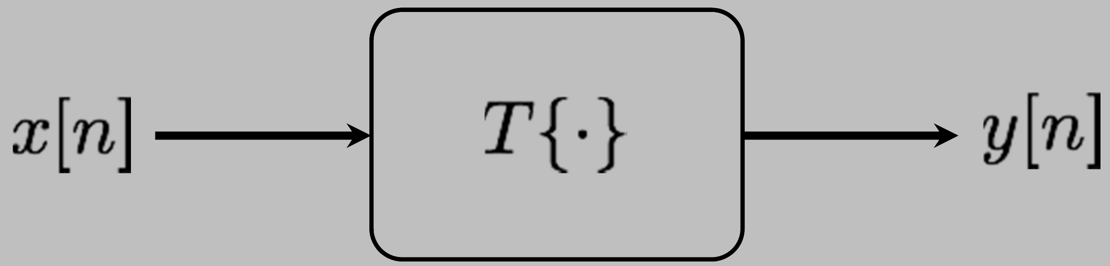
EE123
Digital Signal Processing

Lecture 3

A couple of things

- Read Ch 2 2.0-2.9 (2nd edition is fine)
- Class webcasted in bcourses.berkeley.edu
- Prof. Lustig's office hours: TBA
- Frank Ong
  - M 4p-5, 212 Cory (this week I will cover his OH)
  - Lab Bash – Cancelled this week
- My office hours
  - Th 5-6pm, 212 Cory
- HW1 due this Friday (1/30), 11:59 pm
  - Submit on bCourses
- Lab0 due next Friday (2/6)

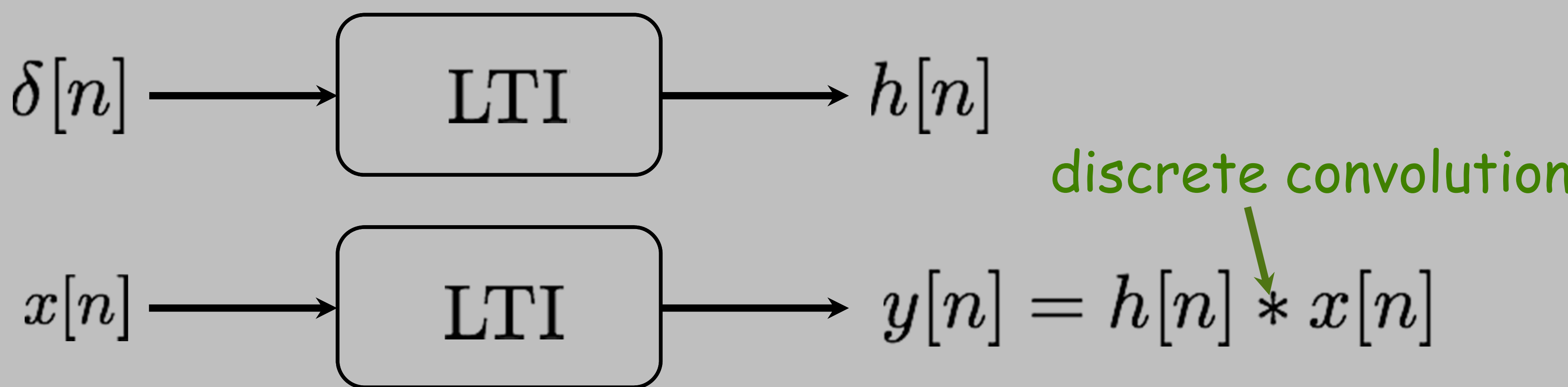
Discrete Time Systems



- Causality
- Memoryless
- Linearity
- Time Invariance
- BIBO stability

Discrete-Time LTI Systems

- The impulse response h[n] completely characterizes an LTI system "DNA of LTI"



$$y[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$

Sum of weighted, delayed impulse responses!

