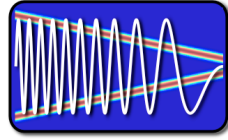


EE123



Digital Signal Processing

Lecture 13
DWT

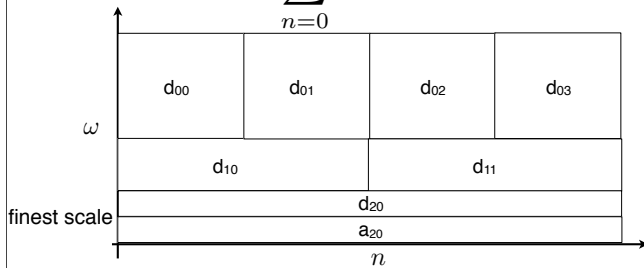
Back to Discrete

- Early 80's, theoretical work by Morlett, Grossman and Meyer (math, geophysics)
- Late 80's link to DSP by Daubechies and Mallat.
- From CWT to DWT not so trivial!
- Must take care to maintain properties

Discrete Wavelet Transform

$$d_{s,u} = \sum_{n=0}^{N-1} x[n] \Psi_{s,u}[n]$$

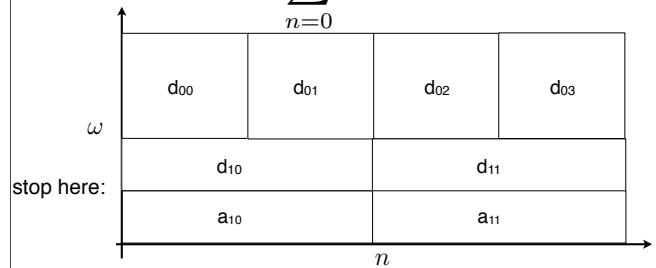
$$a_{s,u} = \sum_{n=0}^{N-1} x[n] \Phi_{s,u}[n]$$



Discrete Wavelet Transform

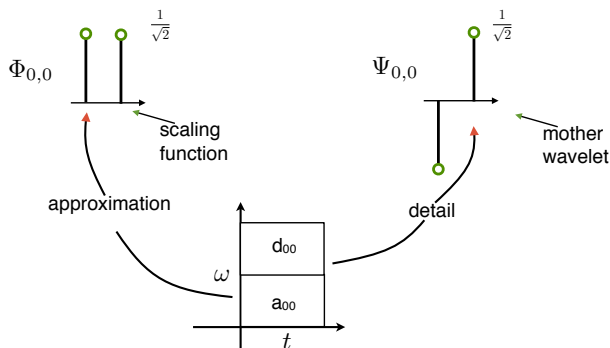
$$d_{s,u} = \sum_{n=0}^{N-1} x[n] \Psi_{s,u}[n]$$

$$a_{s,u} = \sum_{n=0}^{N-1} x[n] \Phi_{s,u}[n]$$



Example: Discrete Haar Wavelet

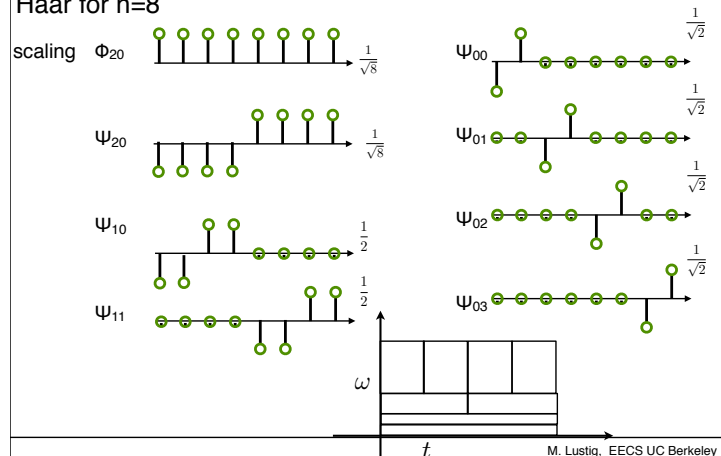
Haar for n=2



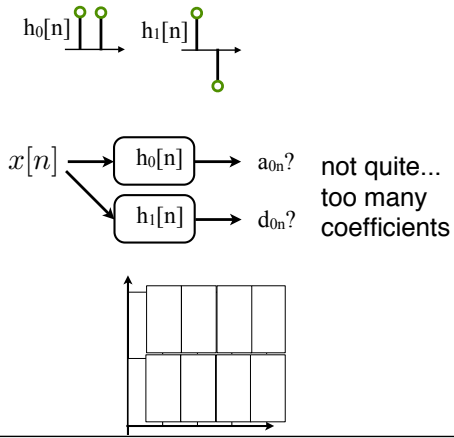
Equivalent to DFT₂!

