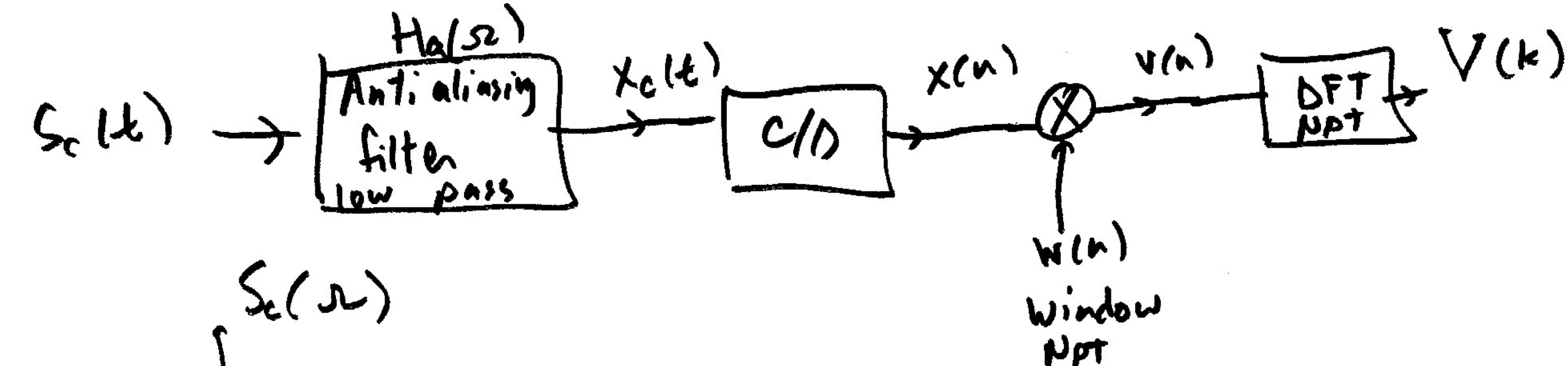
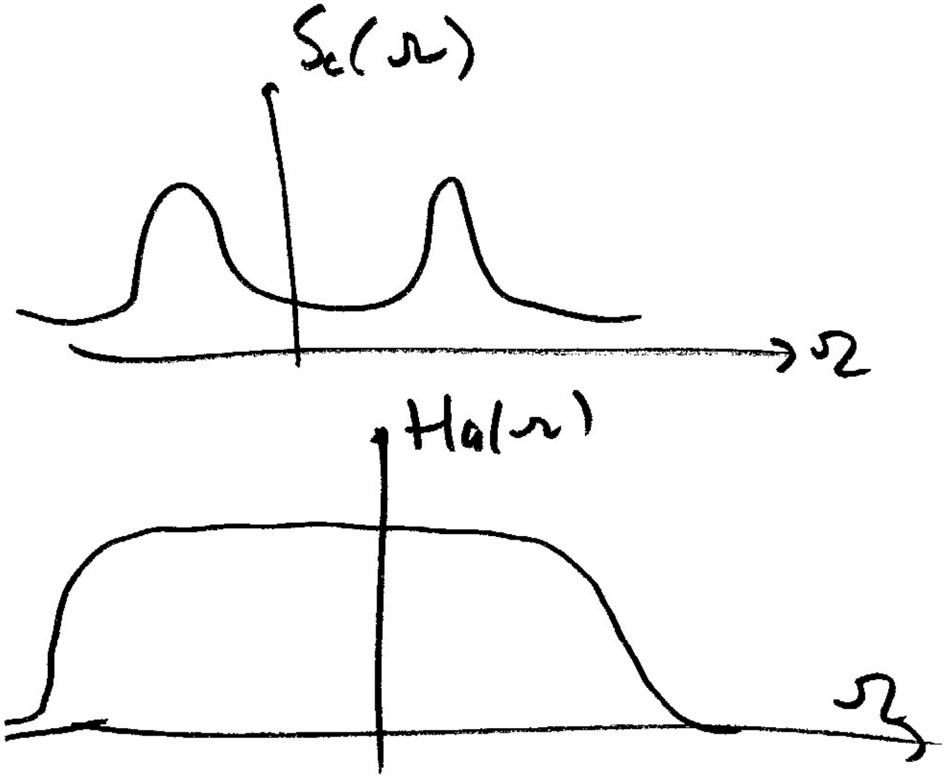
Dec. 5, 2006