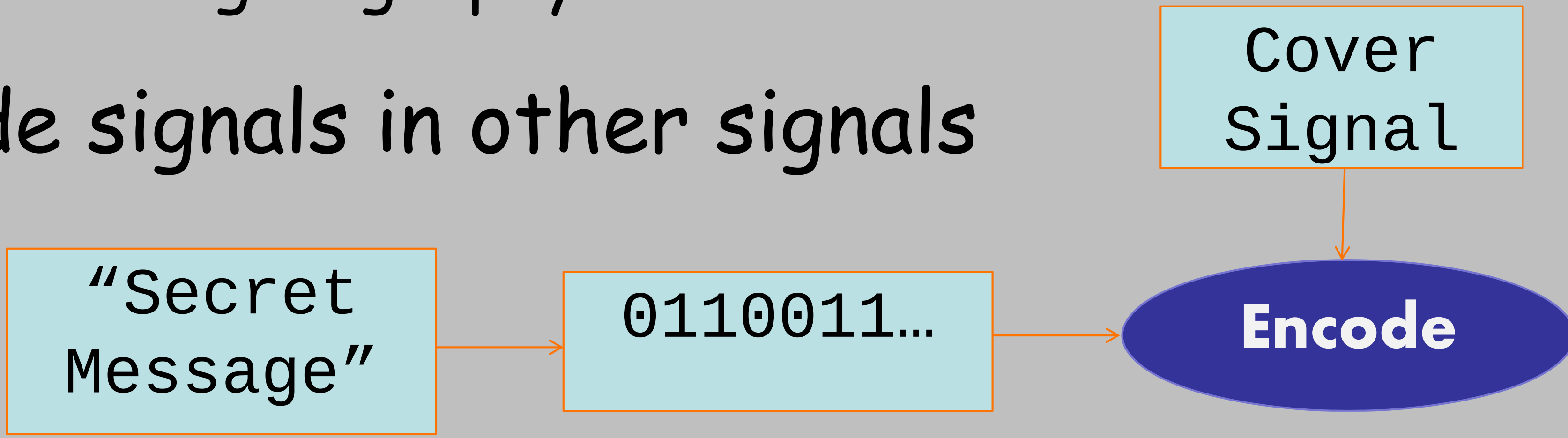
BIBO Stability of LTI Systems

- An LTI system is BIBO stable iff h[n] is absolutely summable

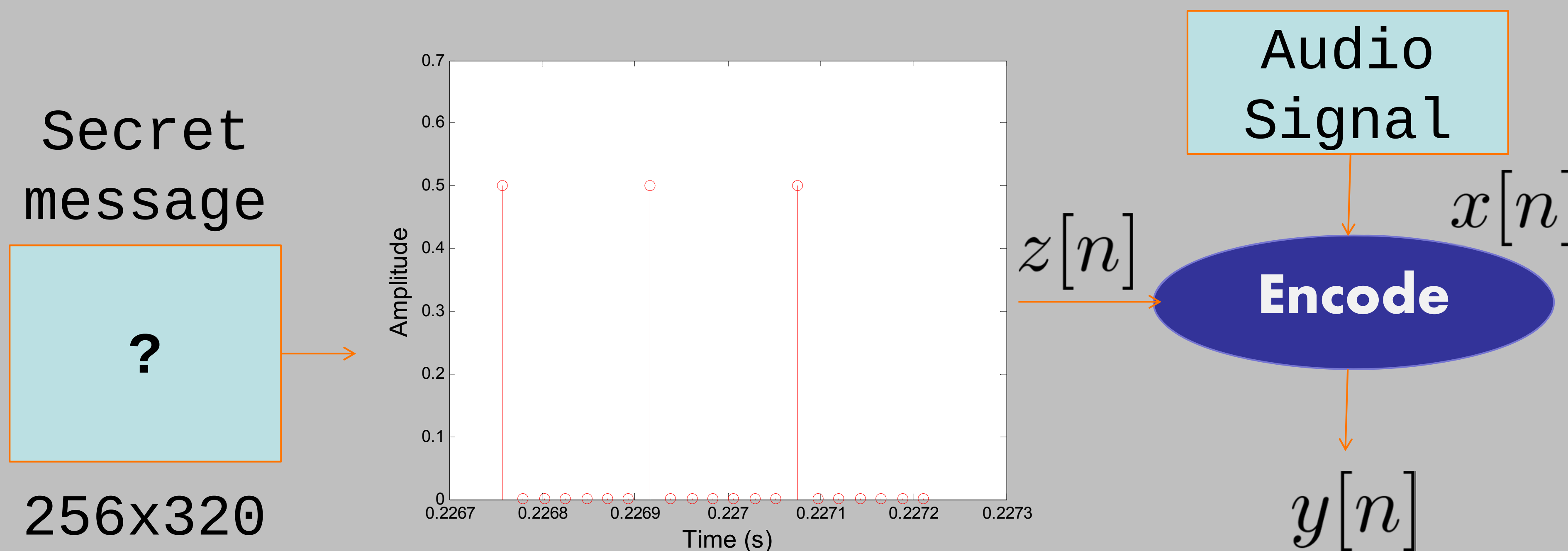
$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

