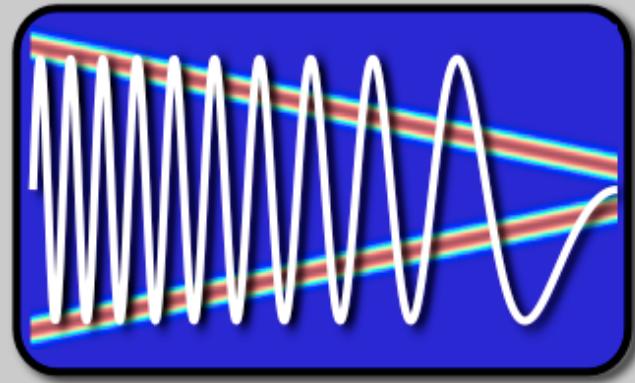


EE123



Digital Signal Processing

Lecture 10

Announcements

- Midterm: Friday next week
 - Open everything
 - ... but cheat sheet recommended instead
 - Who can not stay till 5pm?
- Optional homework next week
 - Will give you midterm and practice questions
- How's lab I going?

How do you know this guy is insane?

Hi, Dr. Elizabeth?

Yeah, uh... I accidentally took
the Fourier transform of my cat...



Spectrum **not** symmetric,
so cat must be imaginary



<http://xkcd.com/26/>

Last Time

- Frequency Analysis with DFT
- Windowing
- Zero-Padding
- Today:
 - Time-Dependent Fourier Transform
 - Heisenberg Boxes

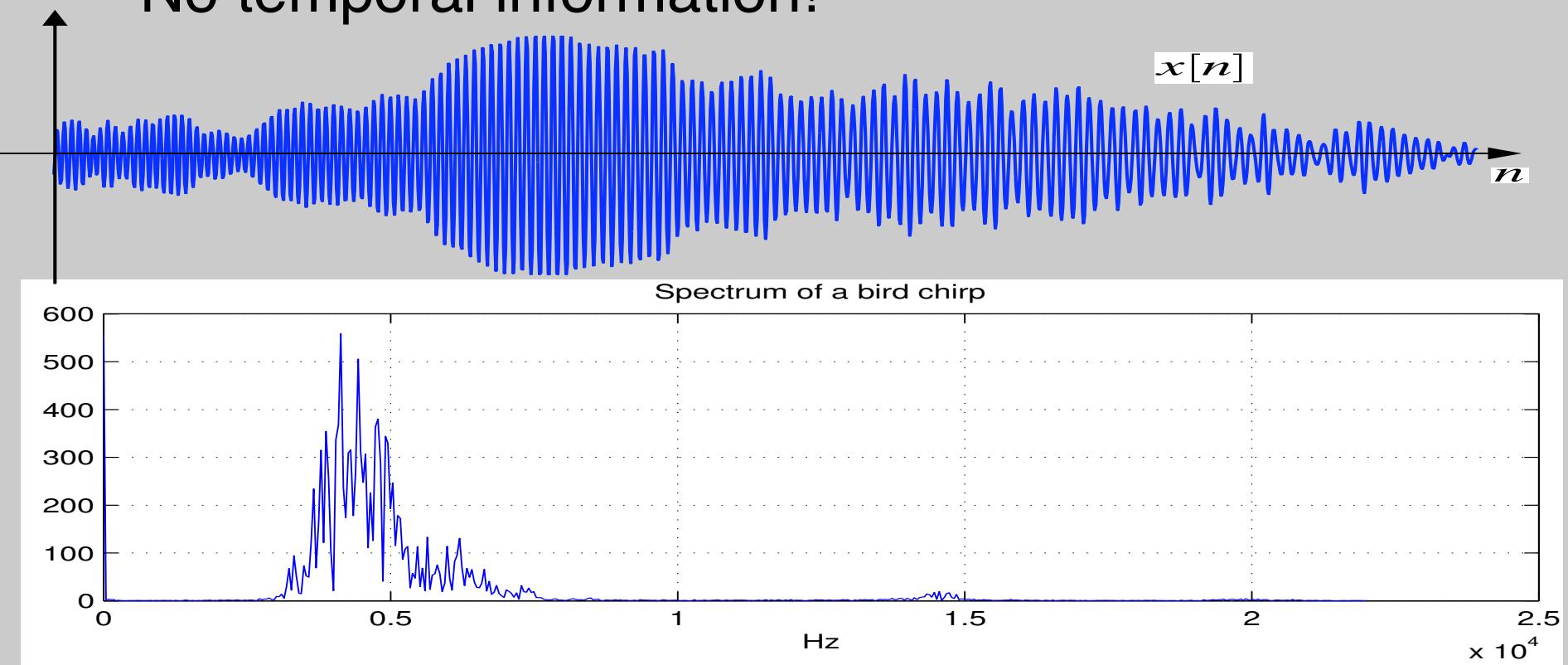
Discrete Transforms (Finite)

- DFT is only one out of a LARGE class of transforms
- Used for:
 - Analysis
 - Compression
 - Denoising
 - Detection
 - Recognition
 - Approximation (Sparse)

Sparse representation has been one of the hottest research topics in the last 15 years in sp

Example of spectral analysis

- Spectrum of a bird chirping
 - Interesting,... but...
 - Does not tell the whole story
 - No temporal information!



Time Dependent Fourier Transform

- To get temporal information, use part of the signal around every time point

$$X[n, \omega] = \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-j\omega m}$$

*Also called Short-time Fourier Transform (STFT)

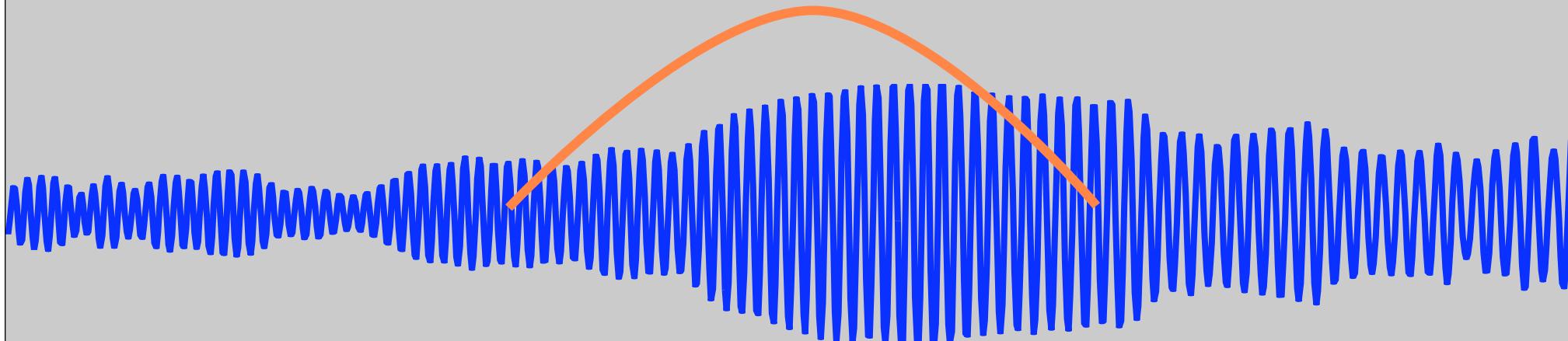
- Mapping from 1D \Rightarrow 2D, n discrete, w cont.
- Simply slide a window and compute DTFT