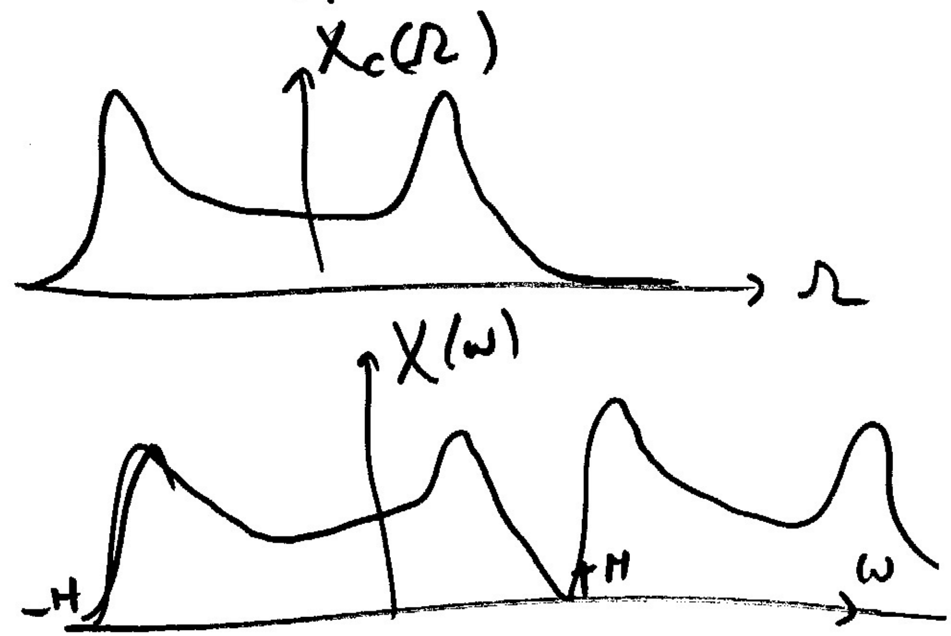
Fourier Analysis of Signals using DFT:

Given analog continuous time signal.

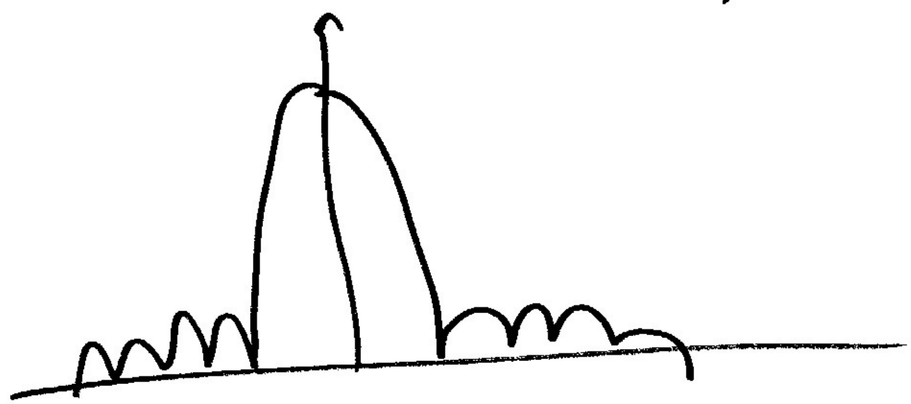
$S_c(t)$

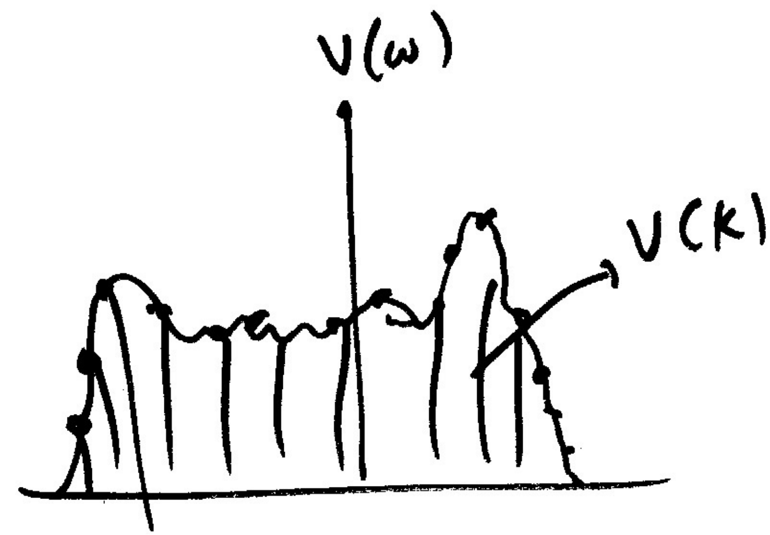






$$F.T.\{w(n)\} = |W(\omega)|$$





Consider Sum of 2 Sinusoids :

$$s_c(t) = A_0 \cos(\Omega_0 t + \theta_0) + A_1 \cos(\Omega_1 t + \theta_1)$$

sample, no quantization, no aliasing.