Cool DSP: Steganography

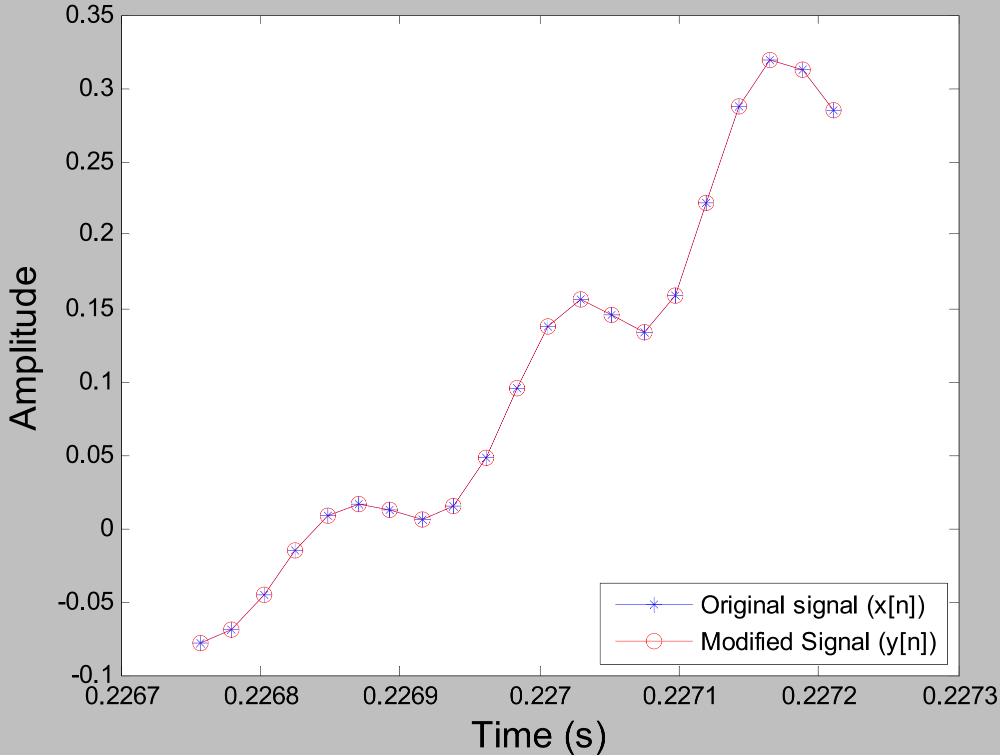
- Hide signals in other signals



- Example: hiding an image in a song



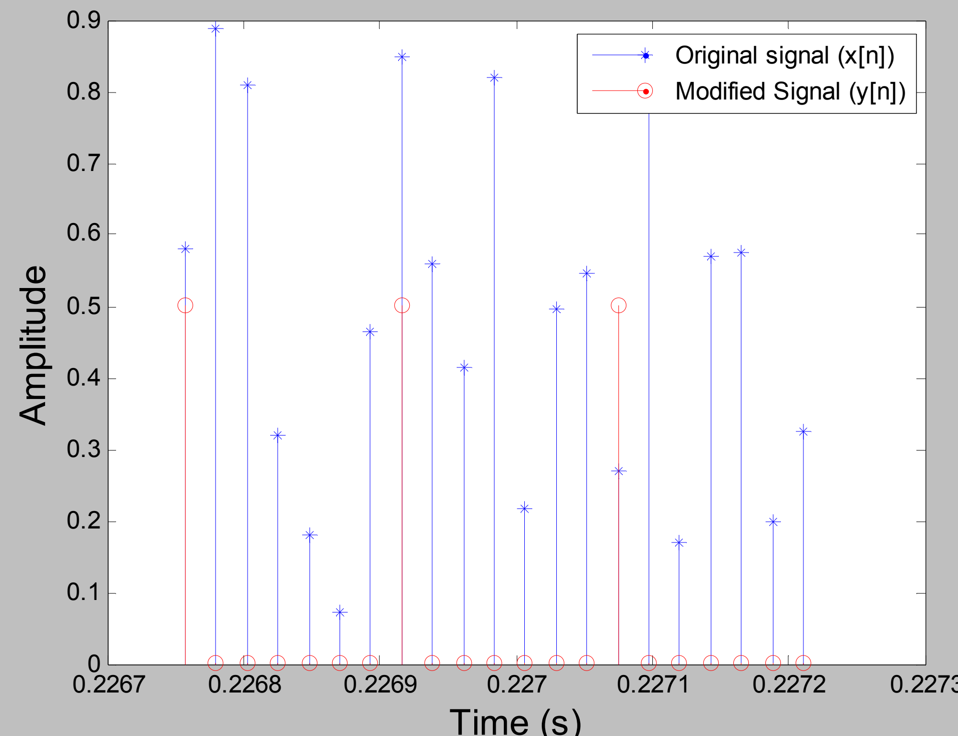
Let's compare the two signals



$y_5[n]$ = Signal y[n] at 5th decimal place

Restricted signals $x_5[n]$ and $y_5[n]$ very different!

