Sep 21, 2006

Effects of round off noise in Digital Filters

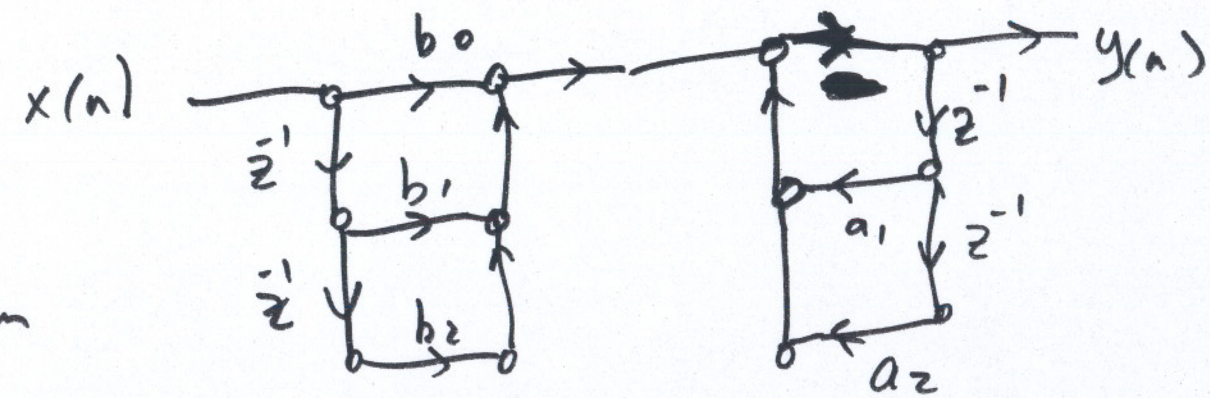
* Direct form IIR.

Nth order D.E.:

$$y(n) = \sum_{k=1}^{N} a_k \, y(n-k) + \sum_{k=0}^{M} b_k \, x(n-k)$$

Direct form 1:

2nd order system



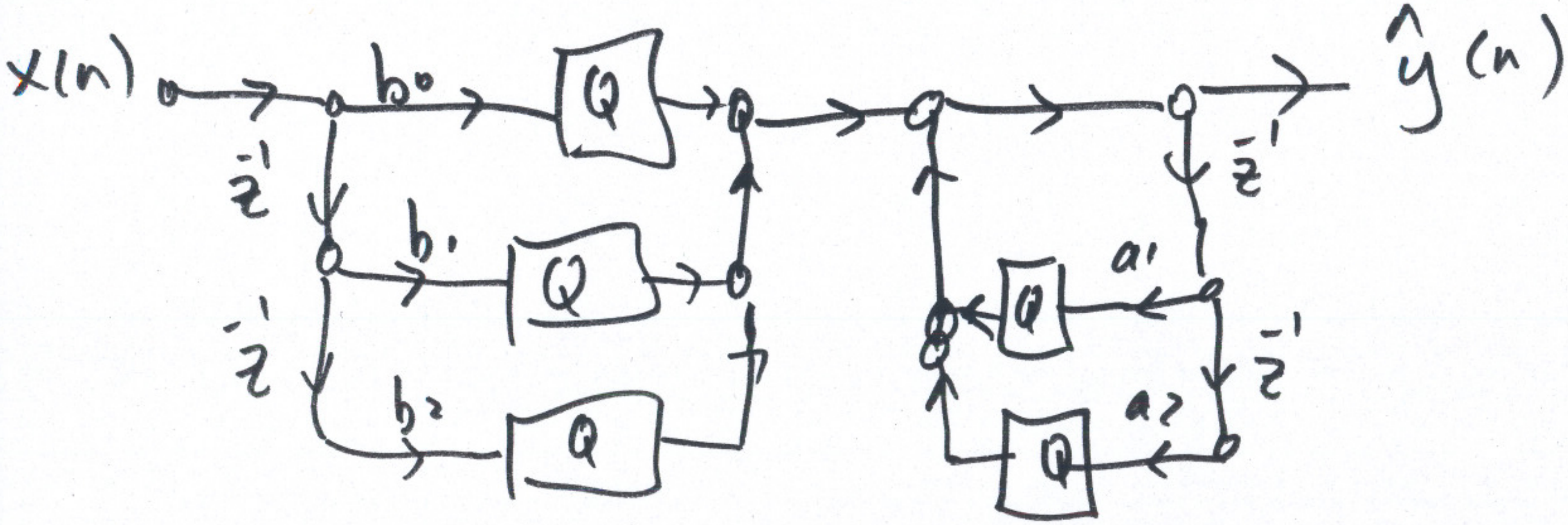
Assume signal values & coeff. are represented by (B+1) bit fixed point binary #s.