$$x(n) = A_0 \cos(\omega_0 n + \theta_0) + A_1 \cos(\omega_1 n + \theta_1)$$

$$\omega_0 = \Omega_0 T \qquad \omega_1 = \Omega_1 T$$

Window $x(n)$ with window fn $w(n)$

$$v(n) = A_0\, w(n) \cos(\omega_0 n + \theta_0) + A_1\, w(n) \cos(\omega_1 n + \theta_1)$$

$$= \frac{A_0}{2} w(n)\, e^{j\theta_0} e^{j\omega_0 n} + \frac{A_0}{2} w(n)\, e^{-j\theta_0} e^{-j\omega_0 n}$$

$$+ \frac{A_1}{2} w(n)\, e^{j\theta_1} e^{j\omega_1 n} + \frac{A_1}{2} w(n)\, e^{-j\theta_1} e^{-j\omega_1 n}$$

$$V(\omega) = \text{D.T.F.T}\{v(n)\} = \frac{A_0}{2} e^{j\theta_0} W(\omega - \omega_0)$$

$$+ \frac{A_0}{2} e^{-j\theta_0} W(\omega + \omega_0)$$

$$+ \frac{A_1}{2} e^{j\theta_1} W(\omega - \omega_1)$$

$$+ \frac{A_1}{2} e^{j\theta_1} W(\omega + \omega_1)$$

<u>Plug in</u>

$A_0 = 1$ $\qquad$ $A_1 = 0.75$

T = sampling rate = 10 KHZ

$\theta_0 = \theta_1 = 0$

length of rectangular window $w(n) = 64$

Fig 10.2

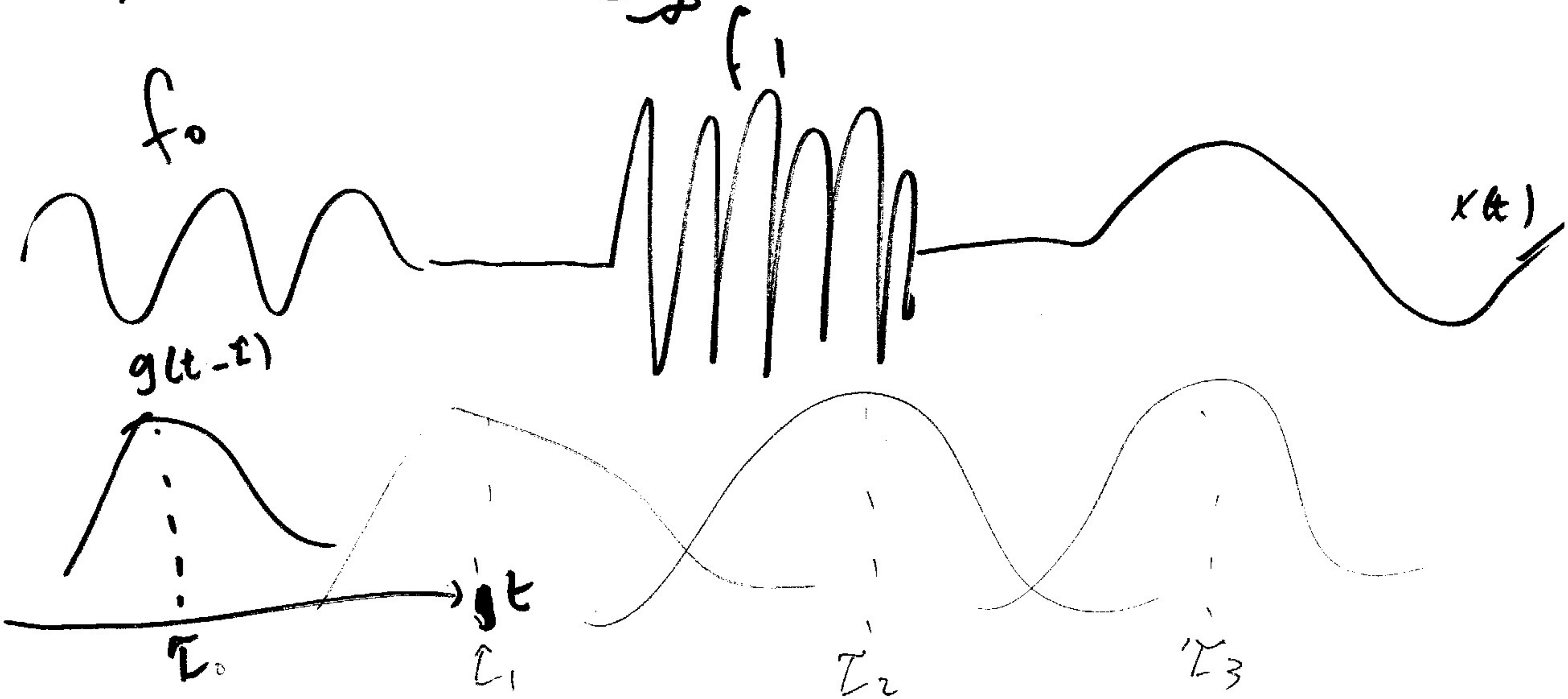
Distance Ω_1 and Ω_0 smaller

$\Omega_1 - \Omega_0 \longrightarrow$ smaller

Short Term Fourier Transform:

C.T. short term F.T. of signal $x(t)$:

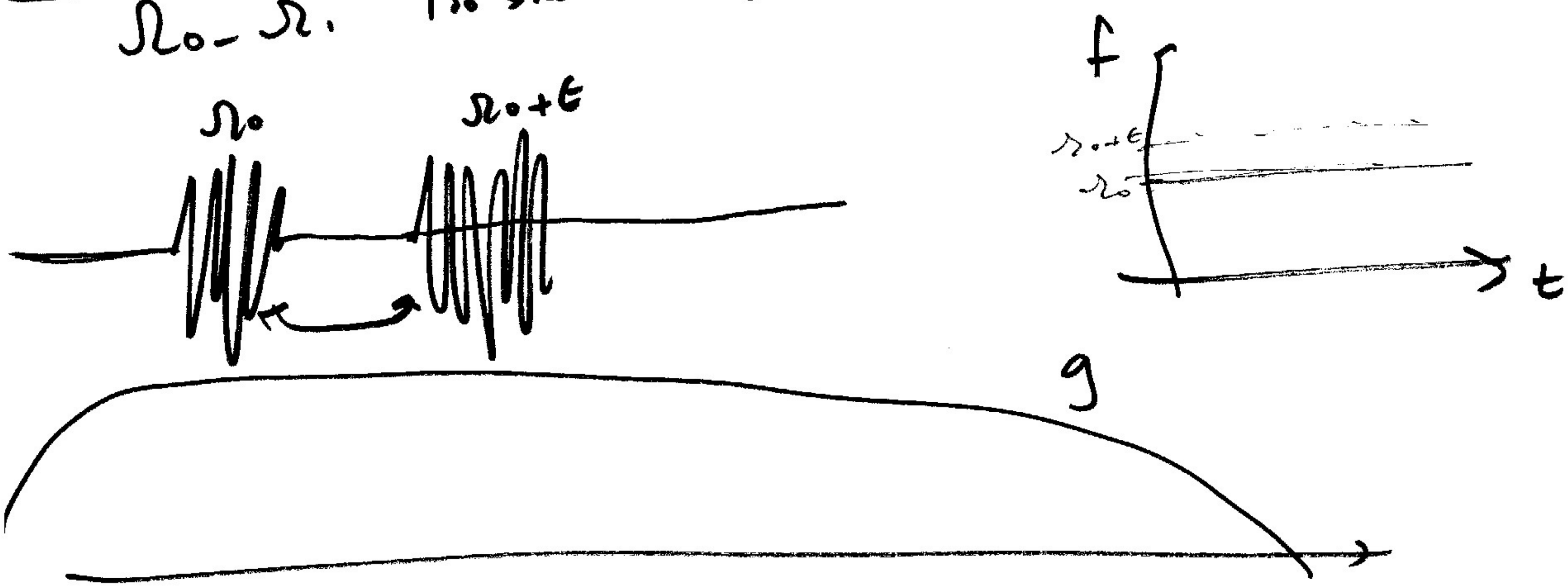
$$STFT_{x(t)}\{\tau, f\} = \int_{-\infty}^{+\infty} x(t)\, g^*(t-\tau)\, e^{-j2\pi f t}\, dt$$