Overall Signals x[n] and y[n] look identical (play the 2 clips)

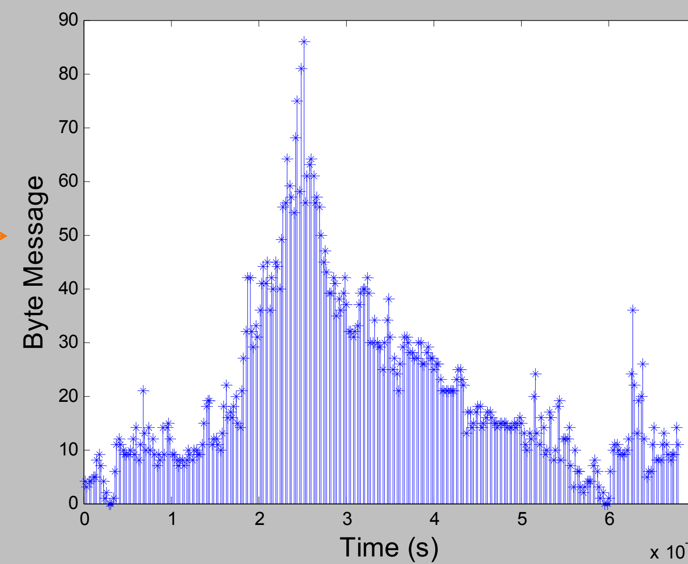


How should we decode the secret message?

$$\hat{z}[n] = \begin{cases} 1 & : y_5[n] > 0 \\ 0 & : y_5[n] = 0 \end{cases}$$

Linear? Time-invariant? BIBO?

$y_5[n]$ → **Decode** → $\hat{z}[n]$ → **Bits to Bytes** →



↓ **Rearrange** ←

256x320

Discrete-Time Fourier Transform (DTFT)

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Why one is sum and the other integral?

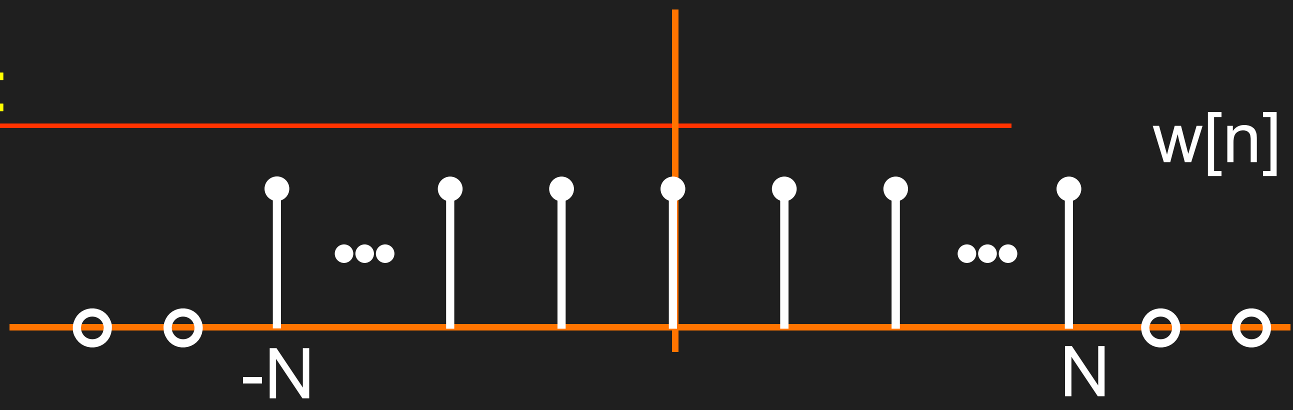
Why use one over the other?

Alternative

$$X(f) = \sum_{k=-\infty}^{\infty} x[k]e^{-j2\pi fk}$$

$$x[n] = \int_{-0.5}^{0.5} X(f)e^{j2\pi fn}df$$

Example 1:



DTFT:

$$W(e^{j\omega}) = \sum_{k=-N}^{N} e^{-j\omega k}$$

$$= e^{-j\omega N}\left(1 + e^{j\omega} + \cdots + e^{j\omega 2N}\right)$$

Recall: $1 + p + p^2 + \cdots + p^M = \frac{1 - p^{M+1}}{1-p}$ $\quad p = e^{j\omega}$, $\quad M = 2N$

Example 1 cont.