when multiply 2 ~~[illegible]~~ #s that are each

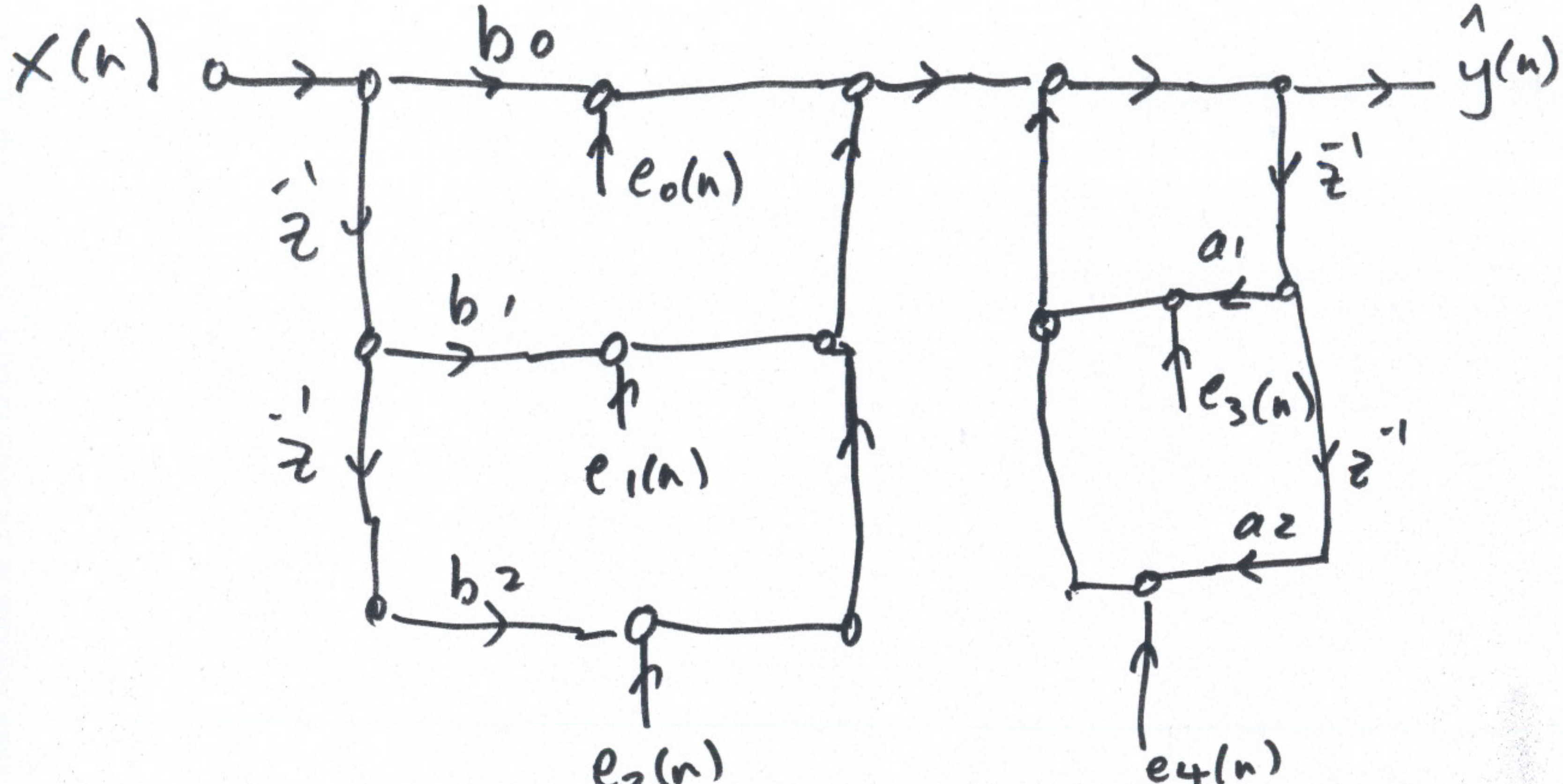
$(B+1)$ bits $\longrightarrow$ $(2B+1)$ bits

