

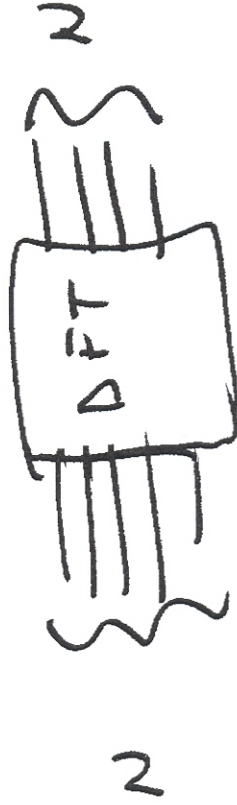
Dec. 5, 2006

DFT = Discrete Fourier Transform

- finite length N pt seq $x(n)$ N non zero values.

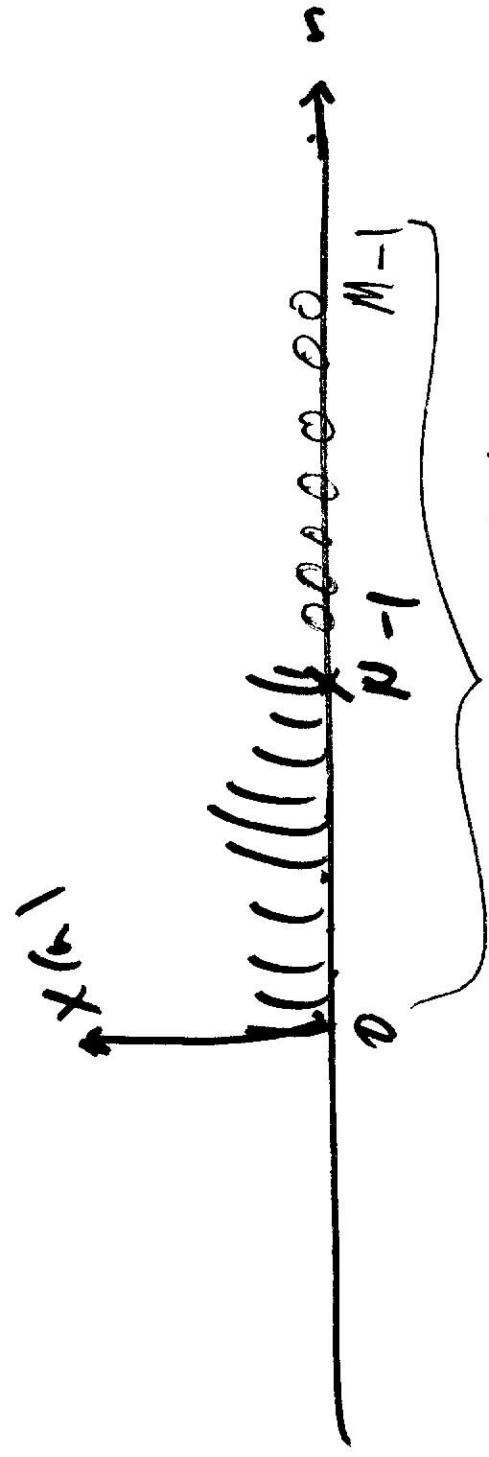
$$DFT \{x(n)\} = X(k) = \begin{cases} \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} & 0 \leq k < N \\ 0 & \text{otherwise} \end{cases}$$

otherwise.



- Possible To pad an N pt seq. with N nonzero values to M points, ie pad it with $M-N$ additional zeros

- Possible To Talk about DFT of M pt seq.

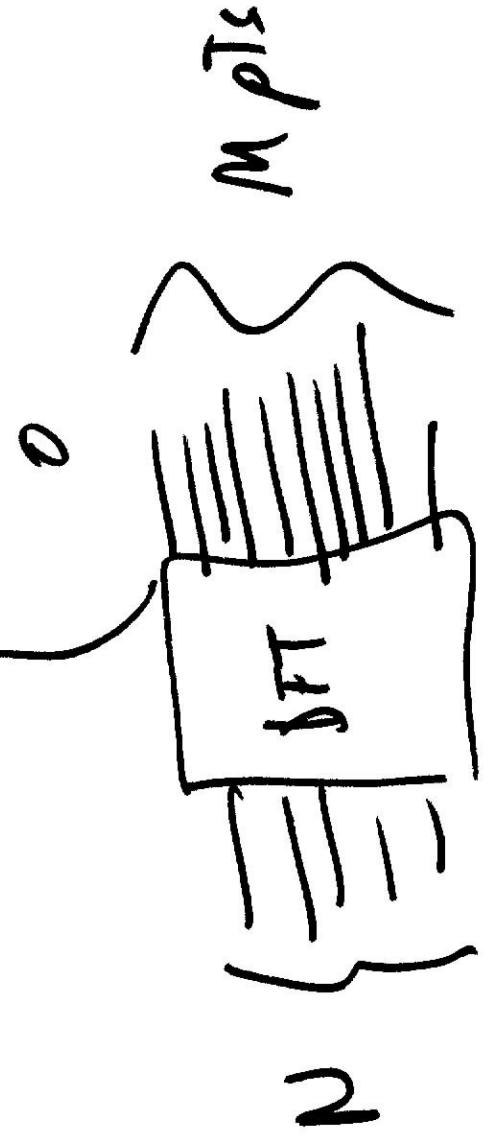


- Can take M PT DFT of $x(n)$:

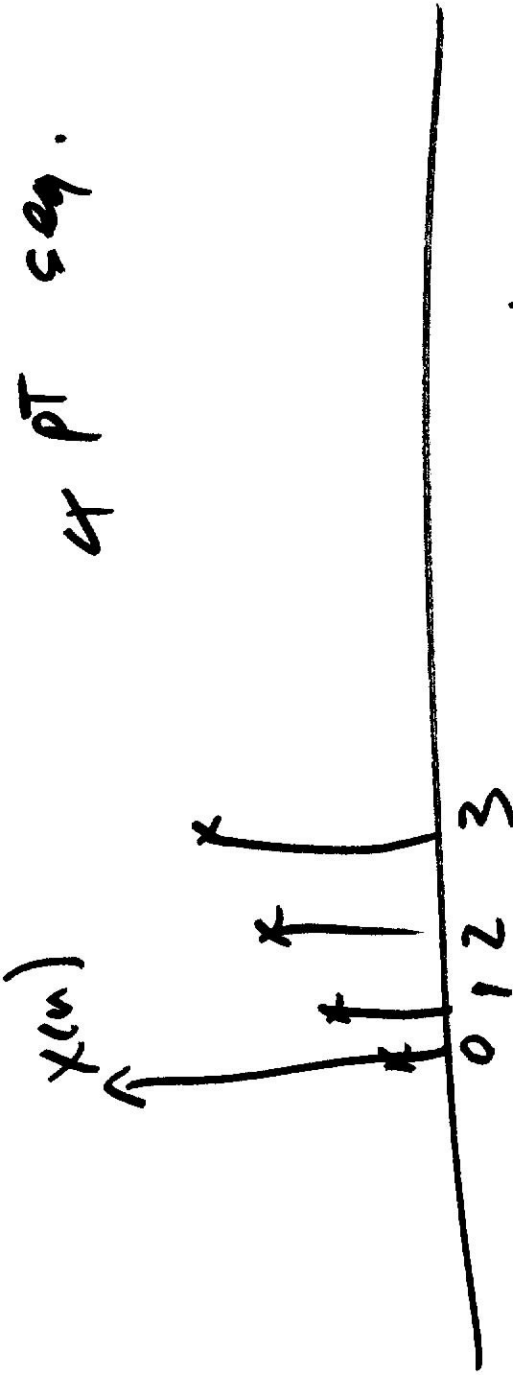
$$X(k) = \sum_{n=0}^{M-1} x(n) e^{-j2\pi nk/M}$$

$$0 \leq k < M$$

otherwise.



Ex



4 #s.

$$4 \text{ pt DFT } x(n) : \sum_{n=0}^3 x(n) e^{-j 2\pi n k / 4}$$

5 #s

$$5 \text{ pt DFT } x(n) : \sum_{n=0}^4 x(n) e^{-j 2\pi n k / 5}$$

1000 #s

$$1000 \text{ pt DFT } x(n) : \sum_{n=0}^{999} x(n) e^{-j 2\pi n k / 1000}$$

8 #s

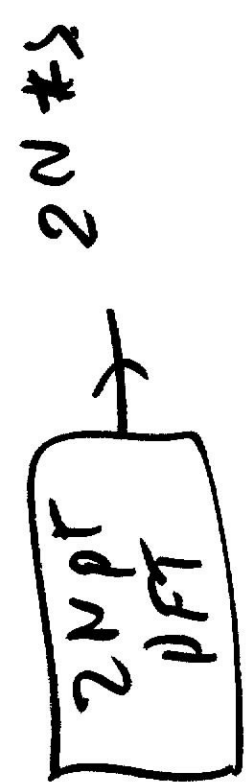
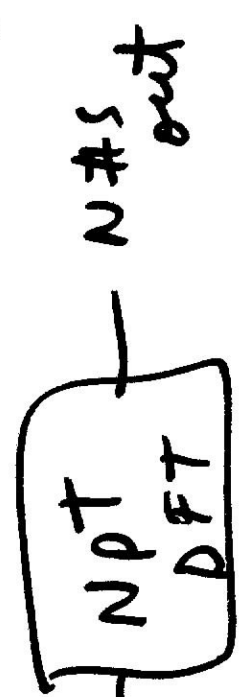
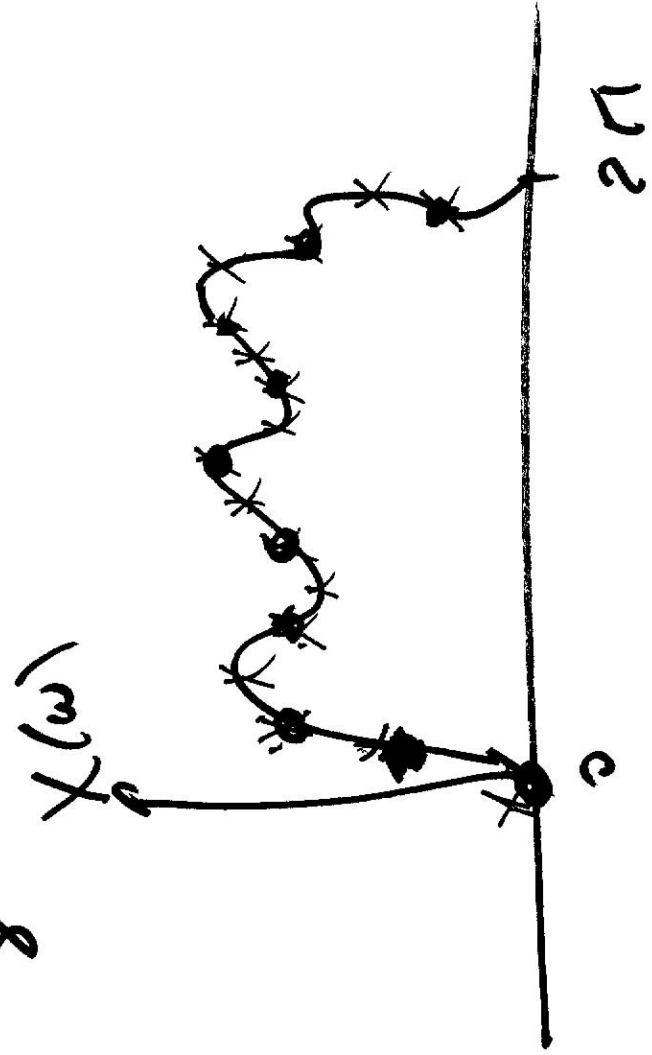
$$8 \text{ pt DFT of } x(n) : \sum_{n=0}^7 x(n) e^{-j 2\pi n k / 8}$$

