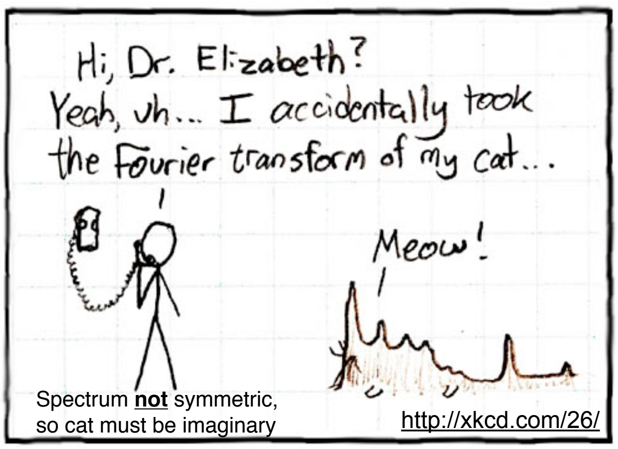
EE123
Digital Signal Processing

Lecture 10

Announcements

- Midterm: Friday next week
  - Open everything
  - ... but cheat sheet recommended instead
  - Who can not stay till 5pm?
- Optional homework next week
  - Will give you midterm and practice questions
- How's lab I going?

How do you know this guy is insane?



Spectrum **not** symmetric,
so cat must be imaginary

http://xkcd.com/26/

Last Time

- Frequency Analysis with DFT
- Windowing
- Zero-Padding
- Today:
  - Time-Dependent Fourier Transform
  - Heisenberg Boxes

Discrete Transforms (Finite)

- DFT is only one out of a LARGE class of transforms
- Used for:
  - Analysis
  - Compression
  - Denoising
  - Detection
  - Recognition
  - Approximation (Sparse)

Sparse representation has been one of the hottest research topics in the last 15 years in sp

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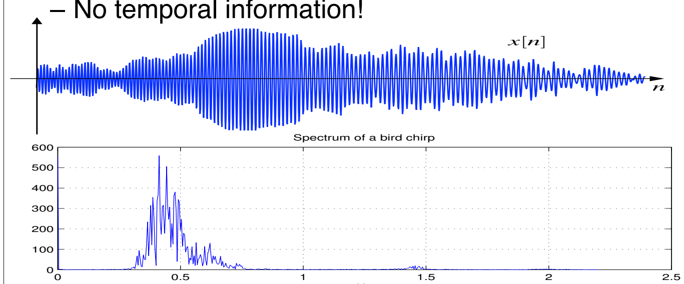
Example of spectral analysis

- Spectrum of a bird chirping
  - Interesting,.... but...
  - Does not tell the whole story
  - No temporal information!



Time Dependent Fourier Transform

- To get temporal information, use part of the signal around every time point

$$X[n,\omega) \;=\; \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-j\omega m}$$

*Also called Short-time Fourier Transform (STFT)

- Mapping from 1D ⇒ 2D, n discrete, *w* cont.
- Simply slide a window and compute DTFT

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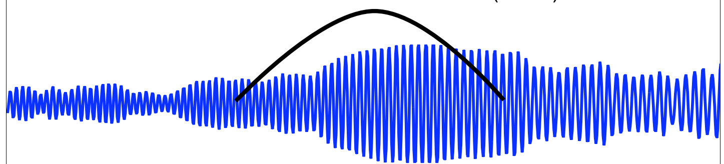
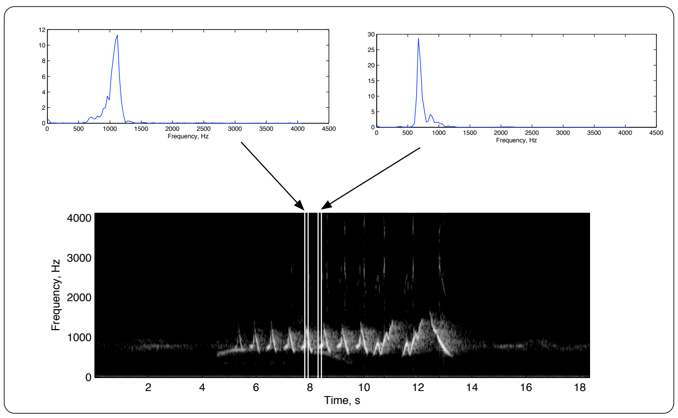