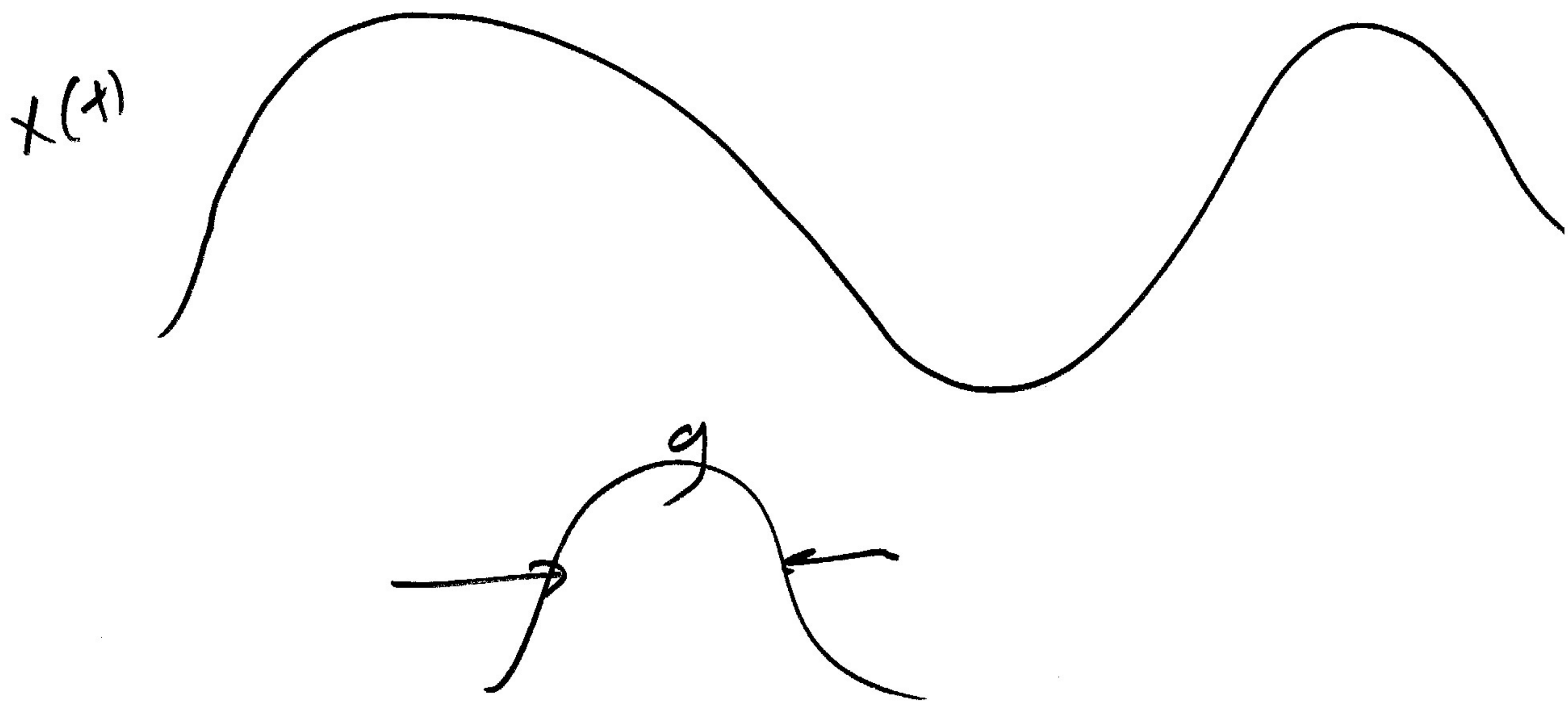
Problem STFT is depending on g and how to choose it.

+ if extent of g is too large $\longrightarrow$ lose temporal resolution.
+ if extent of g is too small $\longrightarrow$ lose freq. resolution

$\Omega_0 - \Omega_1$ too small $\rightarrow$ increase length of g



If extent of g is too small



$g(t) \longrightarrow G(f)$ window fn.

Two sinosoids can be discriminated if They are more Than Δf apart, when

$$(\Delta f)^2 = \frac{\int f^2 |G(f)|^2 \, df}{\int |G(f)|^2 \, df}$$

Two pulses can be discriminated if There are more than Δt apart: when

$$(\Delta t)^2 = \frac{\int t^2 |g(t)|^2 \, dt}{\int |g(t)|^2 \, dt}$$

Uncertainty principle:

Time-bandwidth product $\Delta t \, \Delta f \geq \frac{1}{4\pi}$

Conclusion: Once g is picked, spectral/temporal resolution is fixed for the entire Time-Freq plane.