3

$$3 \text{ pt DFT of } x(n) : \sum_{n=0}^2 x(n) e^{-j 2\pi n k / 3}$$

$x(n)$ $\xrightarrow{\text{MPT}}$ $X(k)$
 MPT

Pad it \rightarrow
 $M-N$ zeros



N point seq.
 padded it N zeros.



Suppose we have nonuniform samples of $X(\omega)$ at N points. We know

$X(\omega)$ is DTFT of an N pt seq.

Can I recover $X(\omega)$?
Linear system of eqns.

Yes solving a linear system of eqns.

$$X(\omega) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}$$
$$[X(\omega_i)]_{i=0, \dots, N-1} = \sum_{n=0}^{N-1} x(n) e^{-j\omega_i n}$$

$$\begin{bmatrix} X(\omega_0) \\ X(\omega_1) \\ \vdots \\ X(\omega_{N-1}) \end{bmatrix} =$$

$$\begin{bmatrix} e^{-j\omega_0 \cdot 0} & e^{-j\omega_0 \cdot 1} & \dots & e^{-j\omega_0 \cdot (N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\omega_{N-1} \cdot 0} & e^{-j\omega_{N-1} \cdot 1} & \dots & e^{-j\omega_{N-1} \cdot (N-1)} \end{bmatrix}$$

$N \times N$

Unknown

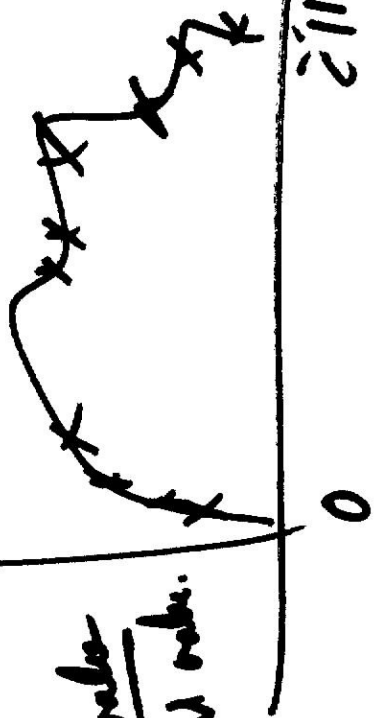
Known:
non unitary
square
NPT \rightarrow X

A

NPT.