DTFT:

Example 1 cont.

DTFT:

$$W(e^{j\omega}) \quad = \quad e^{-j\omega N}\left(1 + e^{j\omega} + \cdots + e^{j\omega 2N}\right)$$

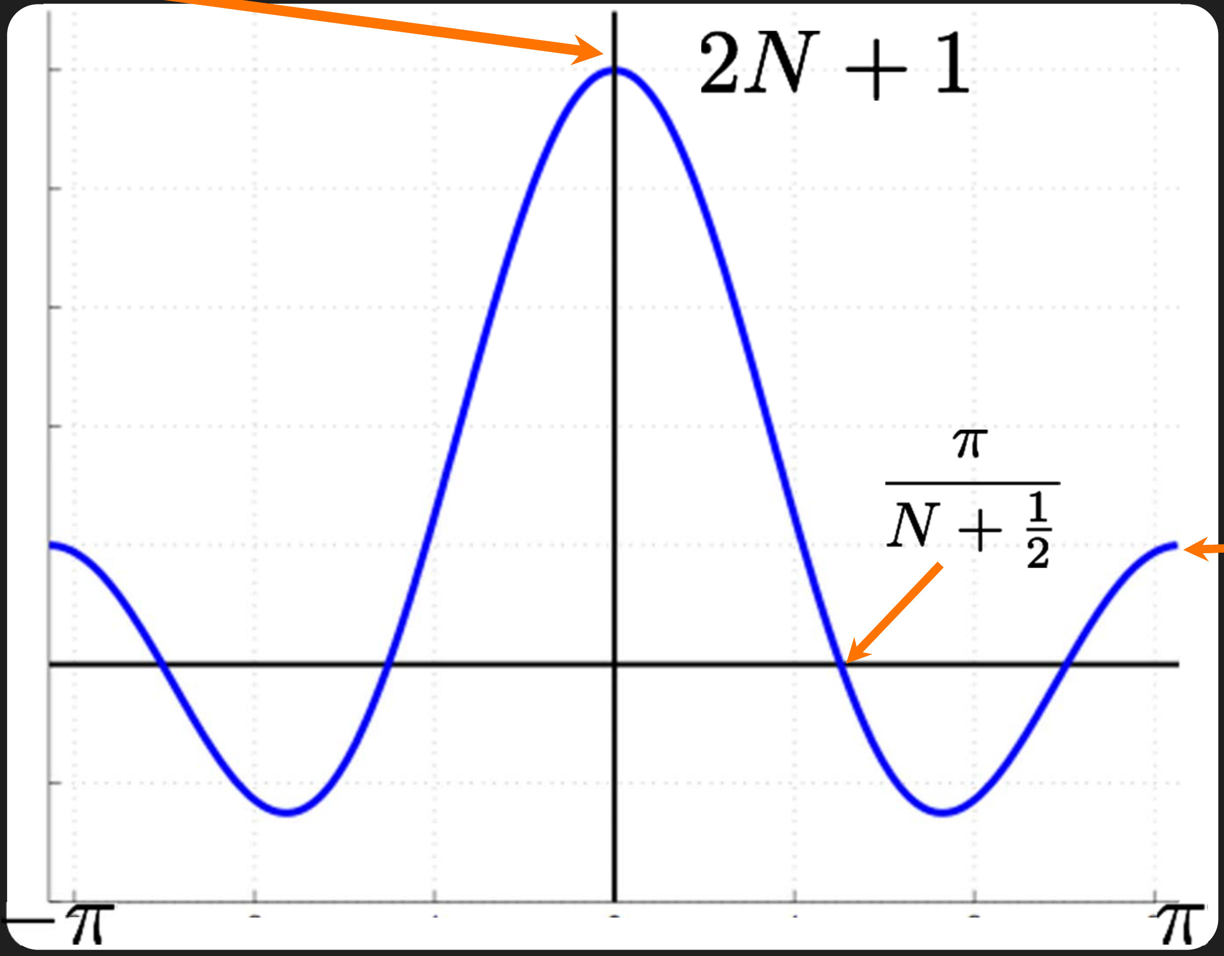
$$\frac{\times \quad e^{-j\frac{\omega}{2}}}{\times \quad e^{-j\frac{\omega}{2}}}$$

Example 1 cont.

$$W(e^{j\omega}) = \frac{\sin[(N+\frac{1}{2})\omega]}{\sin(\frac{\omega}{2})} \rightarrow (2N+1) \text{ as } \omega \rightarrow 0$$

from l'Hôpital

also, Σx[n]



=1, why?