Time Dependent Fourier Transform

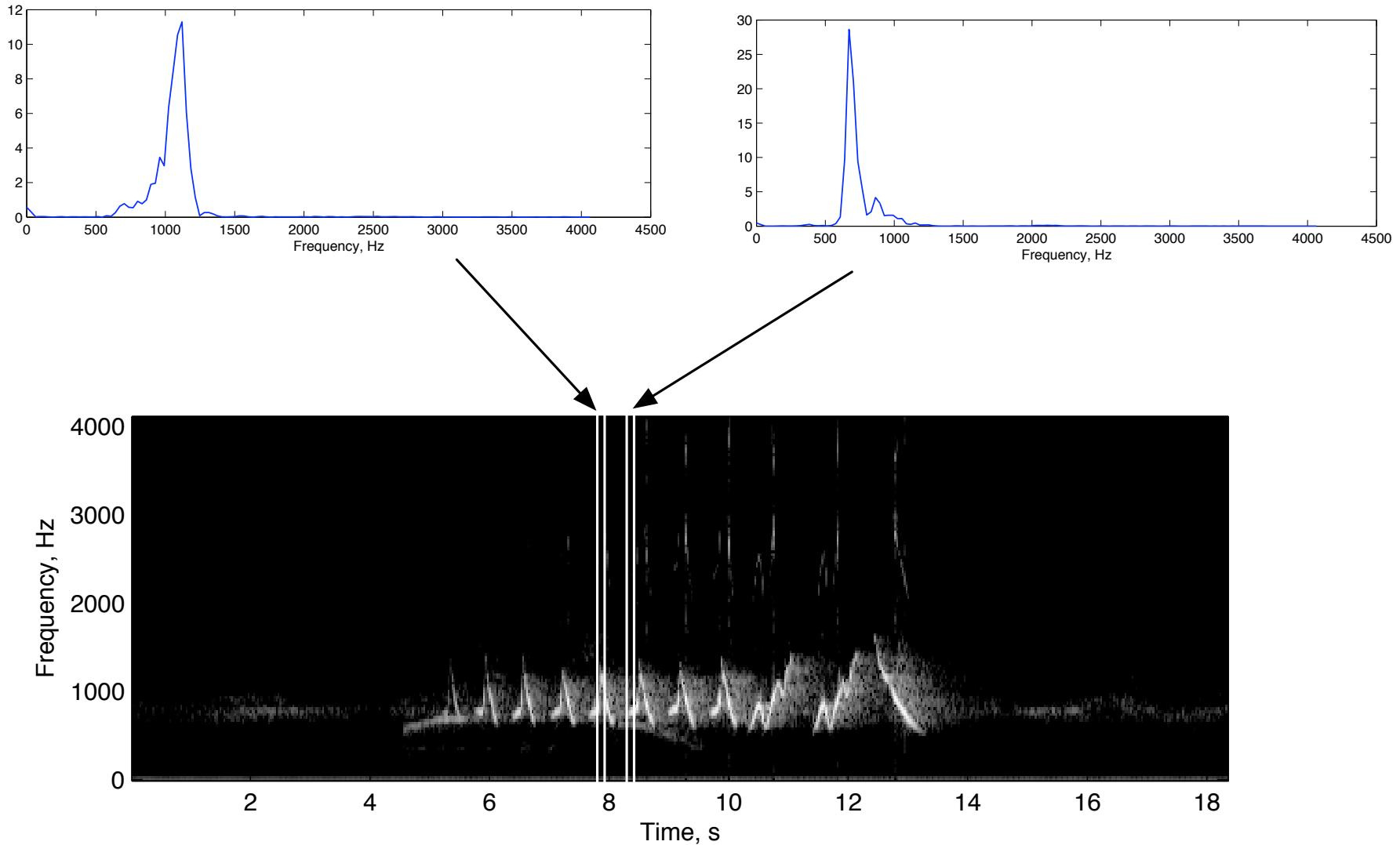
- To get temporal information, use part of the signal around every time point

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*Also called Short-time Fourier Transform (STFT)



Spectrogram



Discrete Time Dependent FT

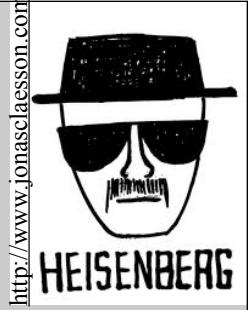
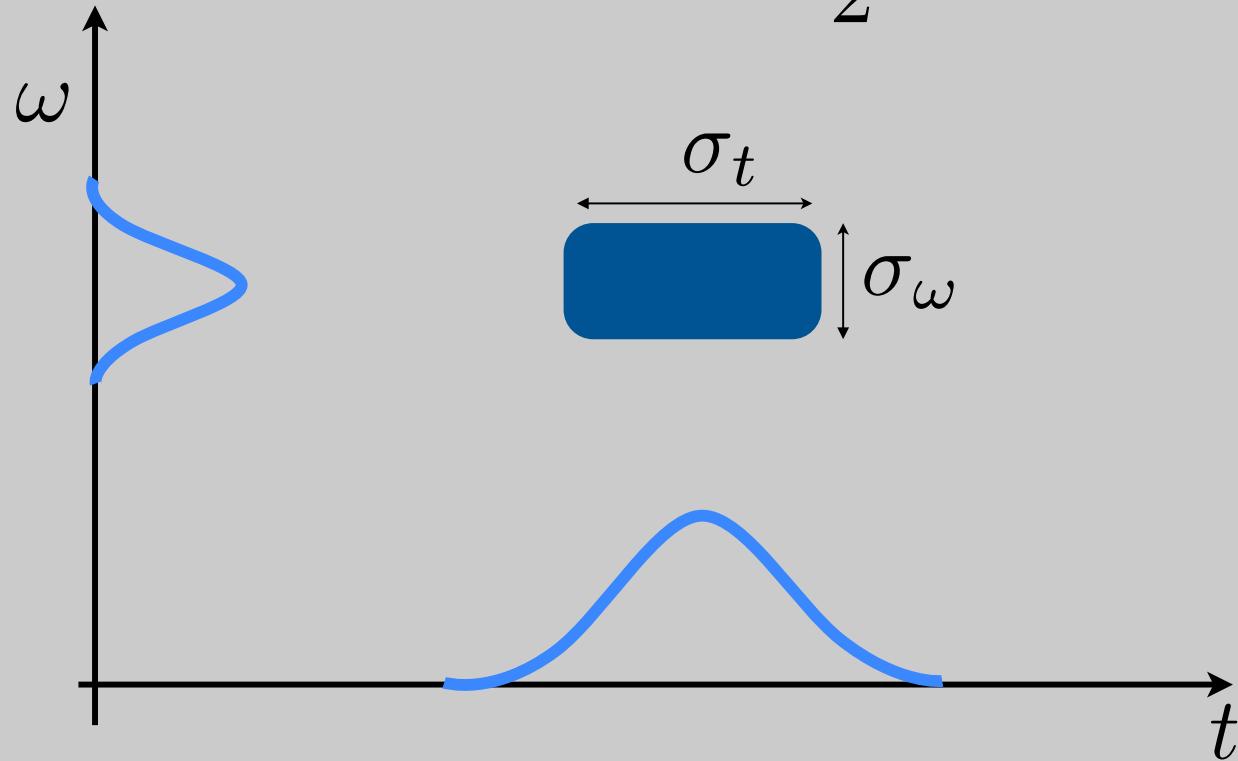
$$X_r[k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N}$$

- L - Window length
- R - Jump of samples
- N - DFT length
- Tradeoff between time and frequency resolution

Heisenberg Boxes

- Time-Frequency uncertainty principle

$$\sigma_t \cdot \sigma_\omega \geq \frac{1}{2}$$



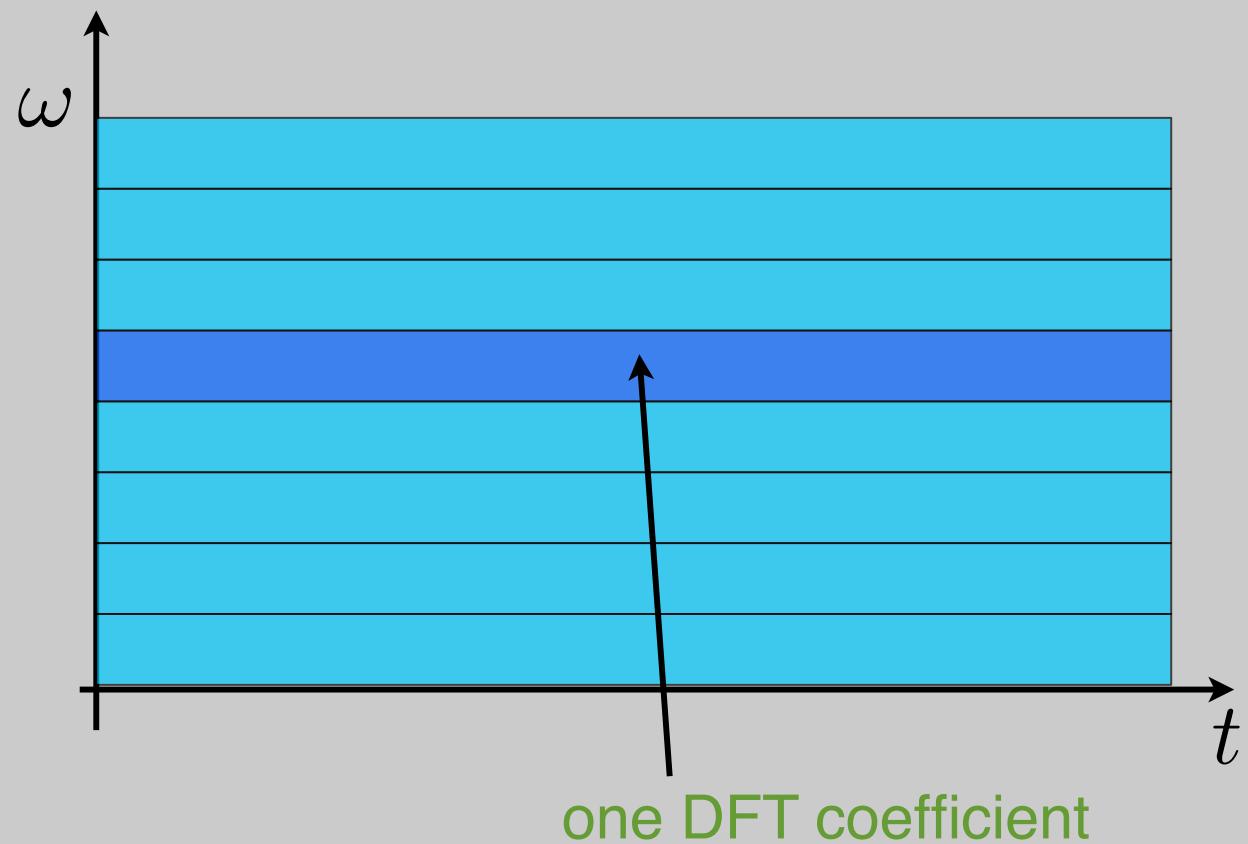
DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$