Discrete Orthogonal Haar Wavelet

Haar for n=8

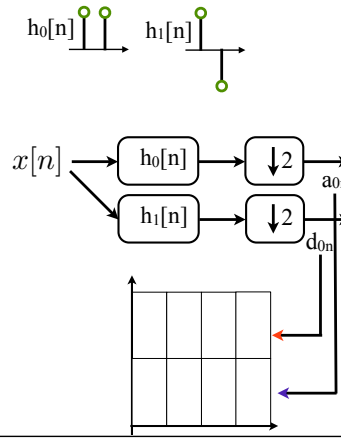


Fast DWT with Filter Banks (more Later!)



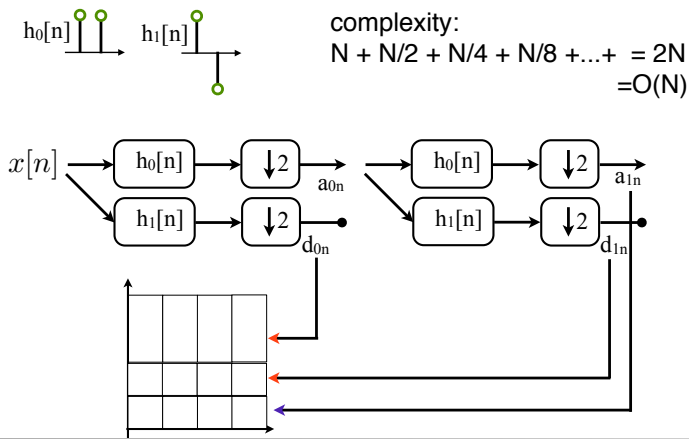
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Fast DWT with Filter Banks



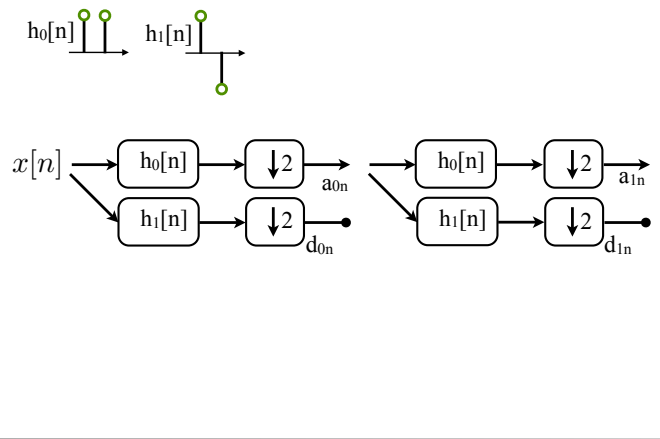
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Fast DWT with Filter Banks



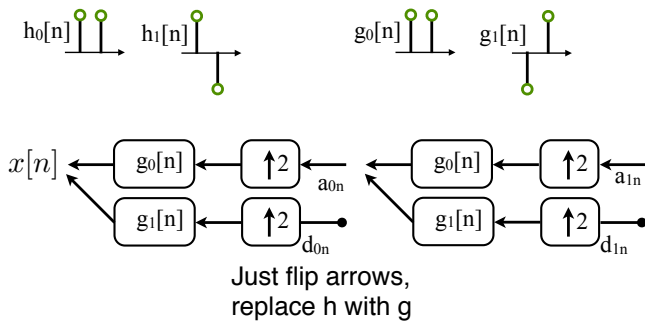
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Decomposition