$\downarrow$ Reduce $\longrightarrow$ Quantization noise

$(B+1)$ bits



$$Q\left[b\,x(n)\right] - b\,x(n) = e(n)$$



x(n)
b0
b1
b2
z^-1
z^-1
e0(n)
e1(n)
e2(n)
a1
a2
z^-1
z^-1
e3(n)
e4(n)
ŷ(n)

Assumption:

1. $e(n)$ wide sense stationary white noise process.

2. Each quantization source has uniform distribution of amplitude over one quantization interval.

3. Each quantization noise source is uncorrelated with its input, all other quantization noise sources and input to system $x(n)$

last time lect. Rounding.

$$-\frac{1}{2}2^{-B} < e(n) \leq \frac{1}{2}2^{-B}$$

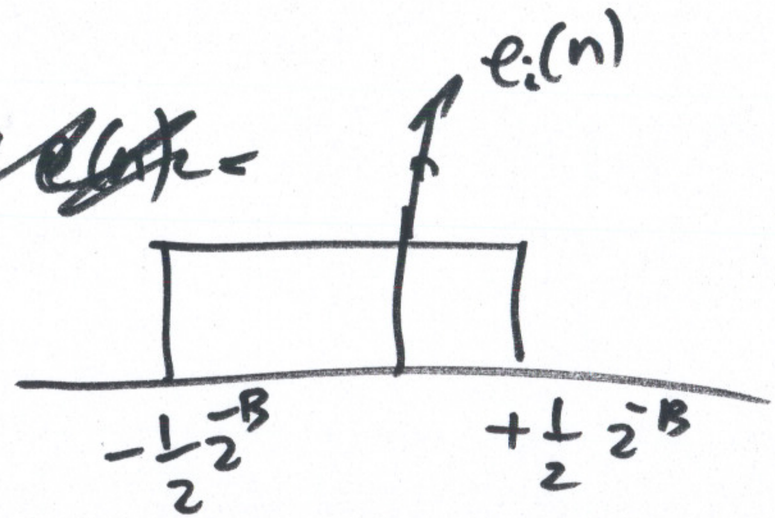