Spectrogram



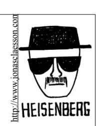
Discrete Time Dependent FT

$$X_r[k] = \sum_{m=0}^{L-1} x[rR+m]w[m]e^{-j2\pi km/N}$$

- L - Window length
- R - Jump of samples
- N - DFT length

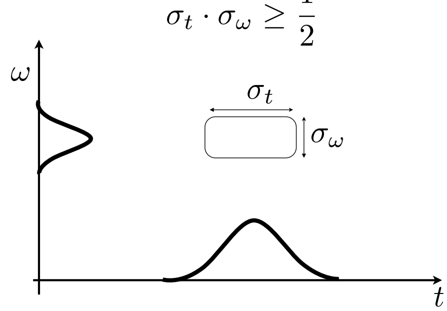
- Tradeoff between time and frequency resolution

Heisenberg Boxes



- Time-Frequency uncertainty principle

$$\sigma_t \cdot \sigma_\omega \geq \frac{1}{2}$$



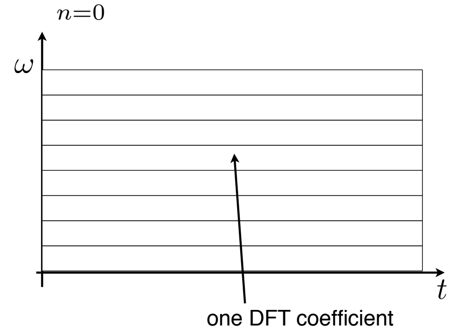
DFT

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$

$$\Delta\omega = \frac{2\pi}{N}$$

$$\Delta t = N$$

$$\Delta\omega \cdot \Delta t = 2\pi$$



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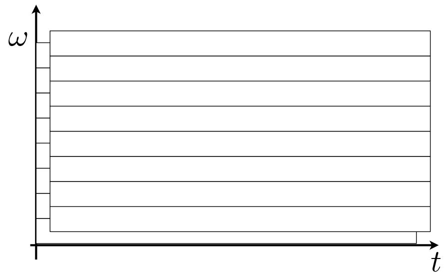


Question: What is the effect of zero-padding?
Answer: Overlapped Tiling!

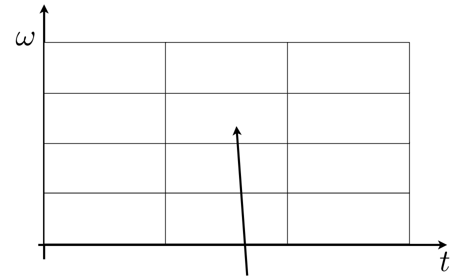
Discrete STFT

optional

$$X[r,k] = \sum_{m=0}^{L-1} x[r{\color{green}R} + m] w[m] e^{-j2\pi km/N}$$

$\Delta\omega = \frac{2\pi}{L}$

$\Delta t = L$



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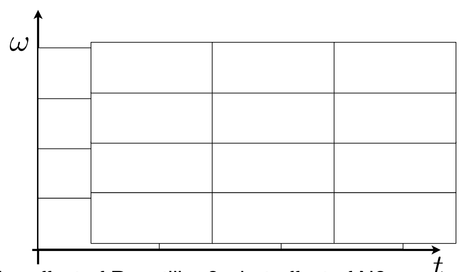
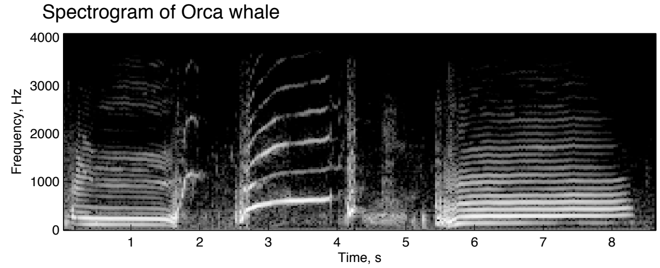


Question: What is the effect of R on tiling? what effect of N?
Answer: Overlapping in time of frequency or both!

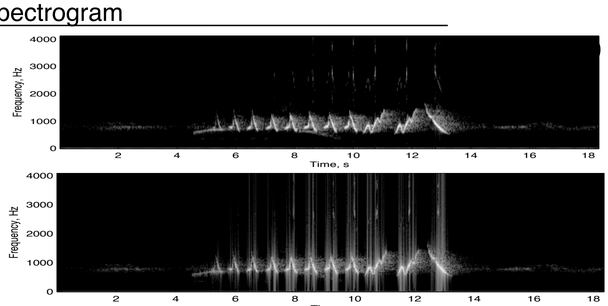
Applications

- Time Frequency Analysis

Spectrogram of Orca whale



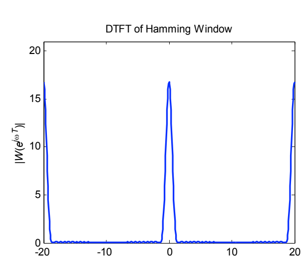
Spectrogram

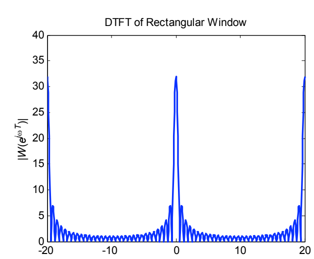


- What is the difference between the spectrograms?