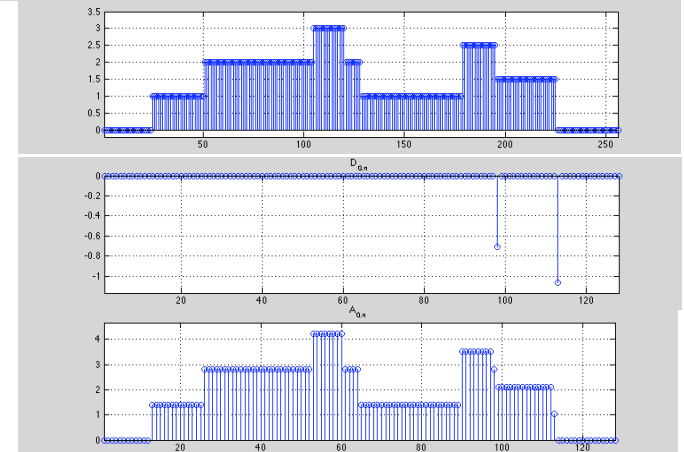
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Reconstruction



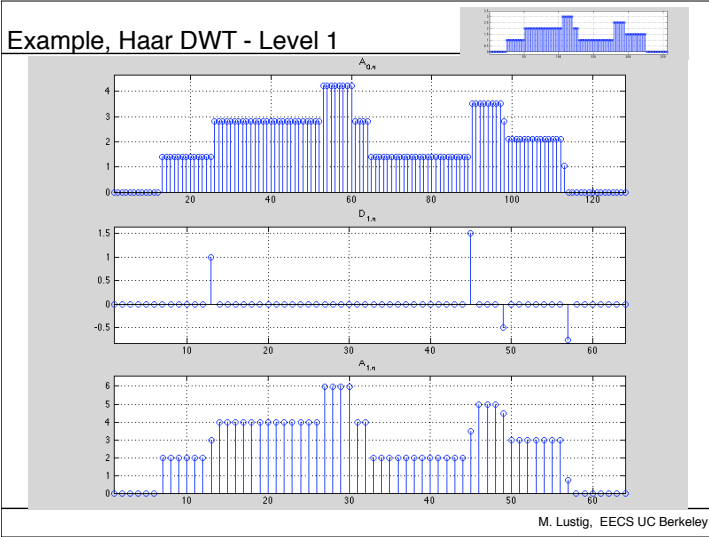
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Example, Haar DWT - Level 0

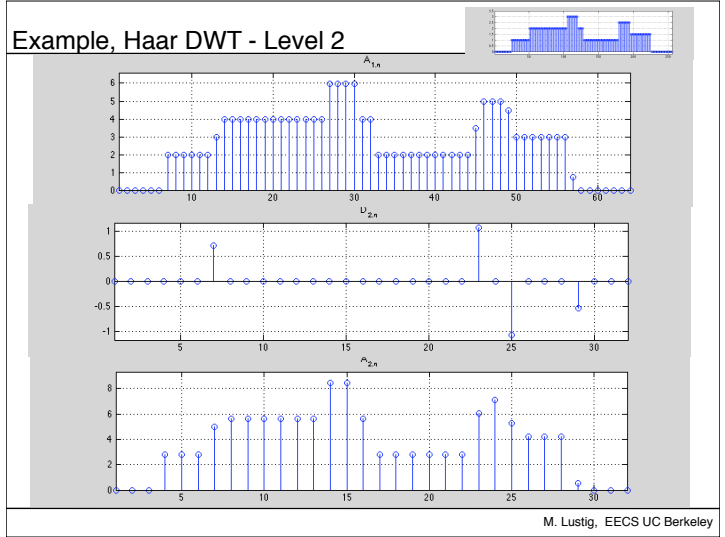


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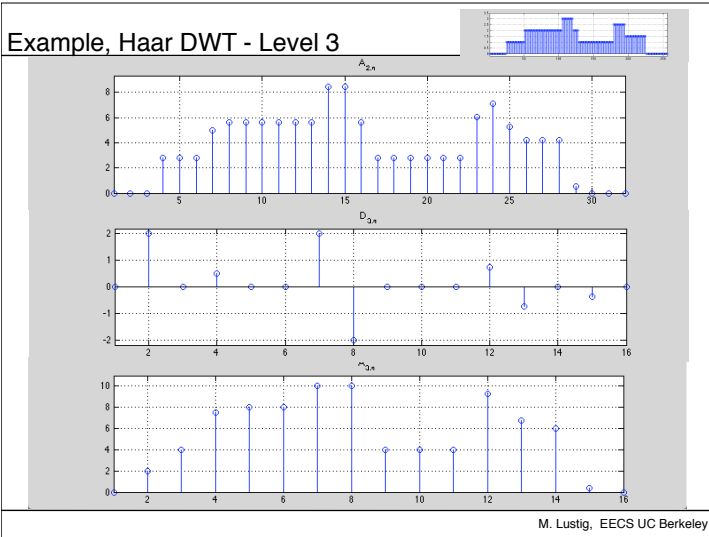
Example, Haar DWT - Level 1