Properties of the DTFT

Periodicity: $$X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$$

Conjugate Symmetry:

$$X^*(e^{j\omega}) = X(e^{-j\omega})$$ if x[n] is real

$$\mathcal{R}e\left\{X(e^{-j\omega})\right\} = \mathcal{R}e\left\{X(e^{j\omega})\right\}$$

$$\mathcal{I}m\left\{X(e^{-j\omega})\right\} = -\mathcal{I}m\left\{X(e^{j\omega})\right\}$$

Big deal for: MRI, Communications, more....

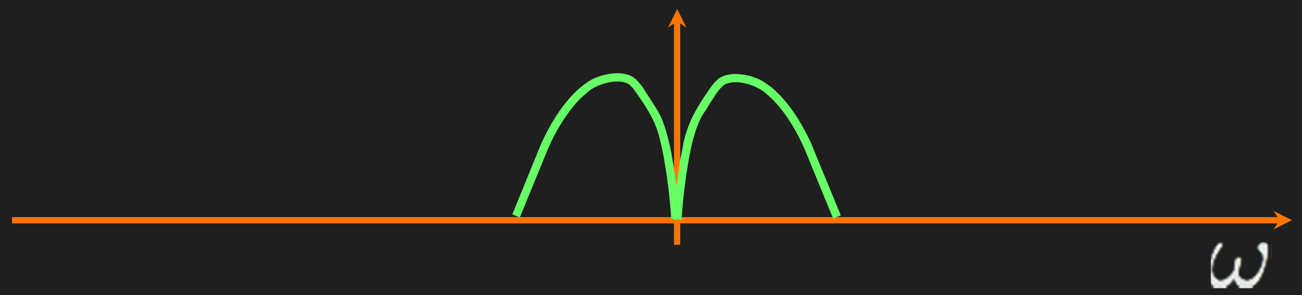
Half Fourier Imaging in MR

k-space (Raw Data)

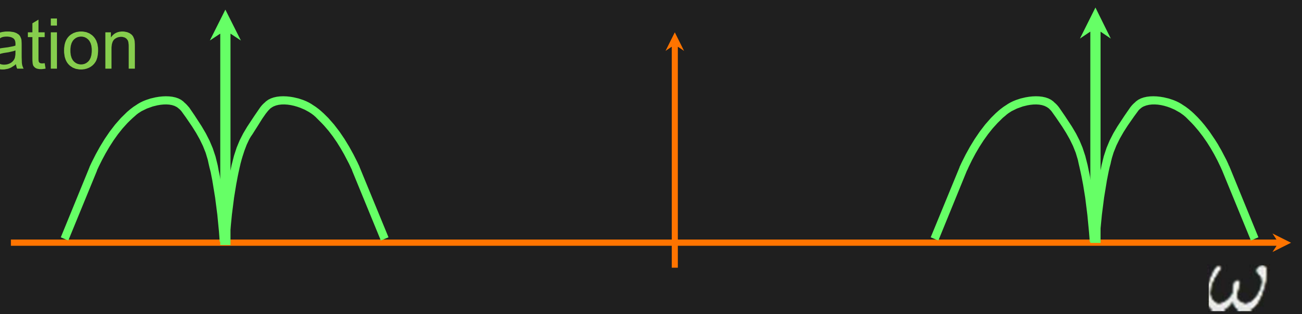
Image

SSB Modulation

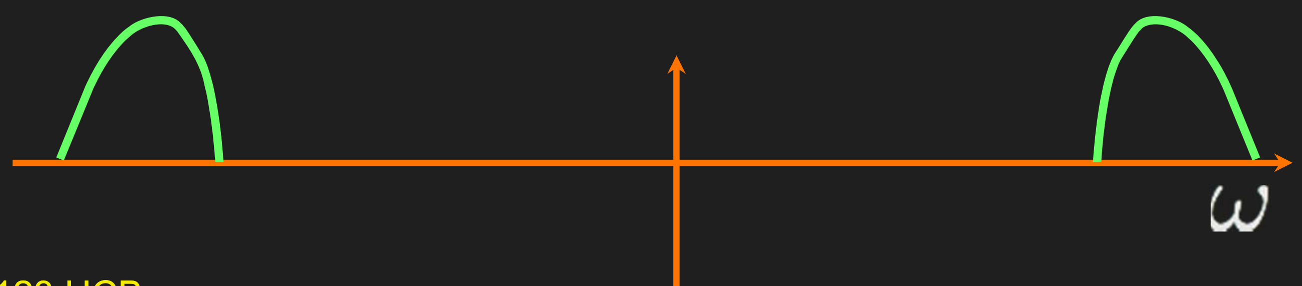
Real Baseband signal has conjugate symmetric spectrum



AM modulation



SSB-SC reduced power, half bandwidth



SSB

http://www.youtube.com/watch?v=y0qi9Fr2j6Y&list=PLA5FE5E811C57CF77

Properties of the DTFT cont.