$\Delta/2 = \frac{1}{2}2^{-B}$

Truncation

$$-2^{-B} < e(n) \leq 0$$

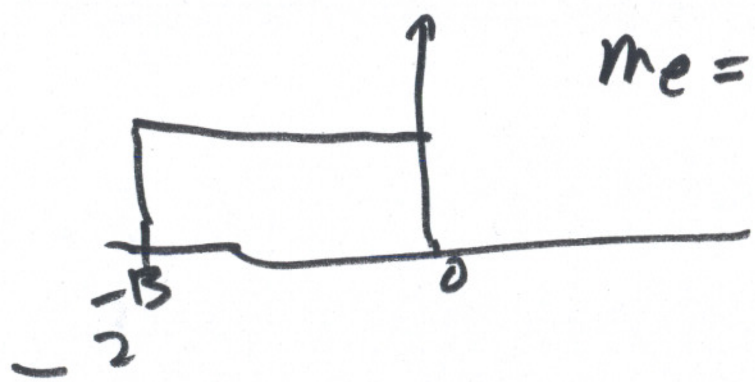
Rounding ~~m_{e_i} = mean of e_i~~

Assume uniform dist



$m_{e_i} = 0$ $\sigma^2_{e_i} = \frac{2^{-2B}}{12}$

Truncation

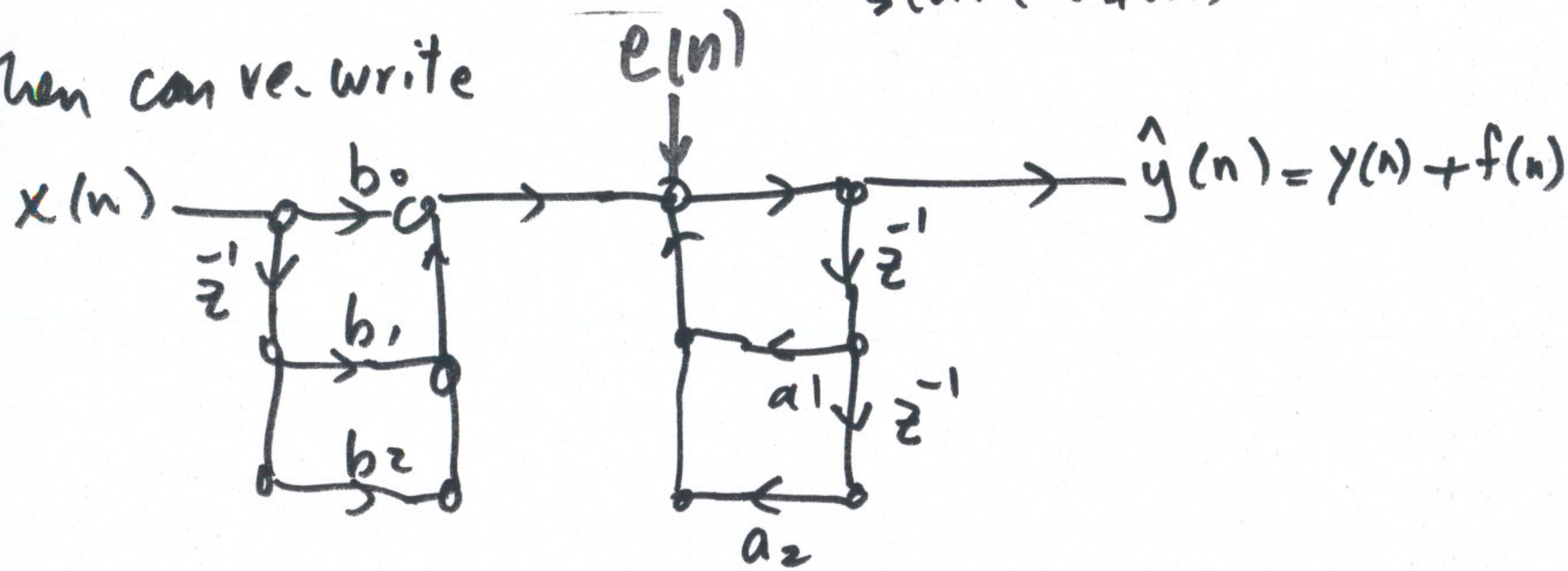


$m_e = -\dfrac{2^{-B}}{2}$

$\sigma_e^2 = \dfrac{2^{-2B}}{12}$

Can show $e(n) = e_0(n) + e_1(n) + e_2(n) + e_3(n) + e_4(n)$

then can re-write



Invoke Assumption 3

$$\mathrm{Var}\{e(n)\} = \sigma_e^2 = \sigma_{e0}^2 + \sigma_{e1}^2 + \sigma_{e2}^2 + \sigma_{e3}^2 + \sigma_{e4}^2$$

$$= 5\,\sigma_{e_i}^2 = 5\,\frac{2^{-2B}}{12}$$

(2nd) order
5 coeffs.