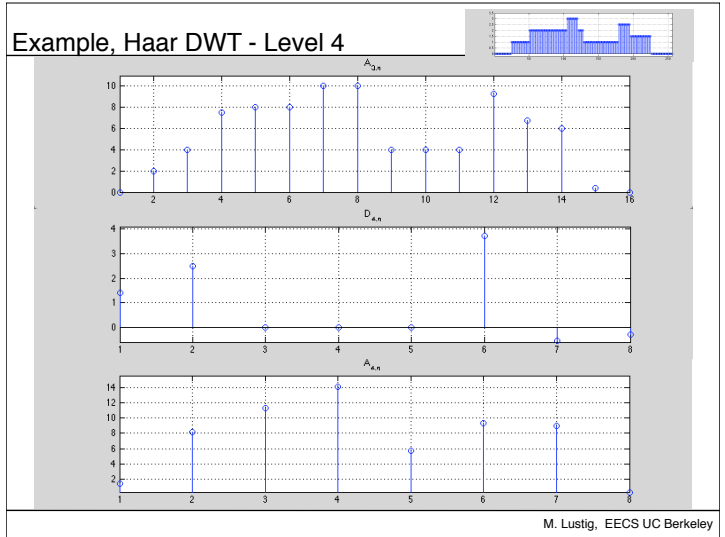
Example, Haar DWT - Level 2



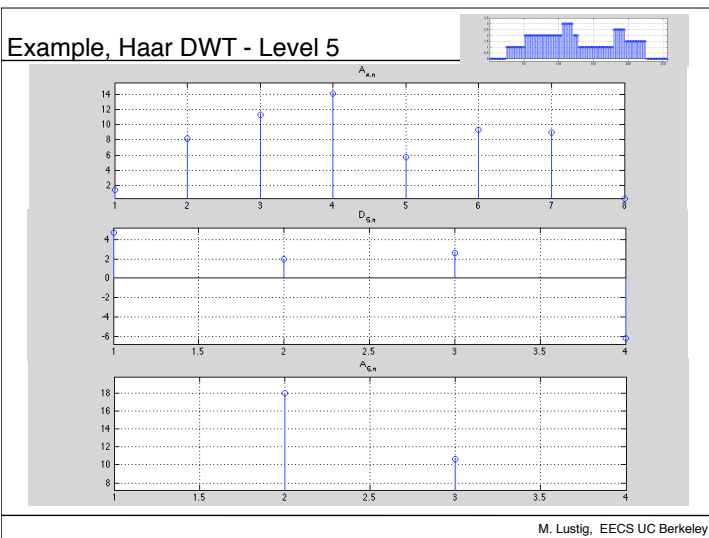
Example, Haar DWT - Level 3



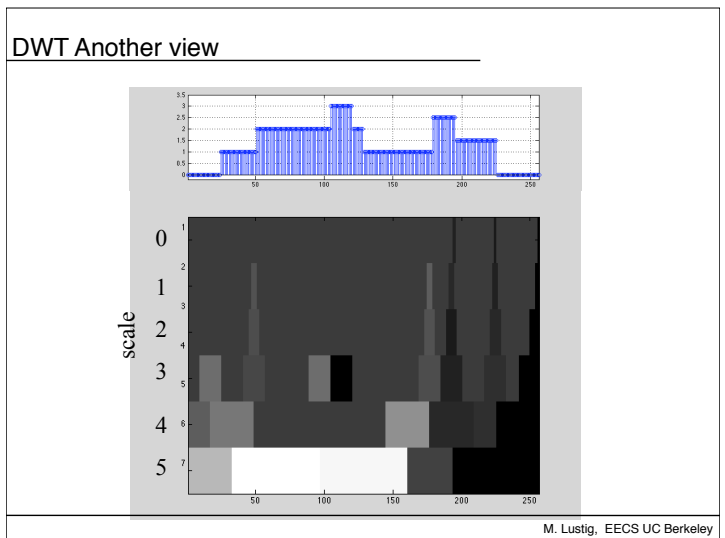
Example, Haar DWT - Level 4



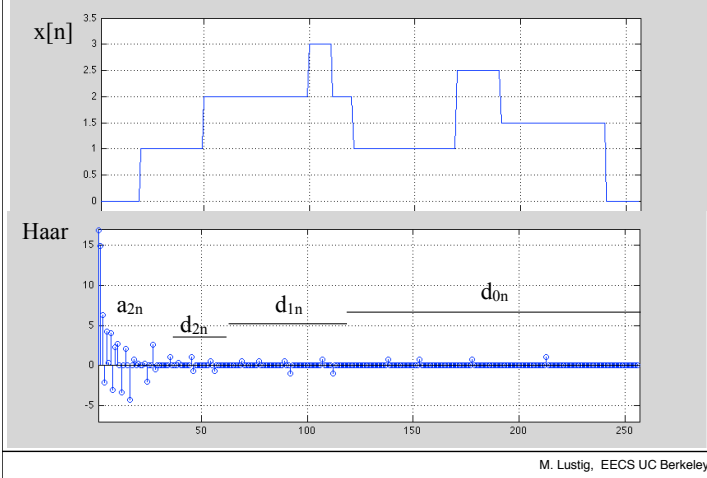