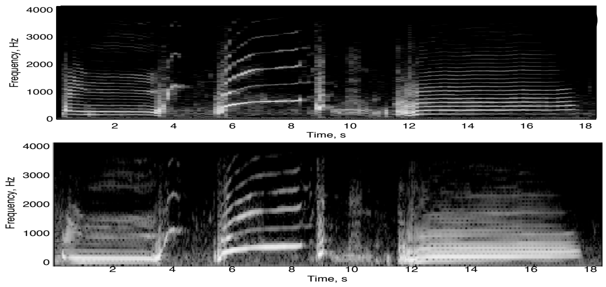
a) Window size B<A
b) Window size B>A
c) Window type is different
d) (A) uses overlapping window

Sidelobes of Hann vs rectangular window





Spectrogram



- What is the difference between the spectrograms?

a) Window size B<A

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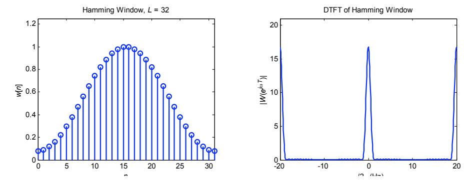
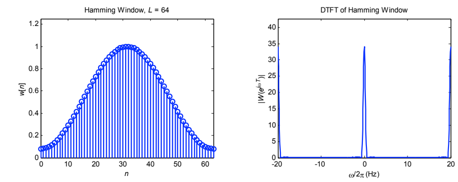
Spectrogram