Can show. General Direct form 1

$$\sigma_e^2 = (M+1+N)\,\frac{2^{-2B}}{12}$$

$$\boxed{f(n) = \sum_{k=1}^{N} a_k \, f(n-k) + e(n)} \rightarrow H_{ef}$$

$$e \rightarrow \boxed{\text{LTI}}_{\,h_{ef}(n)} \rightarrow f.$$

Assuming e is w.s.s. mean of f can be easily related to mean of e.

$$m_f = m_e \sum_{n=-\infty}^{+\infty} h_{ef}(n) = m_e \, H_{ef}(0)$$

$$\sigma_f^2 = \frac{\sigma_e^2}{2\pi} \int_{-\pi}^{+\pi} \left| H_{ef}(\omega) \right|^2 d\omega$$

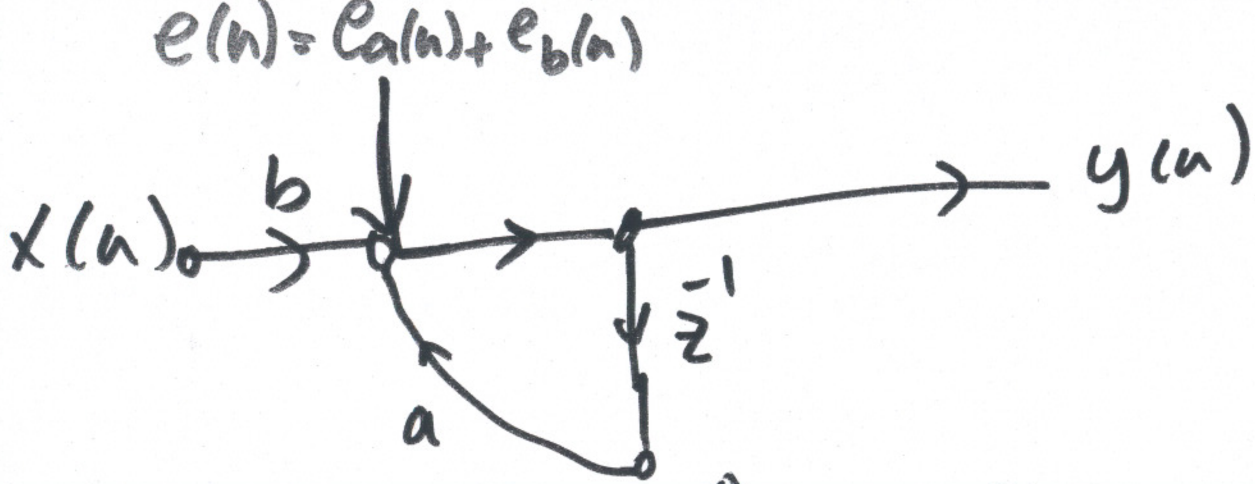
$$\sigma_f^2 = \sigma_e^2 \sum_{n=-\infty}^{+\infty} |h_{ef}(n)|^2$$

$$H_{ef}(z) = \frac{1}{A(z)}$$

$$\boxed{\sigma_f^2 = (M+1+N)\frac{2^{-2B}}{12} \sum_{n=-\infty}^{+\infty} |h_{ef}(n)|^2}$$

$\underline{Ex}$ first order system

$$H(z) = \frac{b}{1 - a\,[redacted]\,z^{-1}} \qquad |a| < 1 \qquad H_{ef}(z) = \frac{1}{1 - a z^{-1}}$$



$$\sigma_f^2 = 2\,\frac{2^{-2B}}{12}\sum_{n=0}^{\infty} |a|^{2n} = 2\,\frac{2^{-2B}}{12}\,\frac{1}{1 - |a|^2}$$

//

Ex 2 : $H(z) = \dfrac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{(1 - re^{j\theta} z^{-1})(1 - re^{-j\theta} z^{-1})}$

$a_1 = 2r\cos\theta \qquad a_2 = -r^2$

$$\sigma_f^2 = 5 \frac{2^{-2B}}{12} \frac{1}{2\pi} \int_{-\pi}^{+\pi} \frac{d\omega}{\left| (1 - re^{j\theta} e^{-j\omega})(1 - re^{-j\theta} e^{-j\omega}) \right|^2}$$

