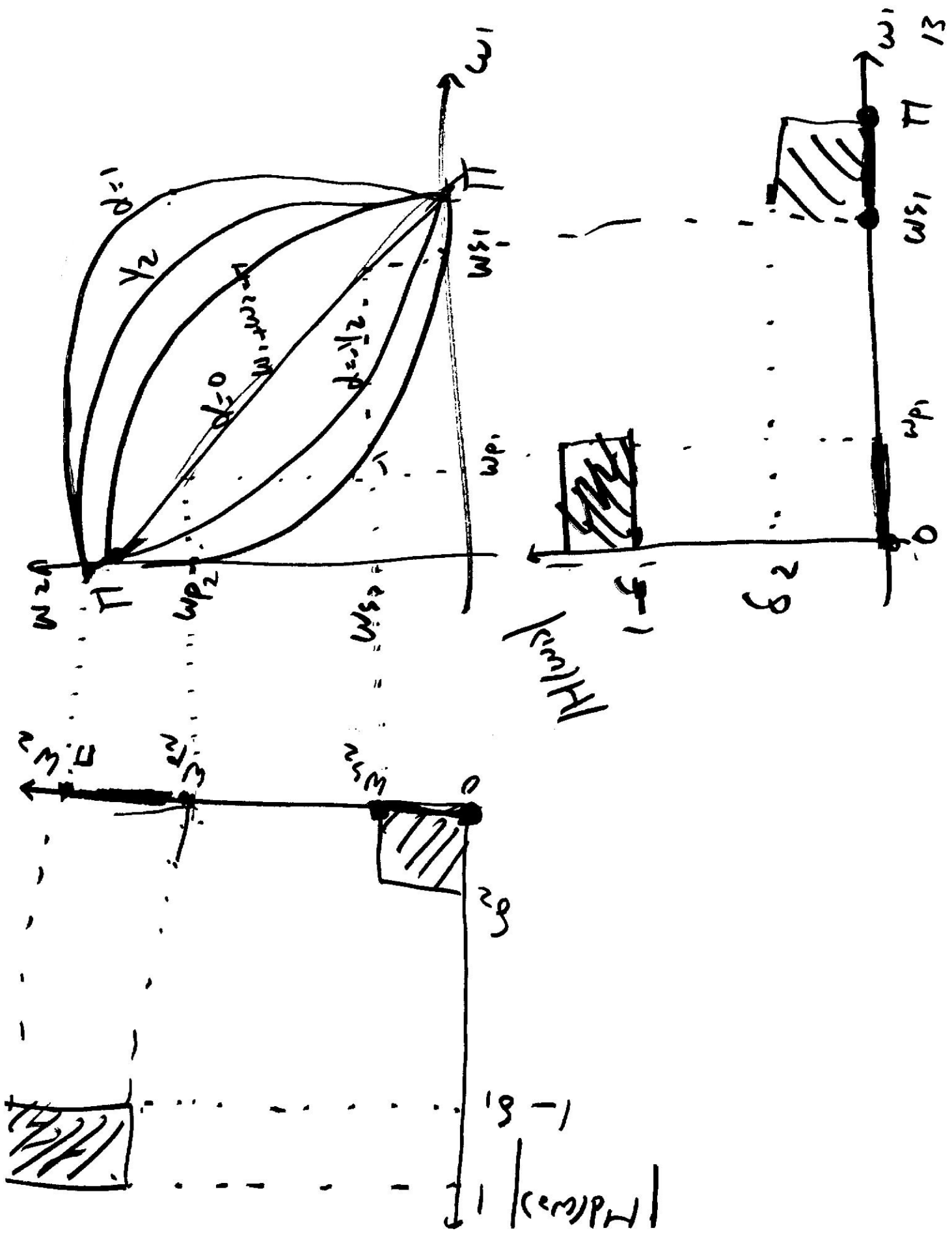
$$e^{j\omega_1} = - e^{j\omega_2} \frac{1 + \alpha e^{j\omega_2}}{1 + \alpha e^{j\omega_2}} = - e^{j\omega_2}$$

$$\omega_2 + \pi = \omega_1 = f(\omega_2, \alpha)$$

$$\Rightarrow f(-\omega_2, \alpha) = -\omega_2 + \pi = \omega_1 \Rightarrow$$

$$\boxed{\omega_1 + \omega_2 = \pi}$$





Steps for HPF, IIR, Discrete Time.
 using. Approach 2 of Bilinear Tr

1. Given specs P.T. HPF. $\xrightarrow{\text{DT LPPF}}$ DT LPPF.
 2. Translate specs DT. HPF $\xrightarrow{\text{Today's lecture}}$

3. Design P.T. LPPF.
 (a) * specs DT LPPF \rightarrow C.T. LPPF.
 (b) + Design C.T. LPPF \rightarrow DT LPPF.
 (c) + Transform C.T. LPPF \rightarrow DT LPPF.