Example, Haar DWT - Level 5



DWT Another view

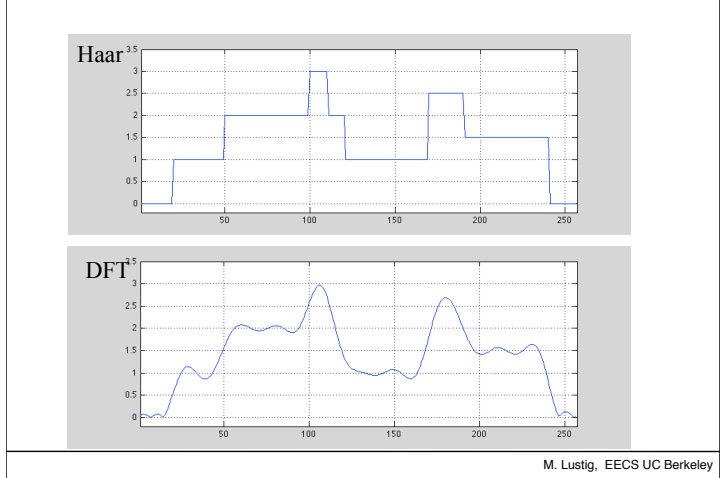


Haar DWT Example



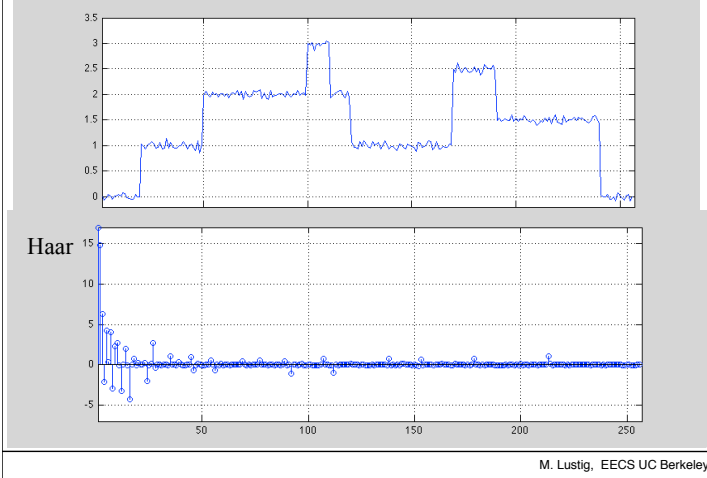
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Approximation from 25/256 coefficients