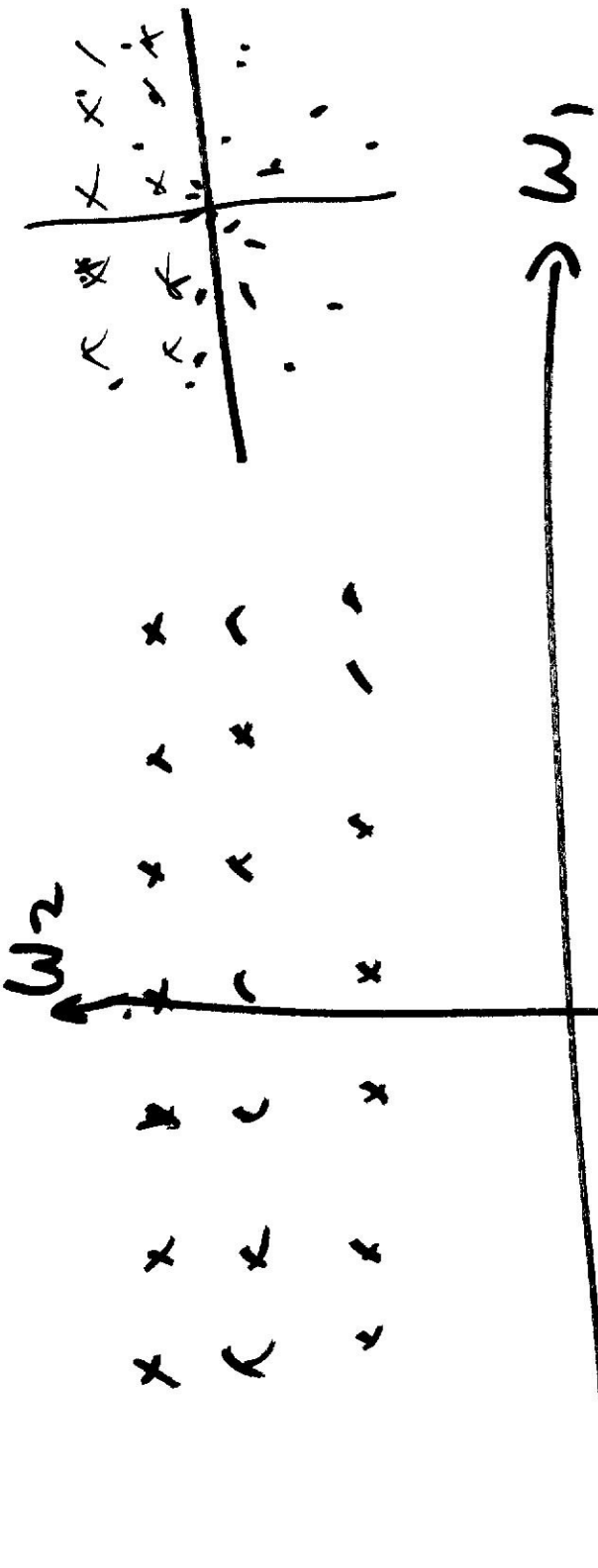
$$Y(\omega) = b$$

$$A \vec{x} = b$$



target signal value
sampled signal value

Condition \neq



Frags:

- 1. X Ray Crystallography
widely used
biology Prog design
- 2. MRI
Protein structure
- 3. Radio
U. Tomography

Graph non-uniform sampling

Thermit Exp:

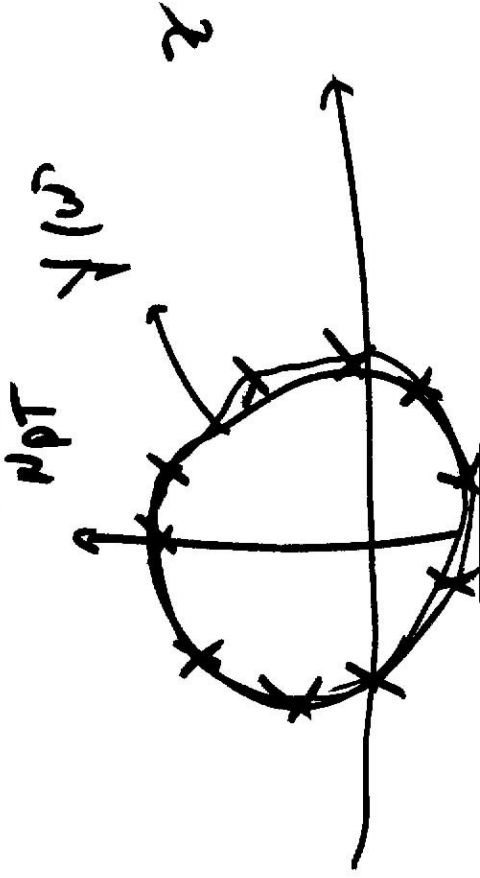
1. $y(n)$: either ∞ extent or finite extent.

2. $Y(\omega) = \text{DTFT} \{ y(n) \}$.

3. Sample $Y(\omega)$ at N equally spaced points.

$$[Y(\omega)]_{\omega = \frac{2\pi k}{N}} = \tilde{X}(k) \quad \swarrow \text{NPT seq.}$$

4. IDTFT $\{ \tilde{X}(k) \} = x(n) \quad \swarrow \text{NPT seq.}$



Answer: periodize $y(n)$ with period N .

$$\tilde{w}(n) = \sum_{k=-\infty}^{+\infty} y(n+kN)$$

extract one period:

$$x(n) = w(n) R_N(n).$$

$$\text{Proof } x(n) = \begin{cases} \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j 2\pi n k / N} & 0 \leq n < N \end{cases}$$

otherwise

$$\sum_{k=0}^{N-1} \left(\sum_m y(m) e^{-j 2\pi k m / N} \right) e^{j 2\pi n k / N}$$

$$0 \leq n < N$$

otherwise

$$x(n) = \begin{cases} \frac{1}{N} & 0 \end{cases}$$

$$x(n) = \begin{cases} \sum_{m=-\infty}^{+\infty} y(m) \frac{N-m}{N} e^{-j2\pi k(m-n)} & 0 \leq n < N \\ 0 & \text{otherwise} \end{cases}$$