$$\Delta\omega = \frac{2\pi}{N}$$

$$\Delta t = N$$

$$\Delta\omega \cdot \Delta t = 2\pi$$



DFT

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$

$$\Delta\omega = \frac{2\pi}{N}$$

$$\Delta t = N$$

$$\Delta\omega \cdot \Delta t = 2\pi$$



Question: What is the effect of zero-padding?

Answer: Overlapped Tiling!

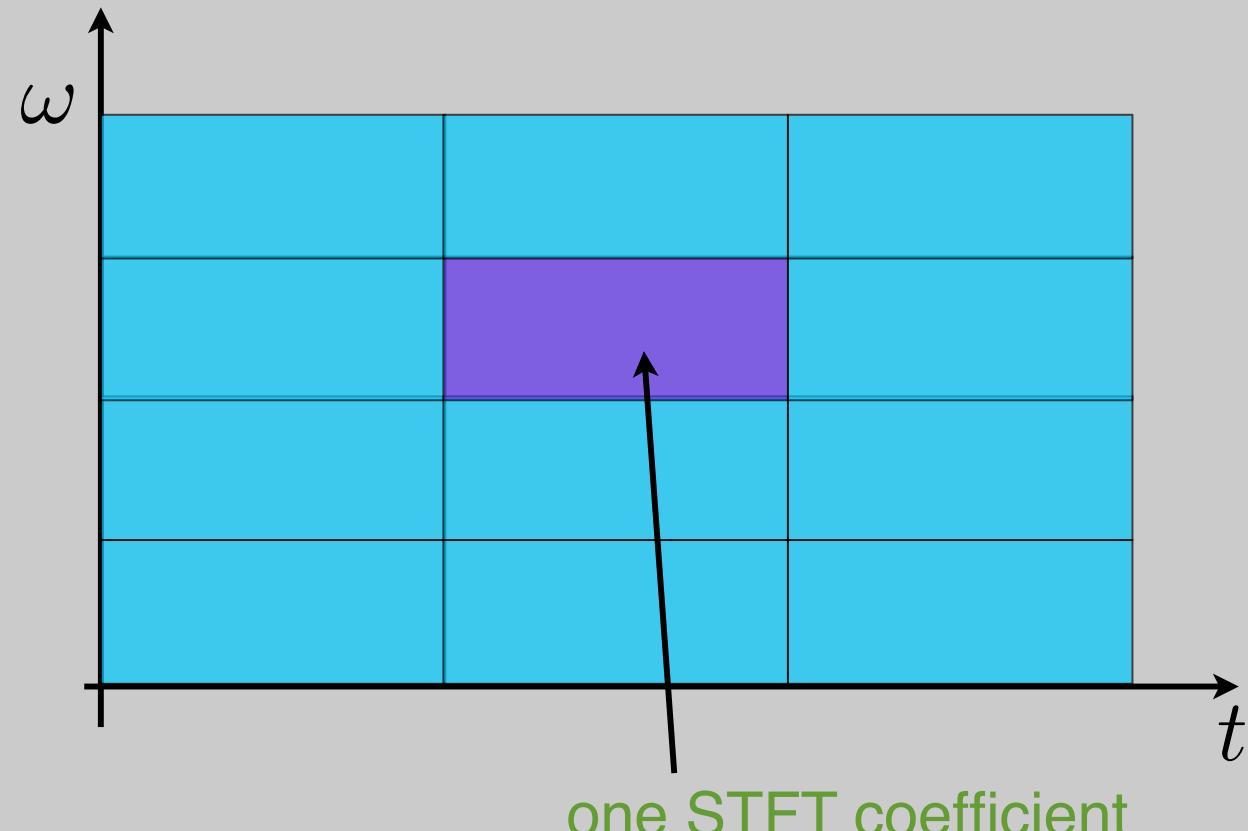
Discrete STFT

$$X[r, k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N}$$

optional
↓

$$\Delta\omega = \frac{2\pi}{L}$$

$$\Delta t = L$$

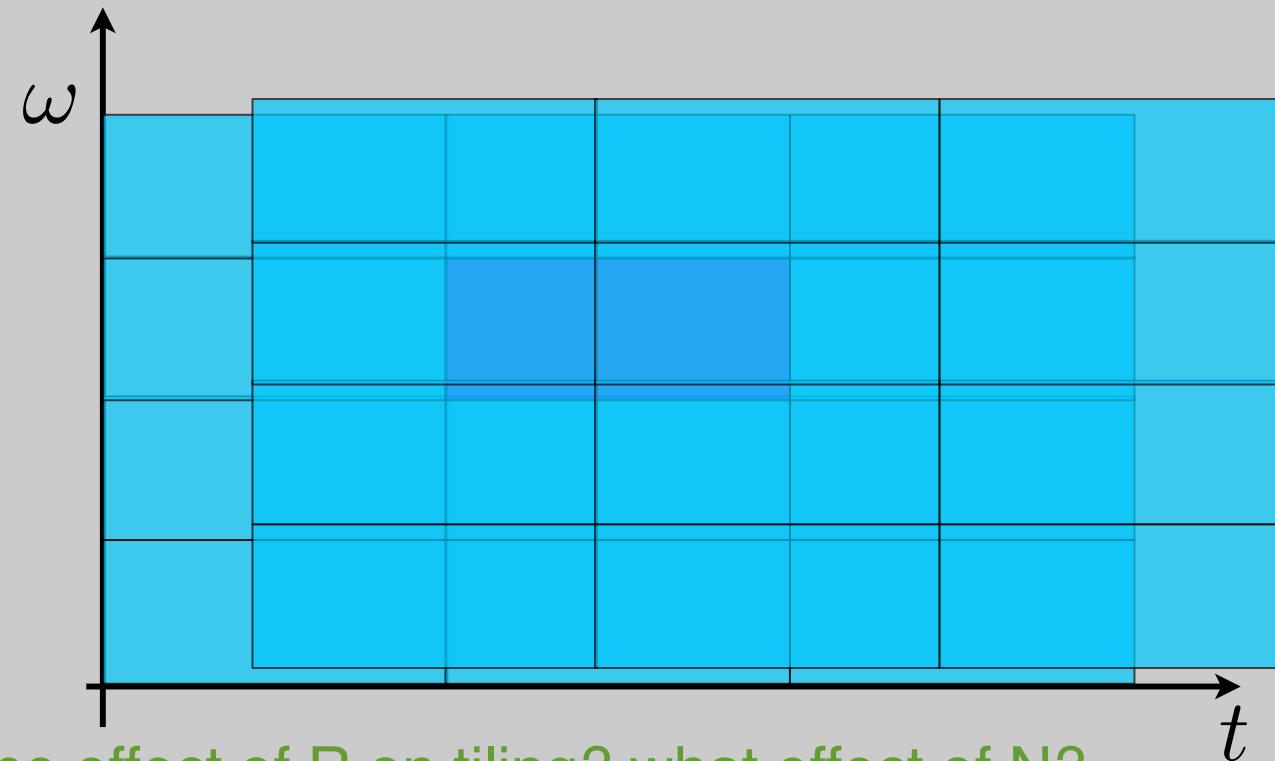


Discrete STFT

$$X[r, k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N}$$

$$\Delta\omega = \frac{2\pi}{L}$$

$$\Delta t = L$$



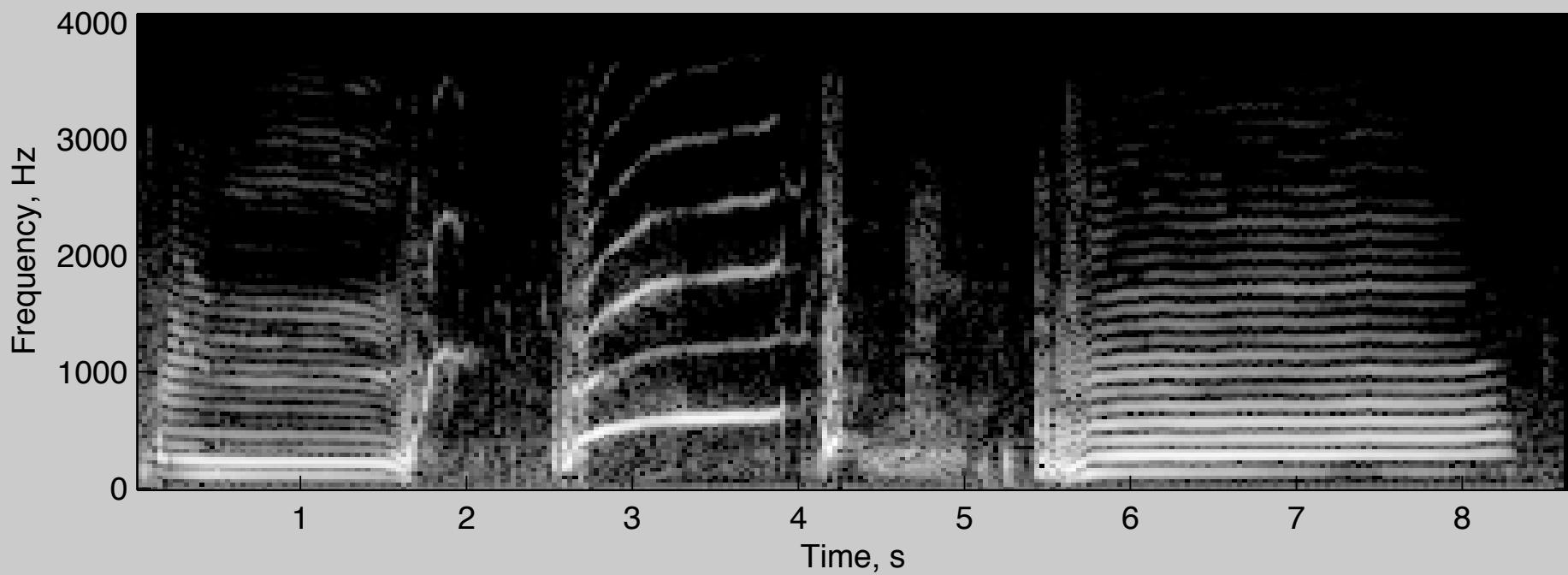
Question: What is the effect of R on tiling? what effect of N?

Answer: Overlapping in time or frequency or both!