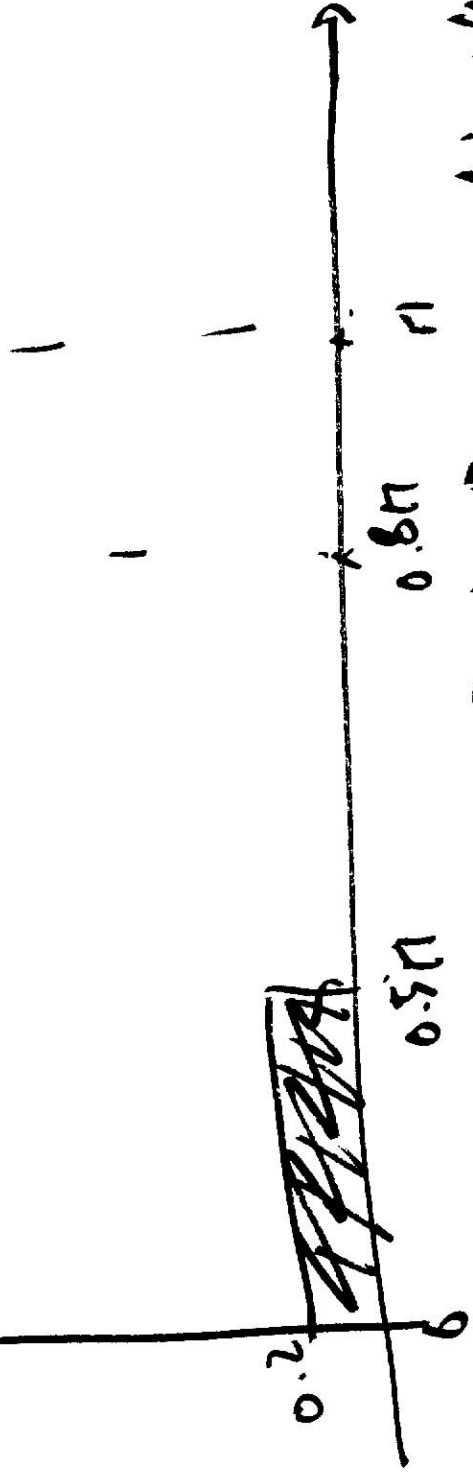
4. Transform DT LPPF $\xrightarrow{\text{P.T. HPF}}$ P.T. HPF.

$$H_d(z_2) = [H(z_1)]_{z_1 = \frac{\alpha + z_2}{1 + \alpha z_2}}$$

D.T. HPF



Ex $\frac{1}{1+0.7s}$



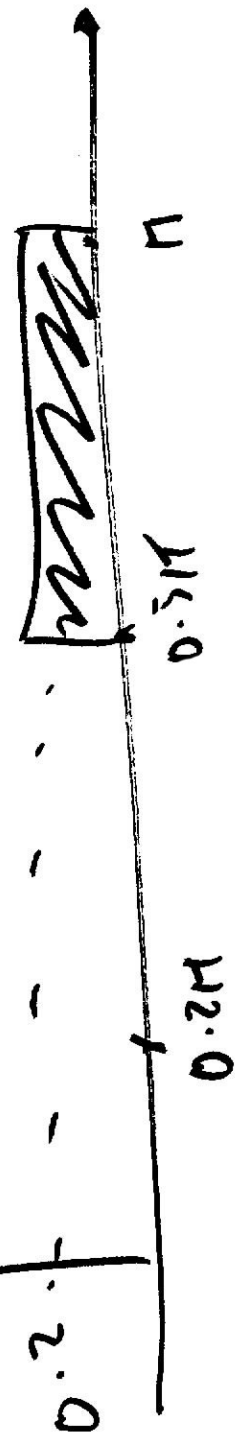
$w_1 + w_2 = 17$

DT LPF

Translate specs onto DT



Specs PT LPF.



- Design LPF, DT.

$$H(z_1) = \frac{0.2416 z_1^{-1}}{1 - 1.16 z_1^{-1} + 0.414 z_1^{-2}}$$

- Transfer LPF D.T \rightarrow HPF P.T.

$$H_f(z_2) = \frac{-0.2416 z_1^{-1}}{1 + 1.16 z_2^{-1} + 0.414 z_2^{-2}}$$