Hamming Window, $L = 32$



Hamming Window, $L = 64$

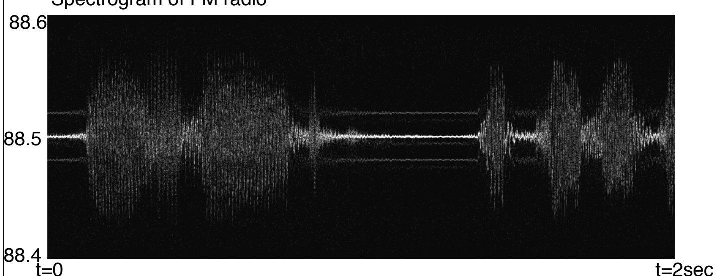


Spectrogram of FM

$$y_c(t) = A\cos\left(2\pi f_c t + 2\pi\Delta f\int_0^t x(\tau)d\tau\right)$$

$$y[n] = y(nT) = A\exp\left(j2\pi\Delta f\int_0^{nT} x(\tau)d\tau\right)$$

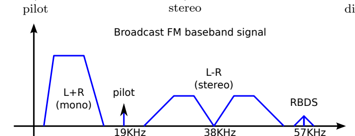
Spectrogram of FM radio



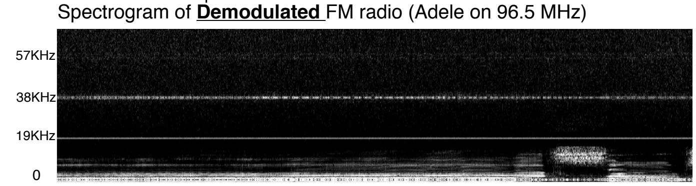
Spectrogram of FM radio Baseband

$$y[n] = y(nT) = A\exp\left(j2\pi\Delta f\int_0^{nT} x(\tau)d\tau\right)$$

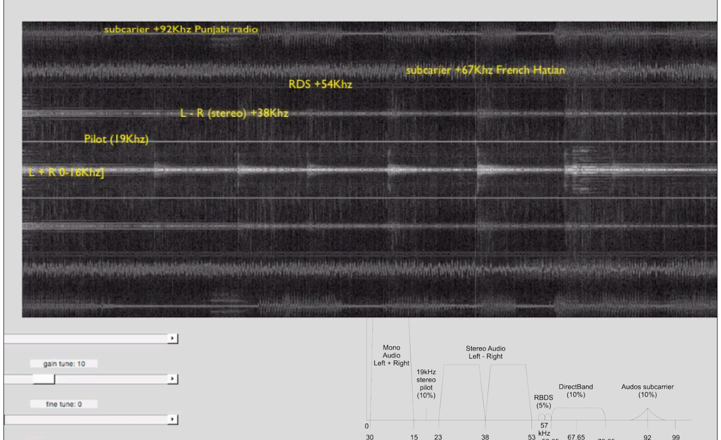
$$x(t) = \underbrace{(L+R)}_{\text{mono}} + \underbrace{0.1\cdot\cos(2\pi f_p t)}_{\text{pilot}} + \underbrace{(L-R)\cos(2\pi(2f_p)t)}_{\text{stereo}} + \underbrace{0.05\cdot\mathrm{RBDS}(t)\cos(2\pi(3f_p)t)}_{\text{digital RBDS}}.$$



Spectrogram of **Demodulated** FM radio (Adele on 96.5 MHz)



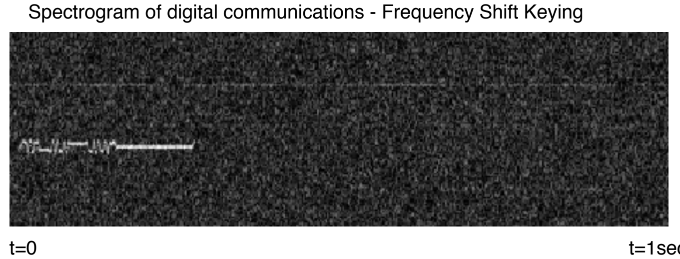
Subcarrier FM radio (Hidden Radio Stations)



Applications

- Time Frequency Analysis

Spectrogram of digital communications - Frequency Shift Keying



STFT Reconstruction

$$x[rR+m]w_L[m] = \frac{1}{N}\sum_{k=0}^{N-1} X[n,k]e^{j2\pi km/N}$$

- For non-overlapping windows, R=L :

$$x[n] = \frac{x[n-rL]}{w_L[n-rL]}$$

$$rL \le n \le (r+1)R-1$$

- What is the problem?

STFT Reconstruction

$$x[rR+m]w_L[m] = \frac{1}{N}\sum_{k=0}^{N-1} X[n,k]e^{j2\pi km/N}$$

- For non-overlapping windows, R=L :

$$x[n] = \frac{x[n-rL]}{w_L[n-rL]}$$

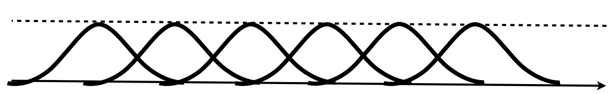
$$rL \le n \le (r+1)R-1$$

- For stable reconstruction must overlap window 50% (at least)

STFT Reconstruction

- For stable reconstruction must overlap window 50% (at least)

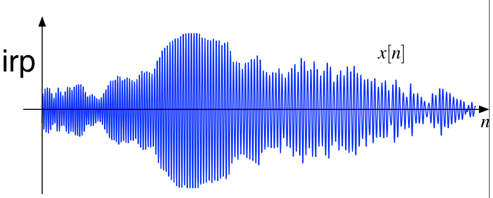
- For Hann, Bartlett reconstruct with overlap and add. No division!

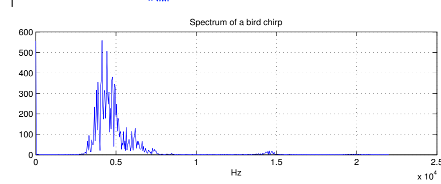


Applications

- Noise removal

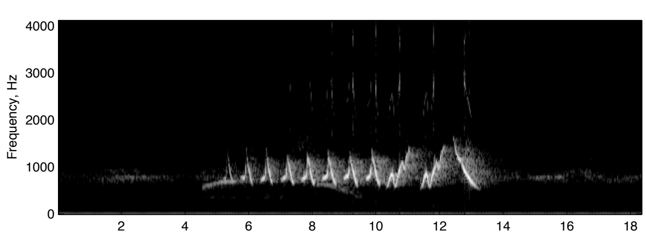
- Recall bird chirp





Application

- Denoising of Sparse spectrograms



- Spectrum is sparse! can implement adaptive filter, or just threshold!

Limitations of Discrete STFT

- Need overlapping ⇒ Not orthogonal

- Computationally intensive O(MN log N)

- Same size Heisenberg boxes

From STFT to Wavelets

- Basic Idea:
  - low-freq changes slowly - fast tracking unimportant
  - Fast tracking of high-freq is important in many apps.
  - Must adapt Heisenberg box to frequency

- Back to continuous time for a bit.....