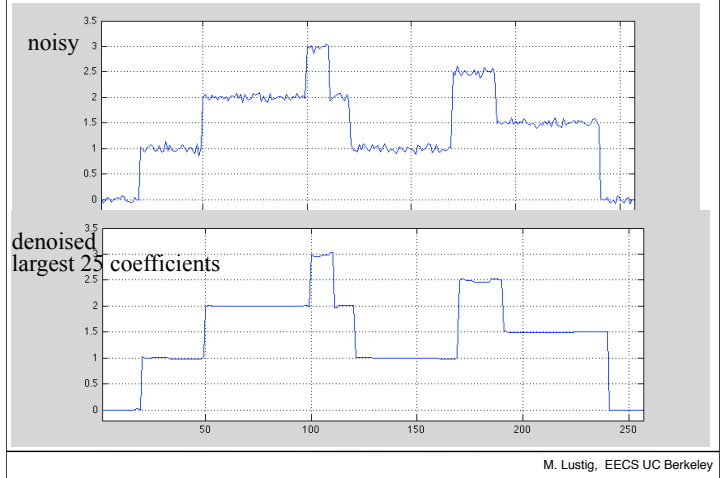
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Example: Denoising Noisy Signals



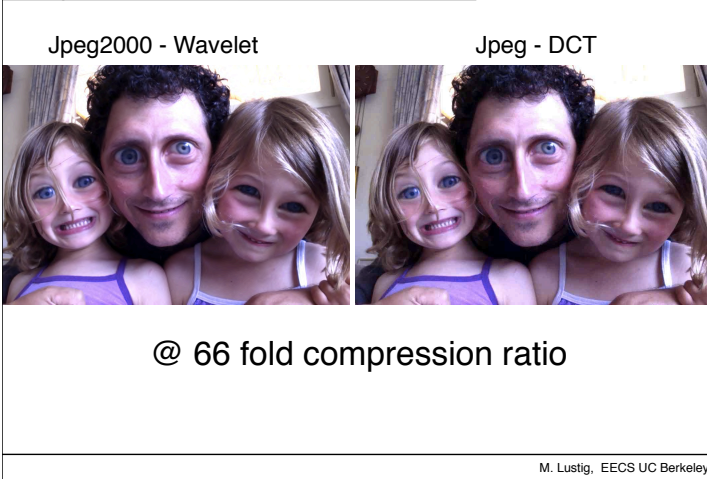
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Example: Denoising by Thresholding



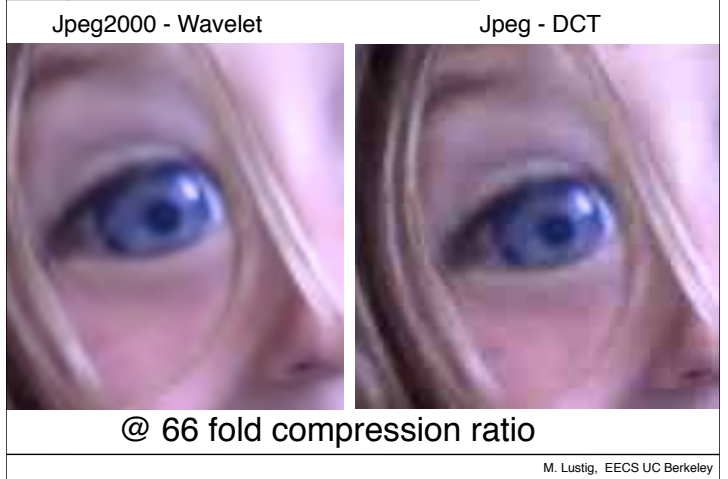
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Compression - JPEG2000 vs JPEG