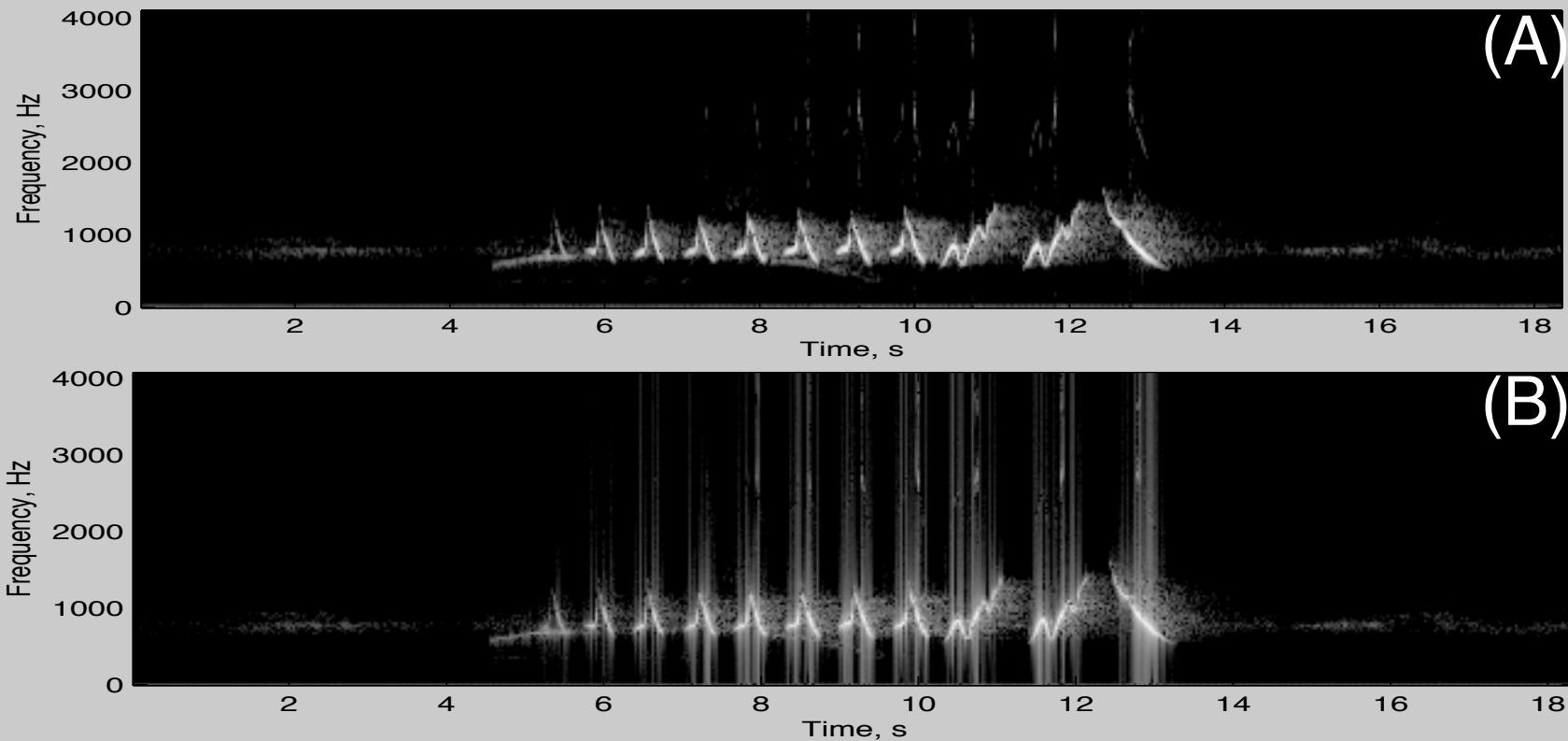
Applications

- Time Frequency Analysis

Spectrogram of Orca whale

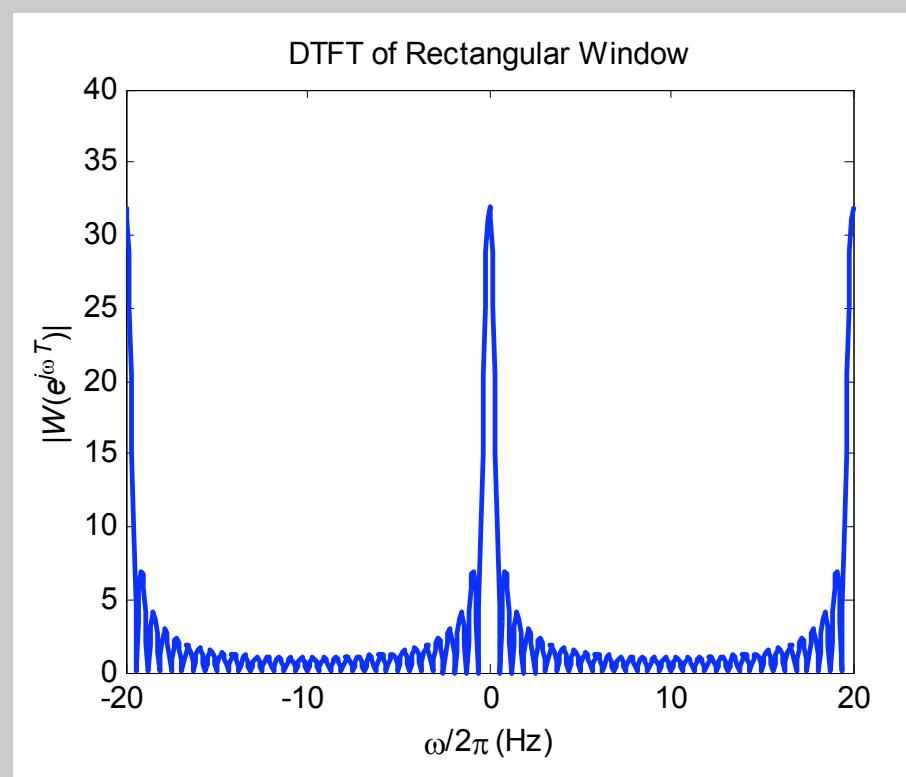
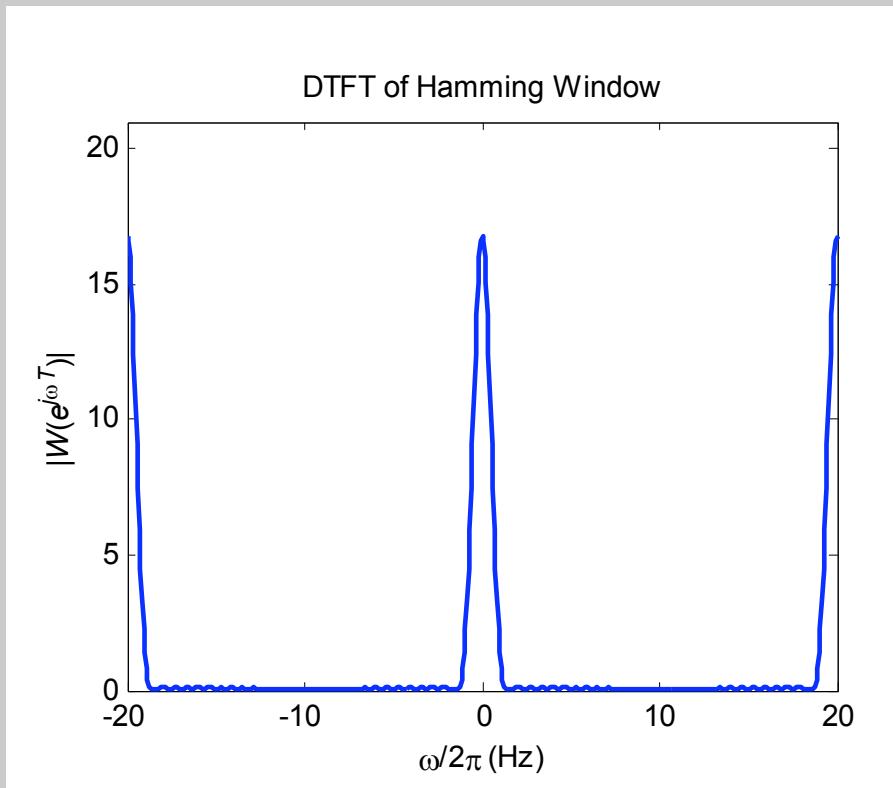


Spectrogram

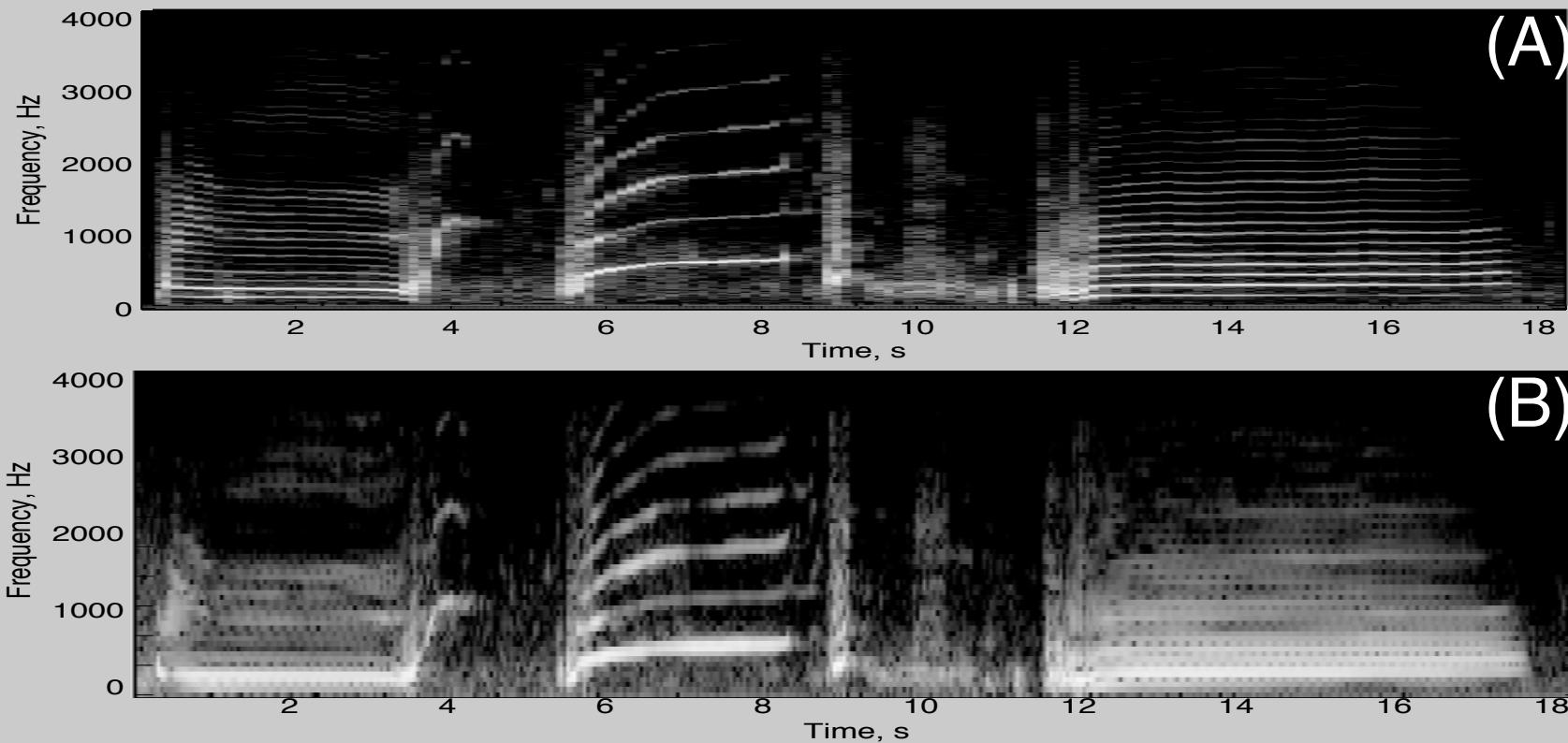


- What is the difference between the spectrograms?