Time-Reversal

$$\begin{aligned} x[n] &\leftrightarrow X(e^{i\omega}) \\ x[-n] &\leftrightarrow X(e^{-i\omega}) \end{aligned}$$

$$= X^*(e^{j\omega}) \quad \text{if } x[n] \in \mathcal{R}eal$$

If x[n] = x[-n] and x[n] is real, then:

$$\begin{aligned} X(e^{j\omega}) &= X^*(e^{j\omega}) \\ &\rightarrow X(e^{j\omega}) \in \mathcal{R}eal \end{aligned}$$

Q: Suppose:

$$x[n] \quad \leftrightarrow \quad X(e^{j\omega})$$

$$? \quad \leftrightarrow \quad \mathcal{R}e\left\{X(e^{j\omega})\right\}$$

A: Decompose x[n] to even and odd functions

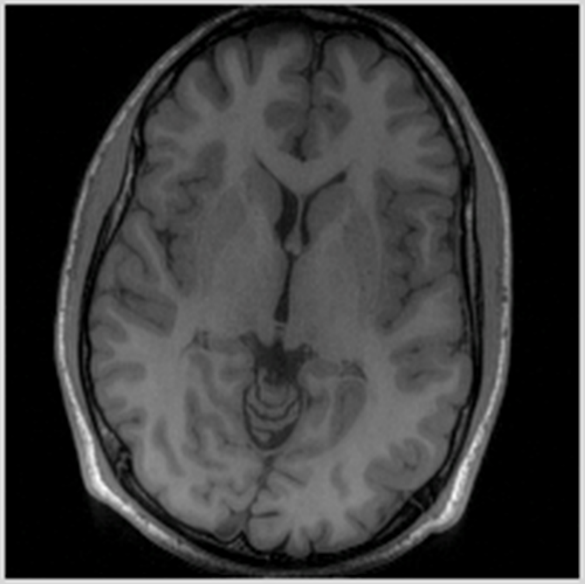
$$x[n] = x_e[n] + x_o[n]$$

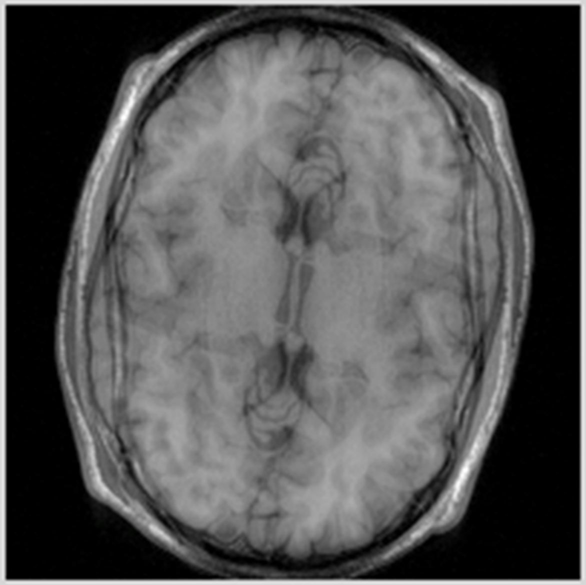
$$x_e[n] \quad := \quad \frac{1}{2}(x[n] + x[-n])$$

$$x_o[n] \quad := \quad \frac{1}{2}(x[n] - x[-n])$$

$$x_e[n] + x_o[n] \rightarrow \mathcal{R}e\left\{X(e^{j\omega})\right\} + j\mathcal{I}m\left\{X(e^{j\omega})\right\}$$

Oops!





Properties of the DTFT cont.