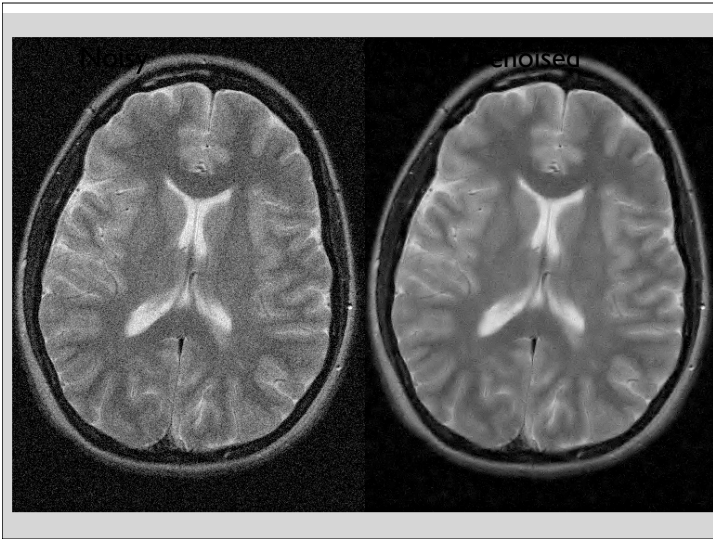
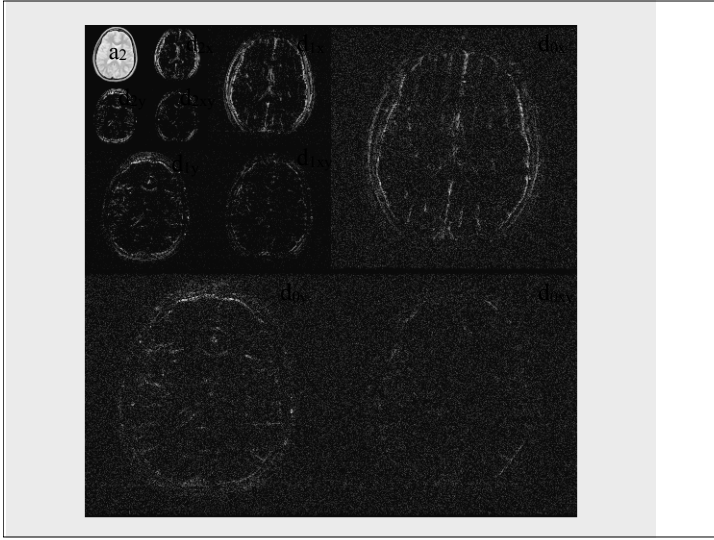
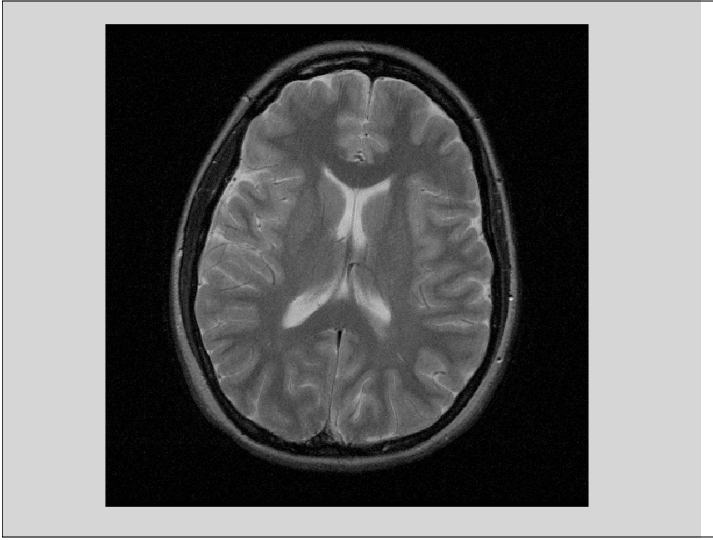


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Compression - JPEG2000 vs JPEG



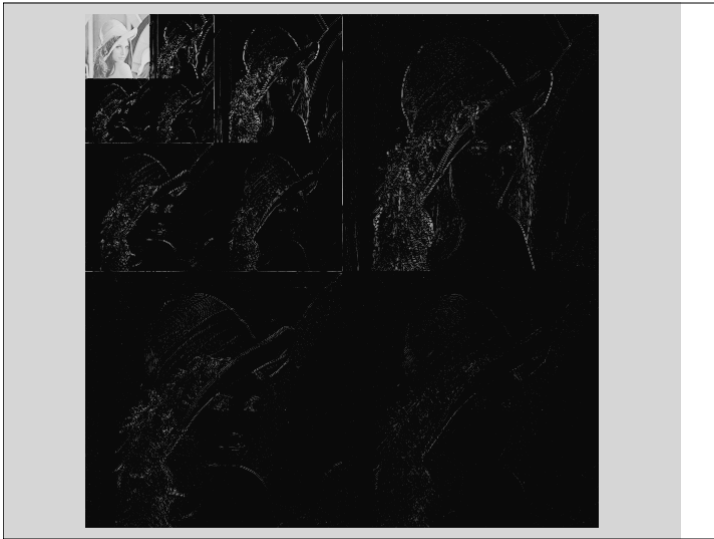
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Approximation/Compression