 - a) Window size $B < A$
 - b) Window size $B > A$
 - c) Window type is different
 - d) (A) uses overlapping window

Sidelobes of Hann vs rectangular window



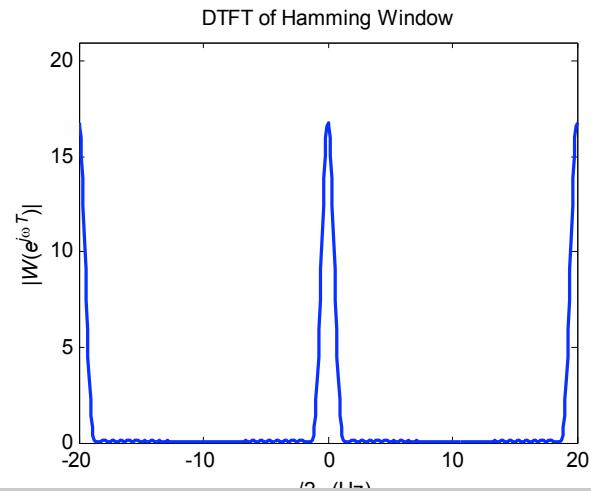
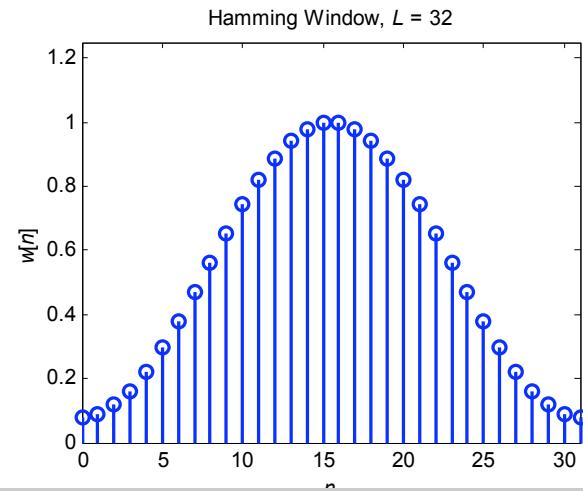
Spectrogram