$$= \sum_{r=-\infty}^{+\infty} \delta(n-m+rN)$$

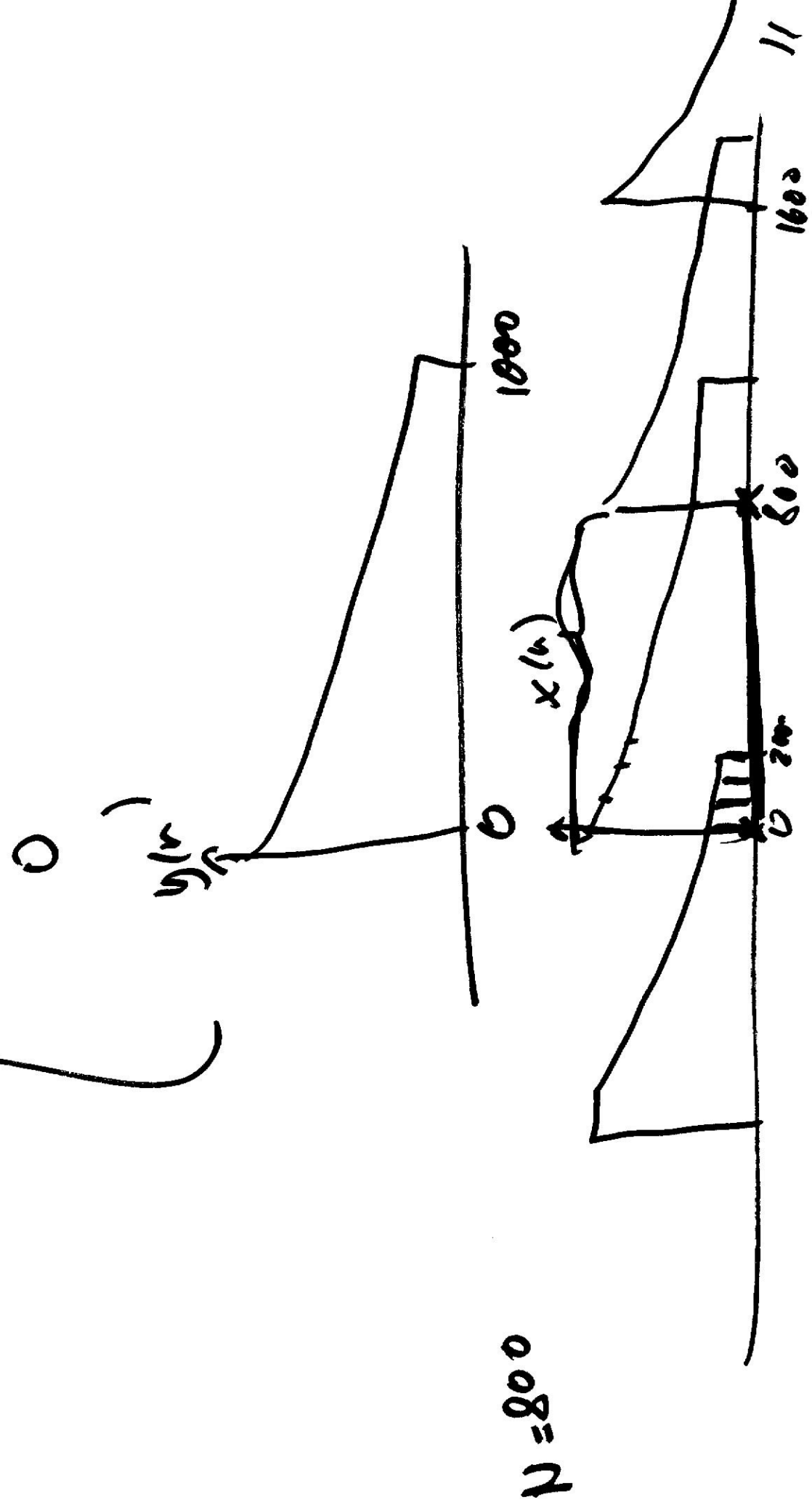
Call $f(n) = \sum_{r=-\infty}^{+\infty} \delta(n+rN)$

$$x(n) = \sum_{m=-\infty}^{+\infty} y(m) f(n-m)$$

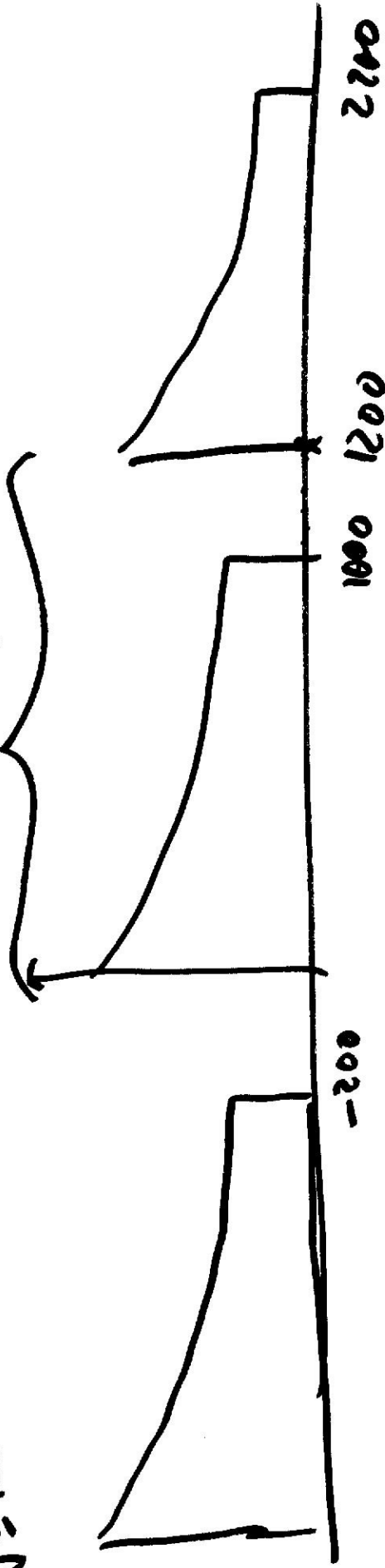
$$0 \leq n < N$$

otherwise...