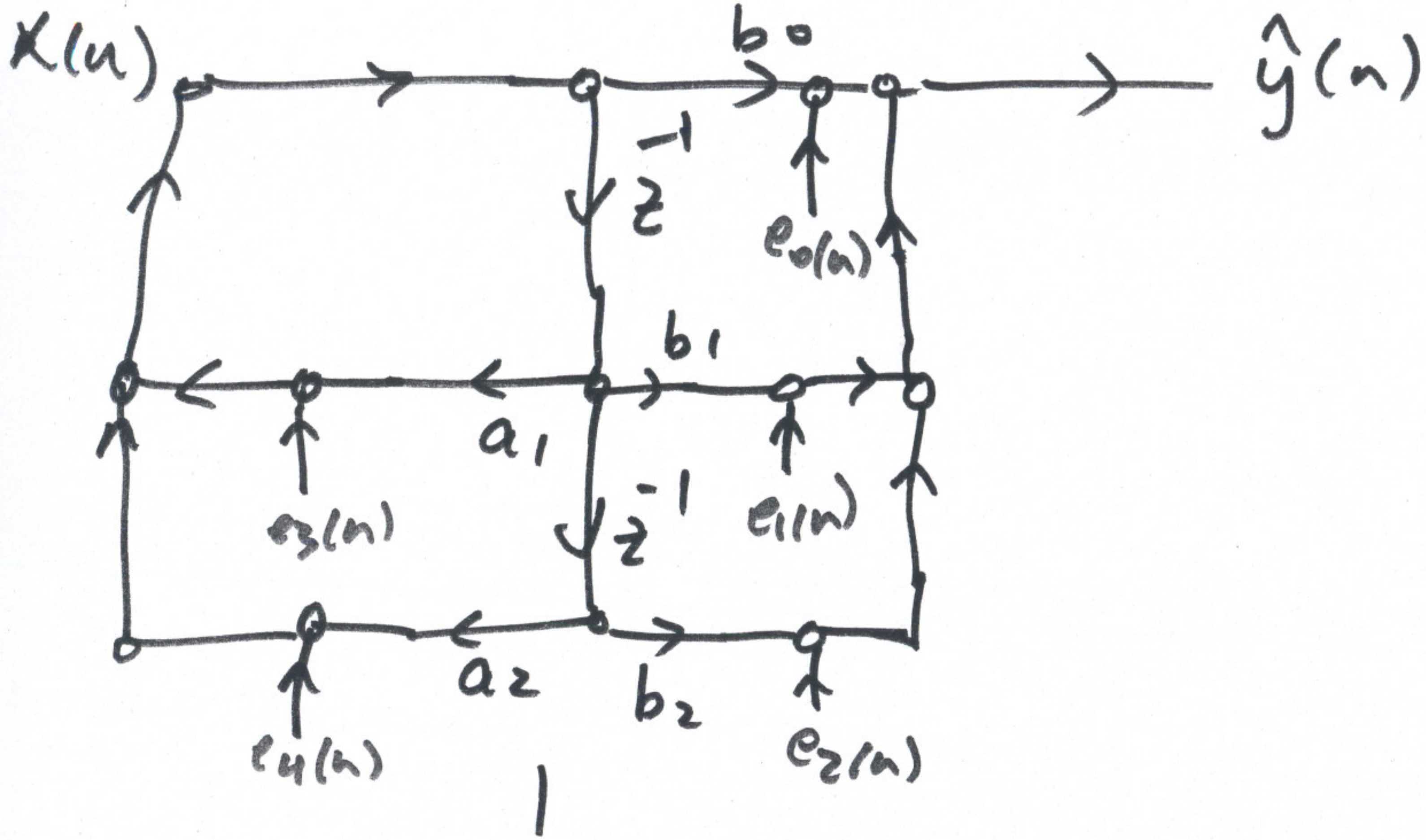
$\therefore$

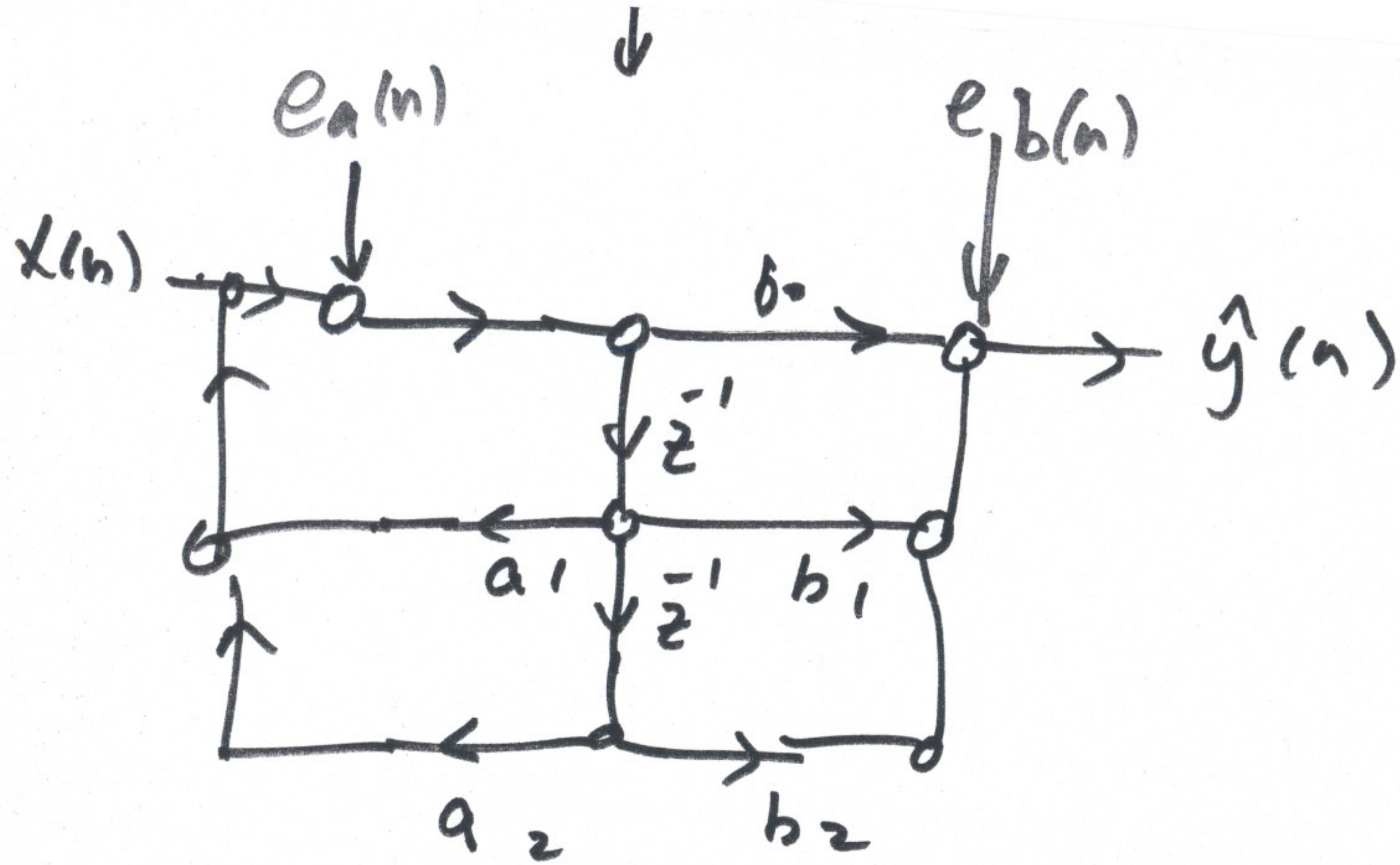
$$\sigma_f^2 = 5 \frac{2^{-2B}}{12} \left(\frac{1+r^2}{1-r^2} \right) \frac{1}{r^4 + 1 - 2r^2\cos\theta}$$

as $r \to 1 \qquad \sigma_f^2 \uparrow \infty$

2nd

Direct form 2 :





$$e_a(n) = e_3(n) + e_4(n)$$
$$e_b(n) = e_0(n) + e_1(n) + e_2(n)$$

Direct form 2

$$\sigma_f^2 = N \frac{2^{-2B}}{12} \sum_{n=-\infty}^{+\infty} |h(n)|^2$$

$$+ \quad (M+1) \frac{2^{-2B}}{12}$$

$M \rightarrow b \rightarrow$ zeros
$N \rightarrow a_i \rightarrow$ poles