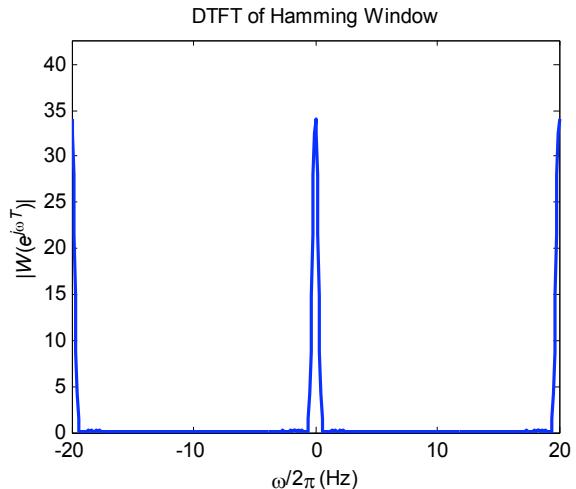
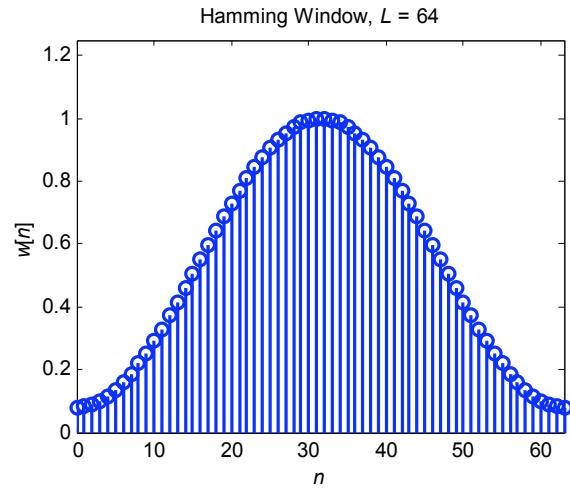
- What is the difference between the spectrograms?
 - a) Window size $B < A$
 - b) Window size $B > A$
 - c) Window type is different
 - d) (A) uses overlapping window

Spectrogram

Hamming Window, $L = 32$



Hamming Window, $L = 64$

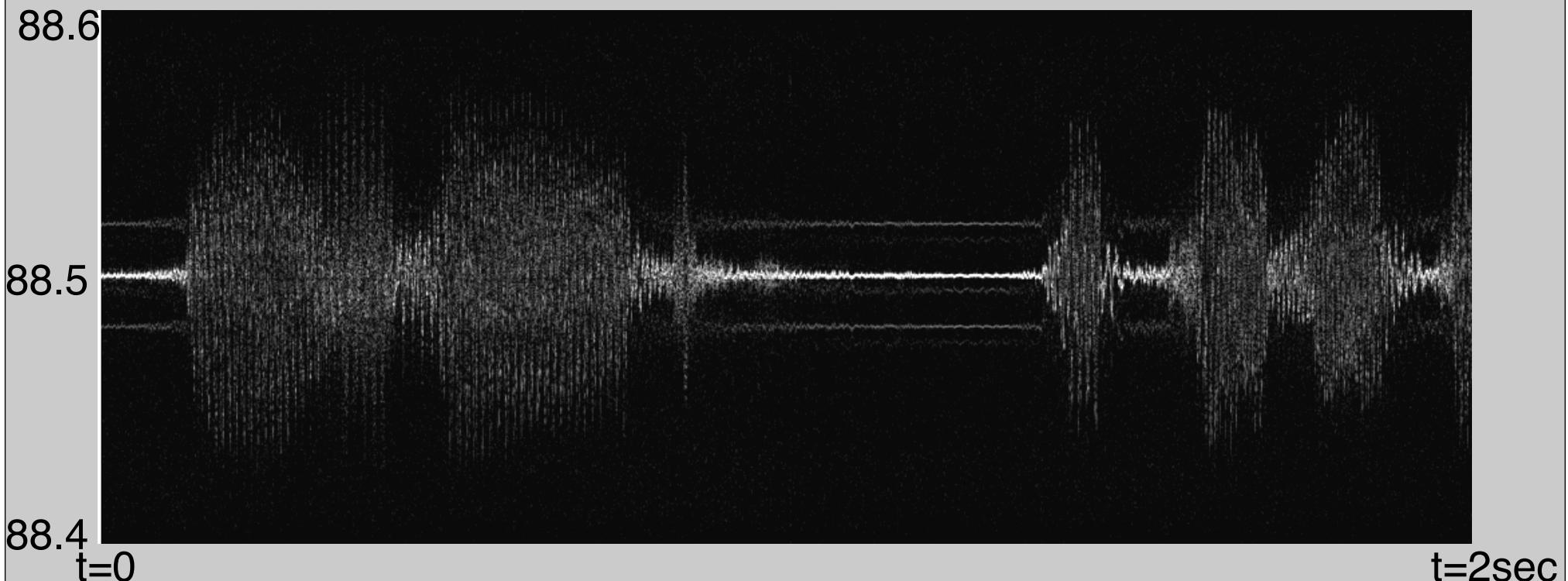


Spectrogram of FM

$$y_c(t) = A \cos \left(2\pi f_c t + 2\pi \Delta f \int_0^t x(\tau) d\tau \right)$$