$$x(n) = \begin{cases} \sum_{r=-A}^{+A} y(n+rN) & 0 \leq n < N \\ 0 & \text{otherwise} \end{cases}$$



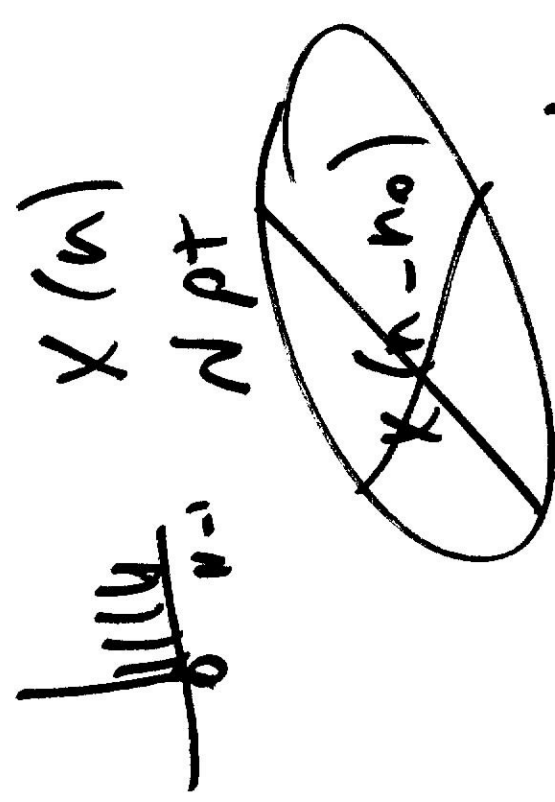
$N = 1200$



To get $|Y(\omega)|$ back precisely, we need
To sample its DTFT, $Y(\omega)$ at a higher
rate than # of samples points in $g(n)$
samples.

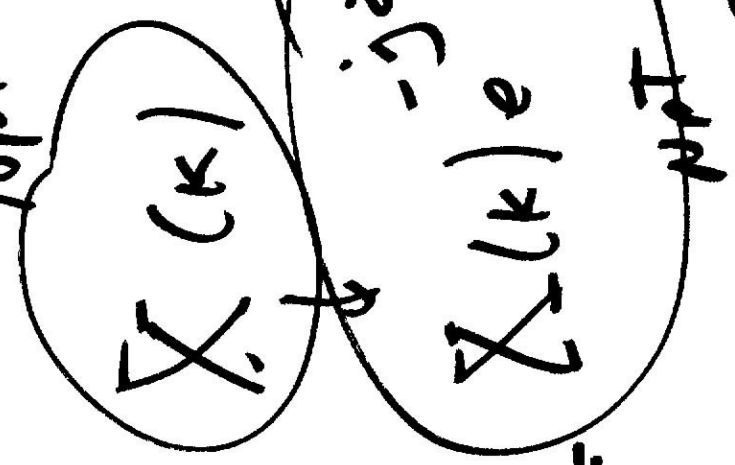
Properties of DFT

① Shift property:

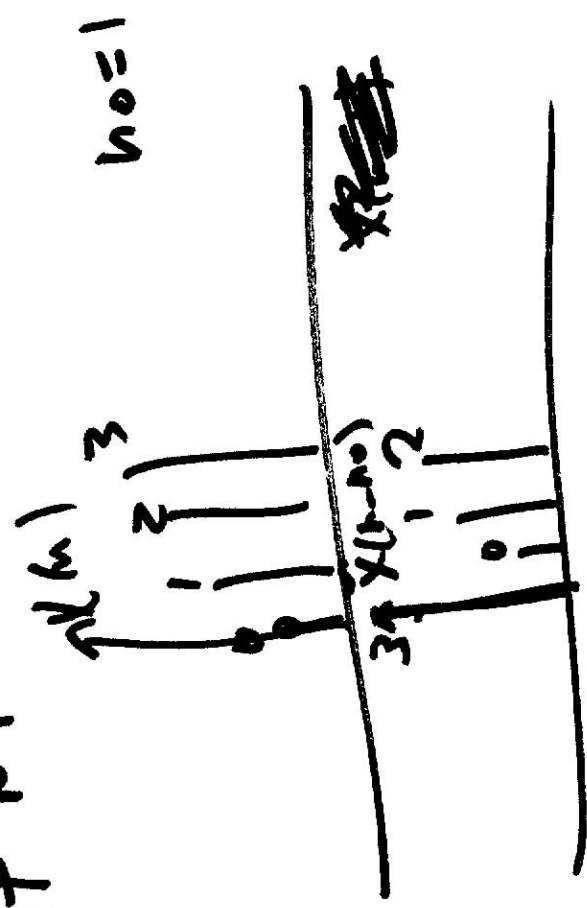


Should only be first N points
nonzero for first N points

$\xrightarrow{\text{DFT}}$



$\xrightarrow{\text{IDFT}}$



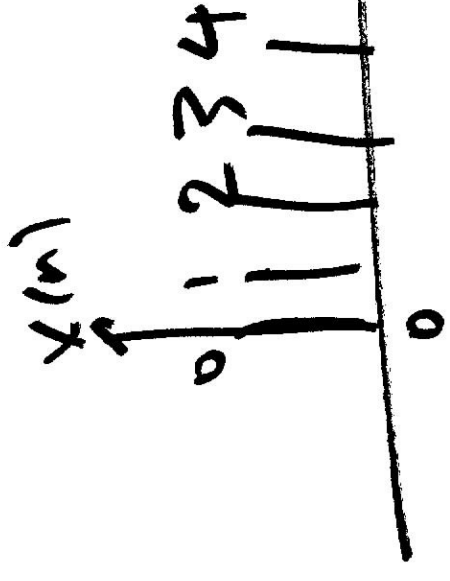
~~Answer~~

$$\tilde{x}(n-n_0) \xleftrightarrow{\text{IDFT}} X(k)e^{-j2\pi n_0 k/N}$$

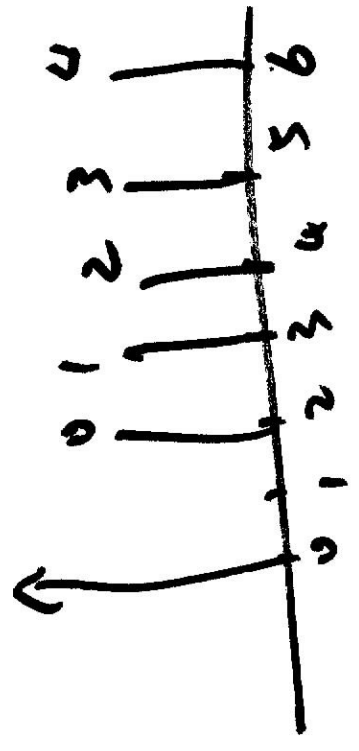
Why?

$$\tilde{x}(n) = \sum_{k=-A}^{+A} X(k+n)$$

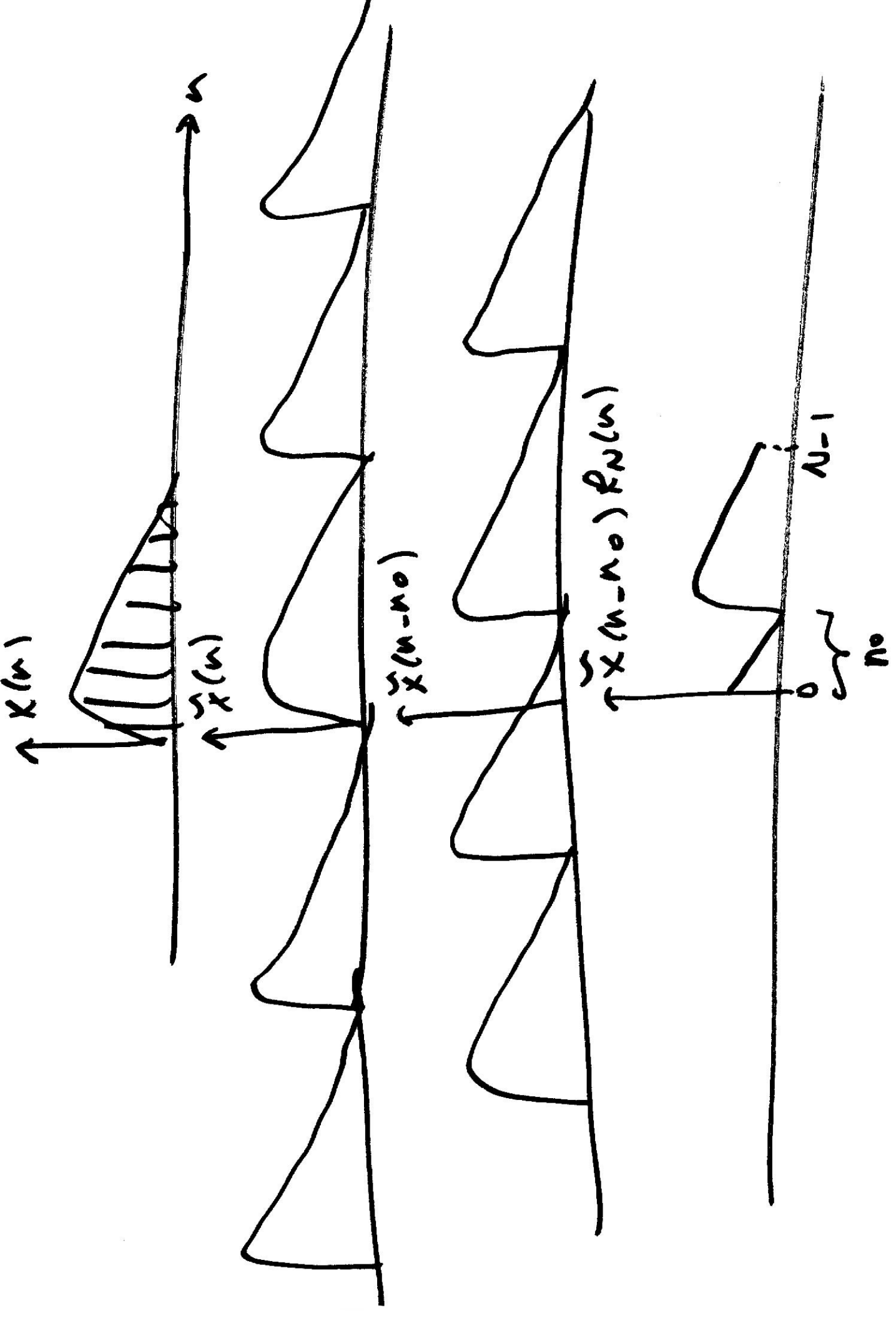
$$X(n) \xrightarrow{\text{Periodicity}}$$