0.000% coefficients

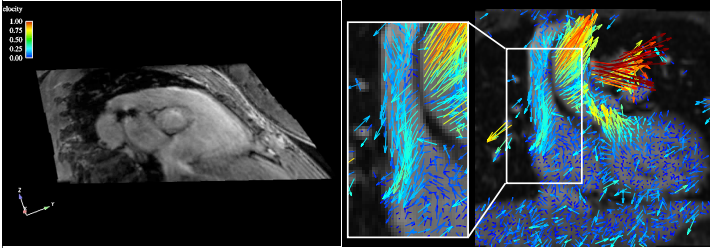
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Example in Research

Robust 4D Flow Denoising using Divergence-free Wavelet Transform

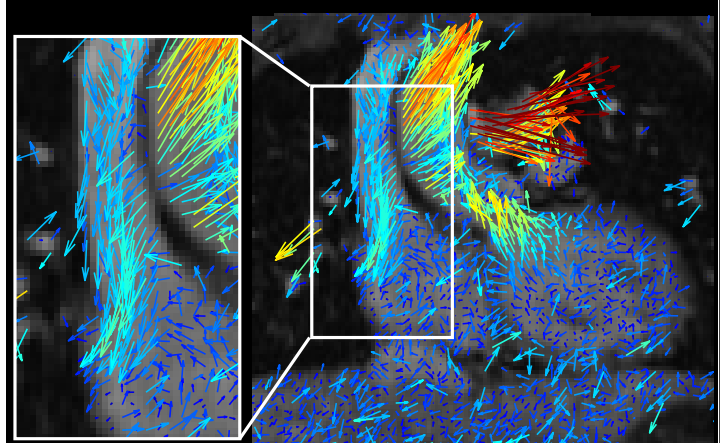
Frank Ong¹, Martin Uecker¹, Umar Tariq², Albert Hsiao², Marcus T Alley²,
Shreyas S Vasanaawala², Michael Lustig¹



courtesy, Frank Ong and Marcus Alley

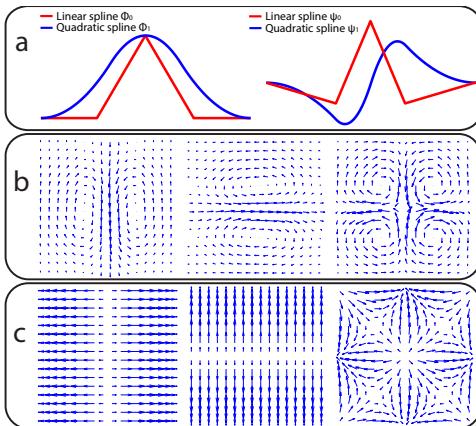
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Noisy Flow Data



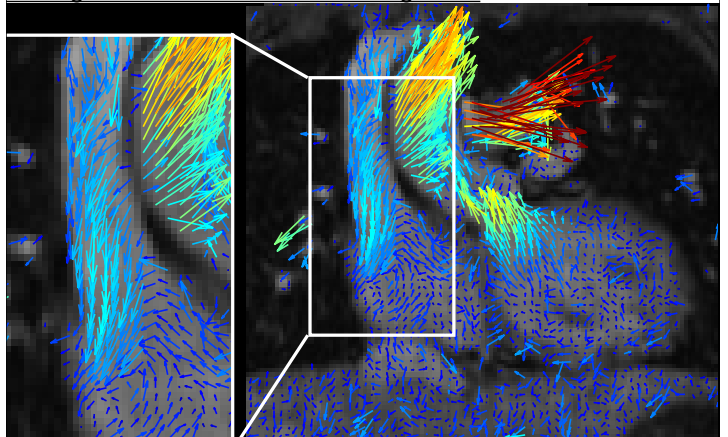
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Divergence Free Wavelets



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Divergence-Free Wavelet Denoising



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