$$y[n] = y(nT) = A \exp \left(j2\pi \Delta f \int_0^{nT} x(\tau) d\tau \right)$$

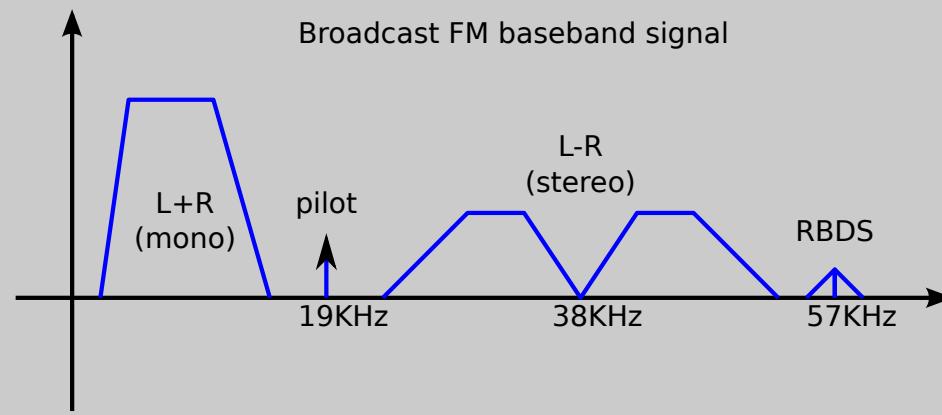
Spectrogram of FM radio



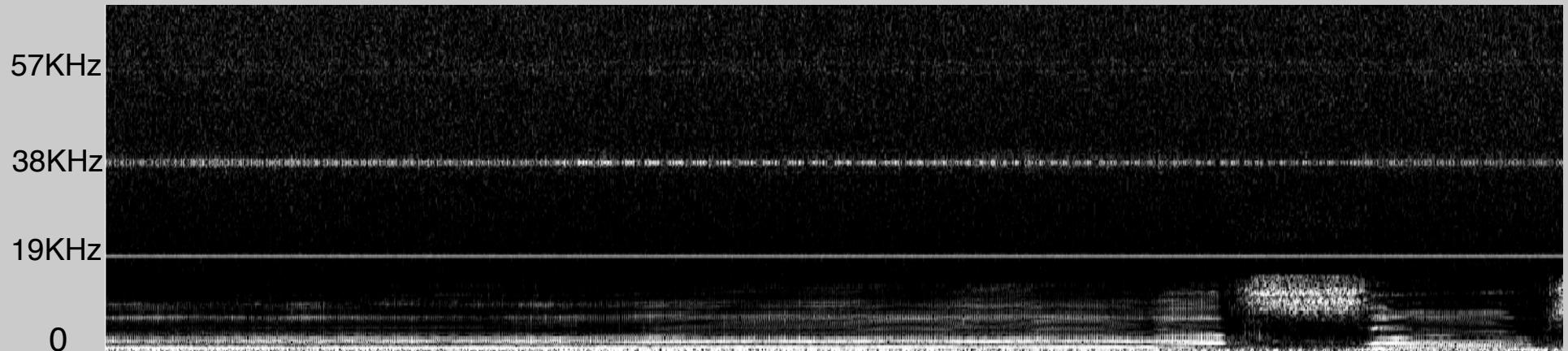
Spectrogram of FM radio Baseband

$$y[n] = y(nT) = A \exp \left(j2\pi\Delta f \int_0^{nT} x(\tau) d\tau \right)$$

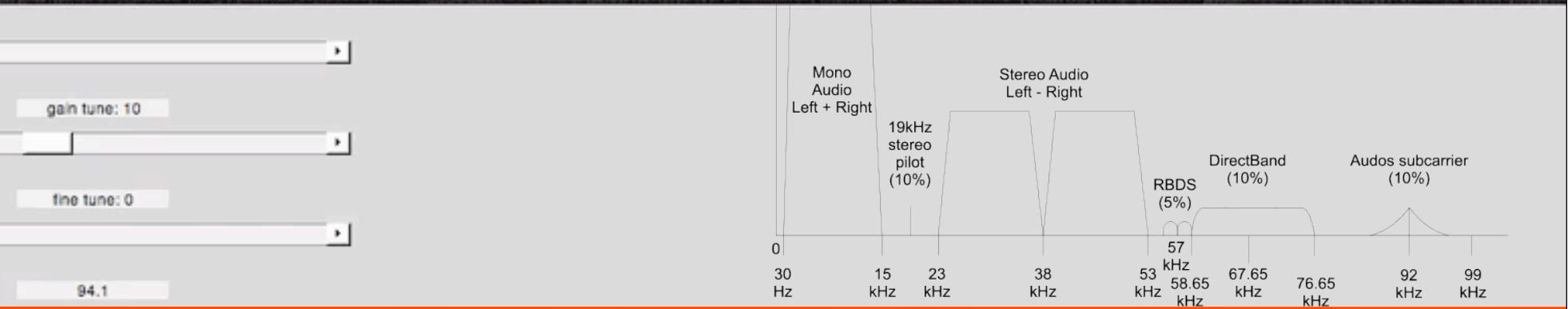
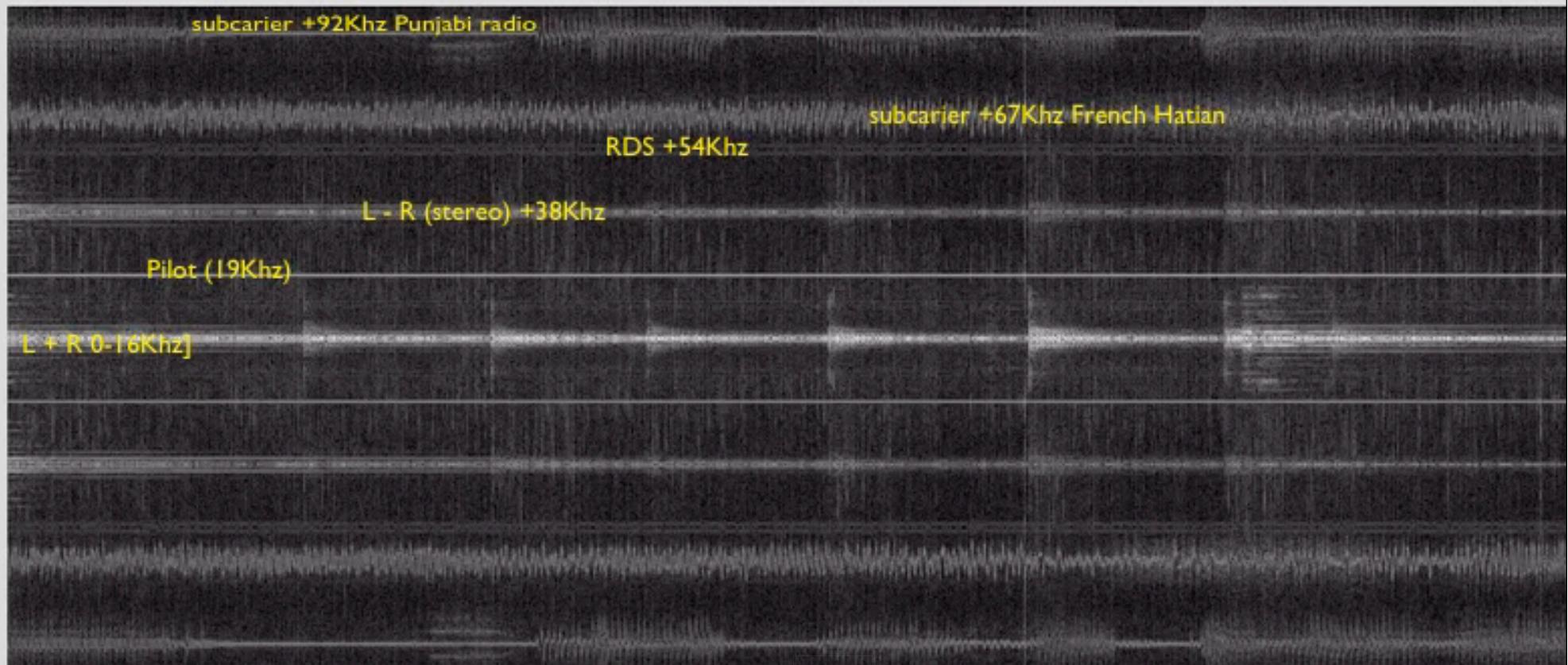
$$x(t) = \underbrace{(L + R)}_{\text{mono}} + \underbrace{0.1 \cdot \cos(2\pi f_p t)}_{\text{pilot}} + \underbrace{(L - R) \cos(2\pi(2f_p)t)}_{\text{stereo}} + \underbrace{0.05 \cdot \text{RBDS}(t) \cos(2\pi(3f_p)t)}_{\text{digital RBDS}}.$$



Spectrogram of Demodulated FM radio (Adele on 96.5 MHz)



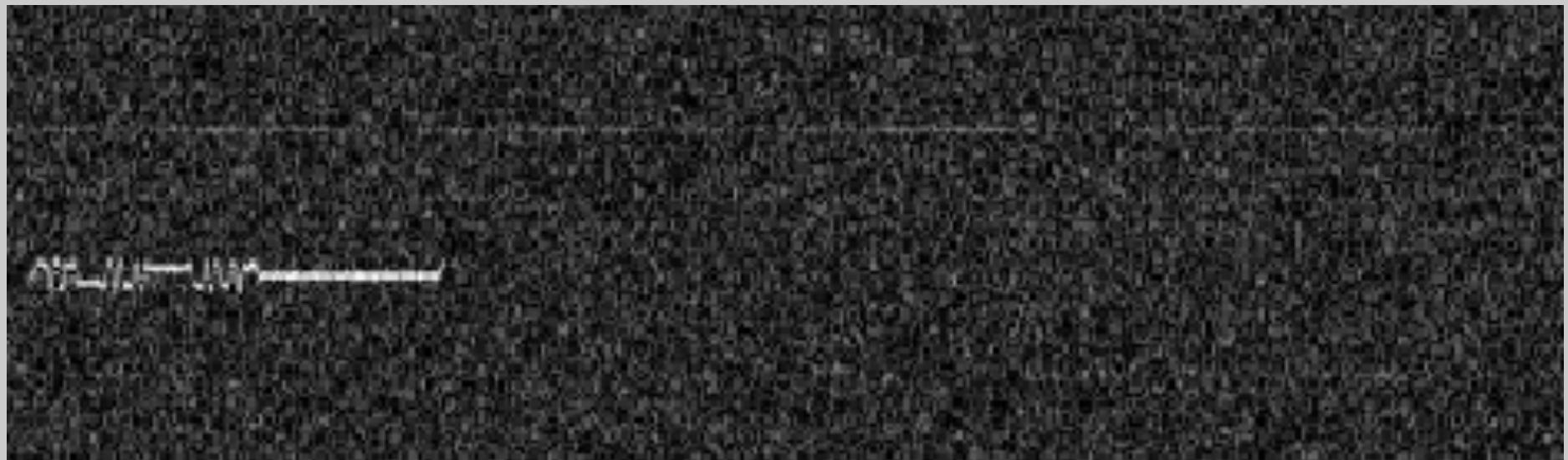
Subcarrier FM radio (Hidden Radio Stations)



Applications

- Time Frequency Analysis

Spectrogram of digital communications - Frequency Shift Keying



t=0

t=1sec

STFT Reconstruction

$$x[rR + m]w_L[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[n, k] e^{j2\pi km/N}$$

- For non-overlapping windows, R=L :

$$x[n] = \frac{x[n - rL]}{w_L[n - rL]}$$

$$rL \leq n \leq (r + 1)R - 1$$

- What is the problem?