$$X(n-n_0) \quad h=2$$

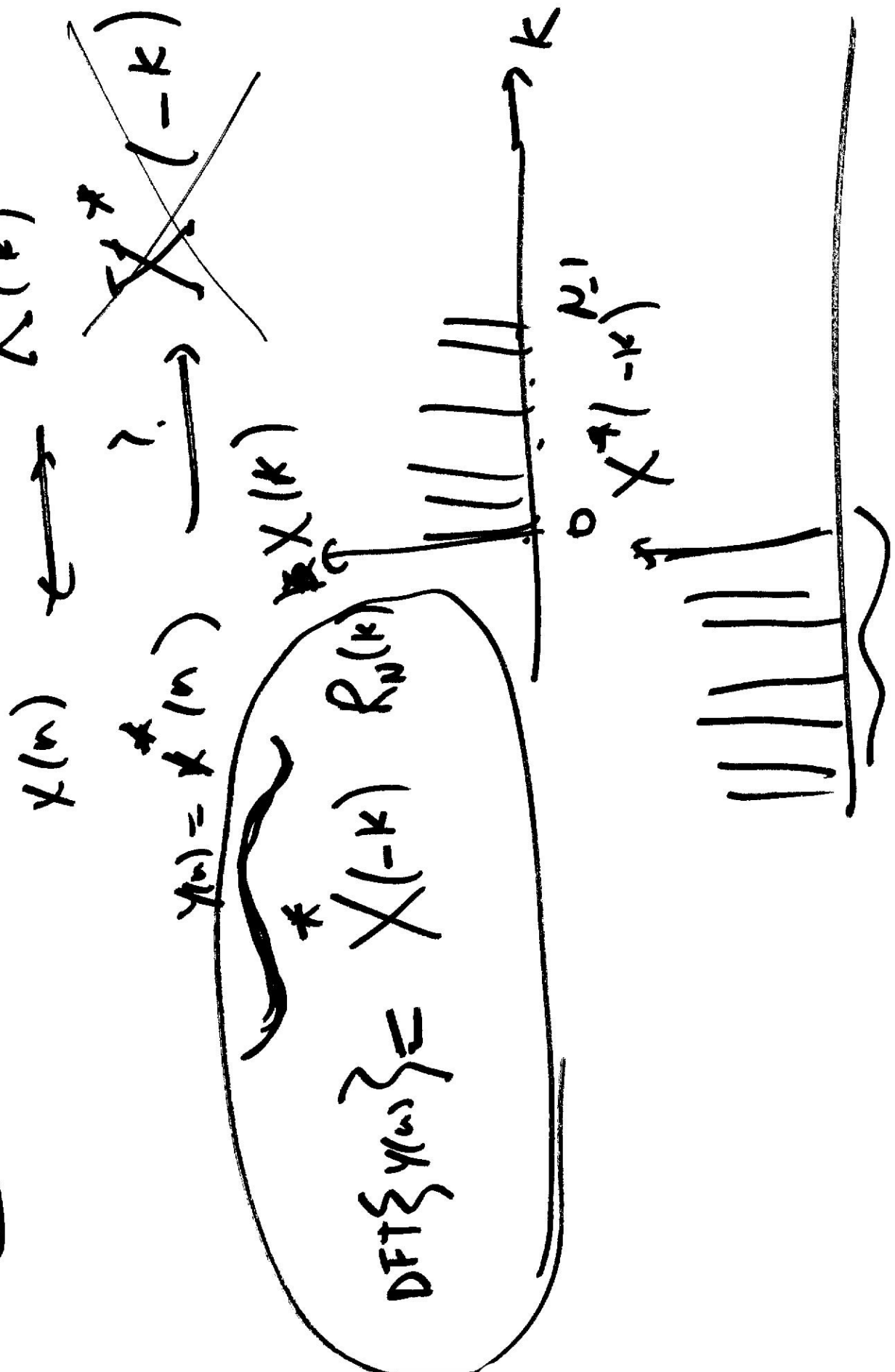


$x(n-n_0)$ is not valid IDFT.



Another property of DFT:

NFT.



Symmetry Prop