Time-Freq Shifting/modulation:

$$x[n] \quad \leftrightarrow \quad X(e^{j\omega})$$

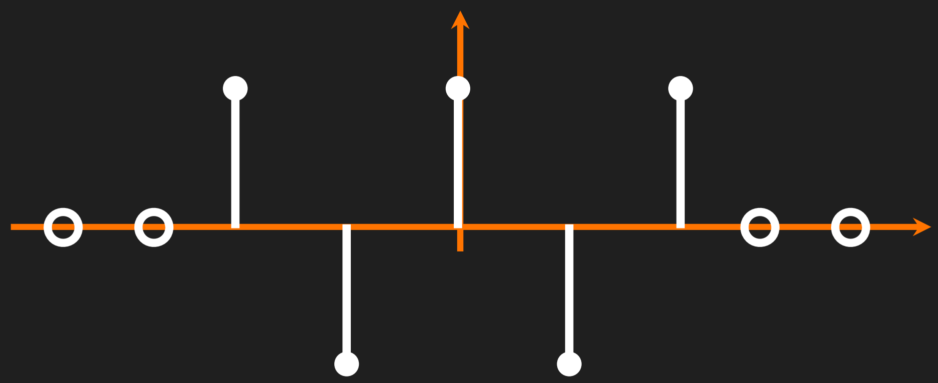
$$x[n-n_d] \quad \leftrightarrow \quad e^{-j\omega n_d}X(e^{j\omega})$$

Good for MRI! Why

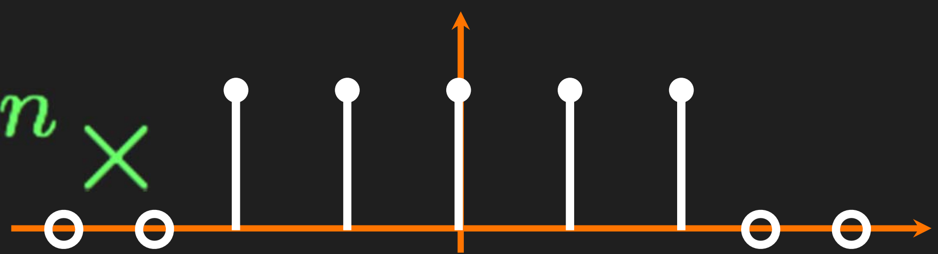
$$e^{j\omega_0 n}x[n] \quad \leftrightarrow \quad X(e^{j(\omega-\omega_o)})$$

Example 2

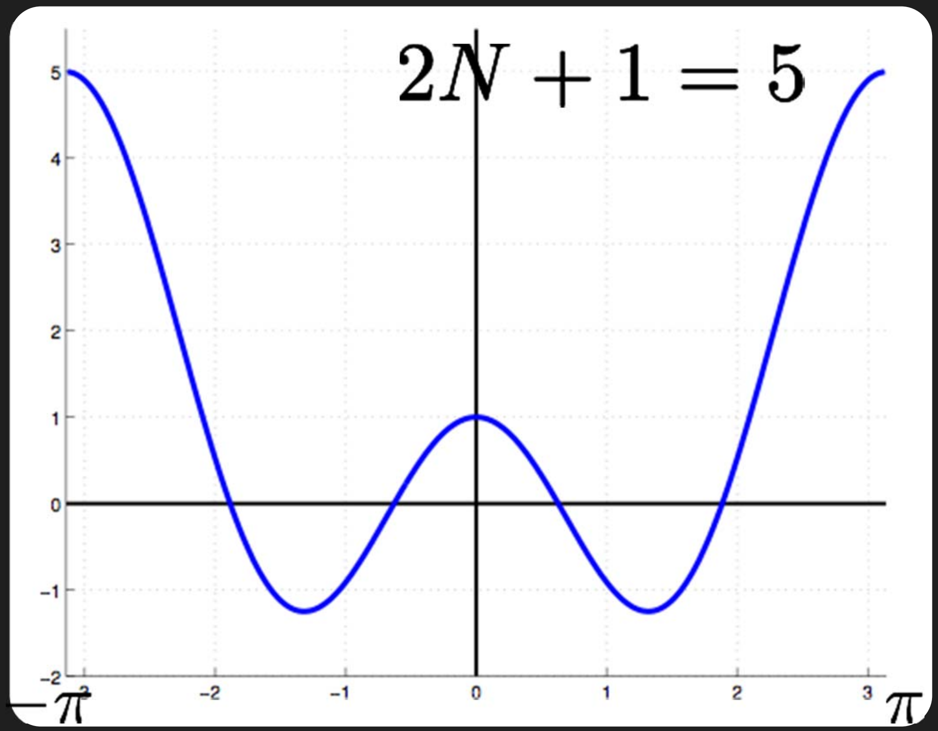
What is the DTFT of:



$e^{j\pi n} \times$



High Pass Filter



See 2.9 for more properties