STFT Reconstruction

$$x[rR + m]w_L[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[n, k] e^{j2\pi km/N}$$

- For non-overlapping windows, R=L :

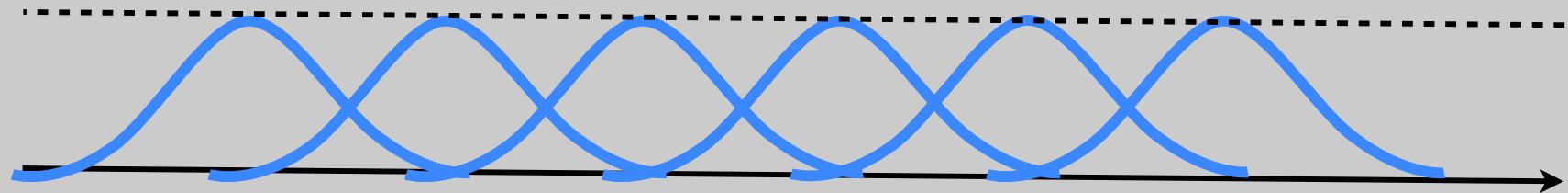
$$x[n] = \frac{x[n - rL]}{w_L[n - rL]}$$

$$rL \leq n \leq (r + 1)R - 1$$

- For stable reconstruction must overlap window 50% (at least)

STFT Reconstruction

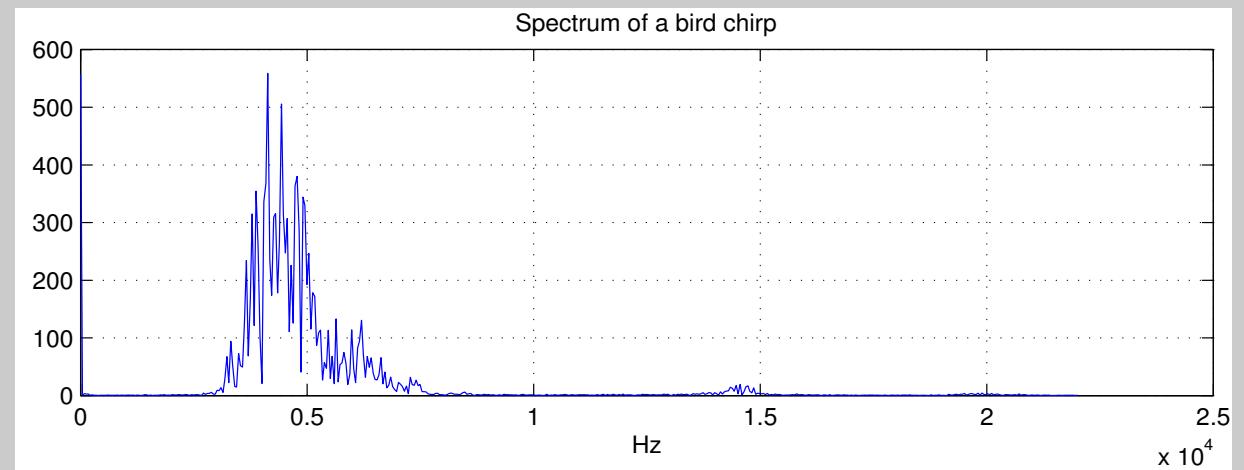
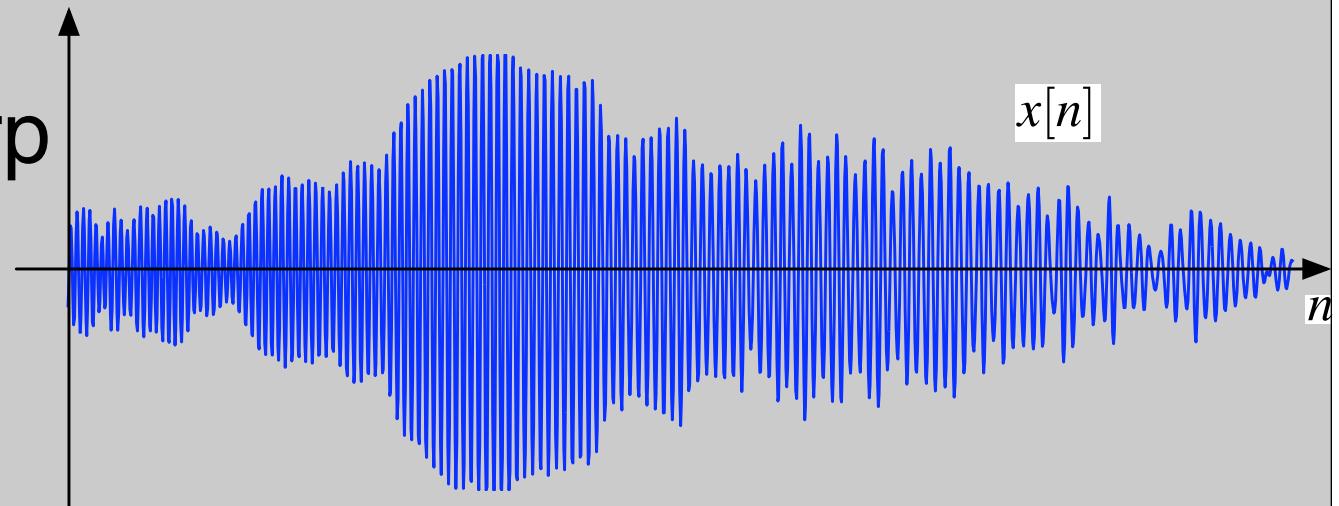
- For stable reconstruction must overlap window 50% (at least)
- For Hann, Bartlett reconstruct with overlap and add. No division!



Applications

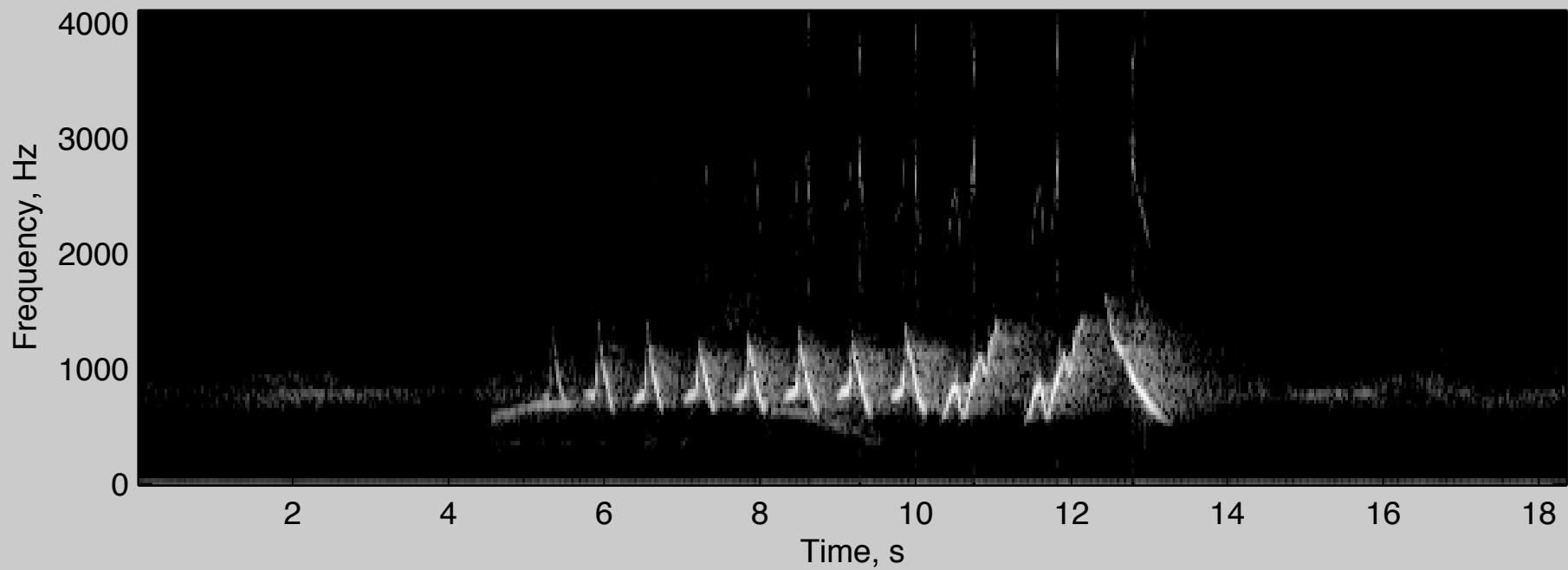
- Noise removal

- Recall bird chirp



Application

- Denoising of Sparse spectrograms



- Spectrum is sparse! can implement adaptive filter, or just threshold!

Limitations of Discrete STFT

- Need overlapping \Rightarrow Not orthogonal
- Computationally intensive $O(MN \log N)$
- Same size Heisenberg boxes

From STFT to Wavelets

- Basic Idea:
 - low-freq changes slowly - fast tracking unimportant
 - Fast tracking of high-freq is important in many apps.
 - Must adapt Heisenberg box to frequency
- Back to continuous time for a bit.....