EE123
Digital Signal Processing

Lecture 13
DWT

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Back to Discrete

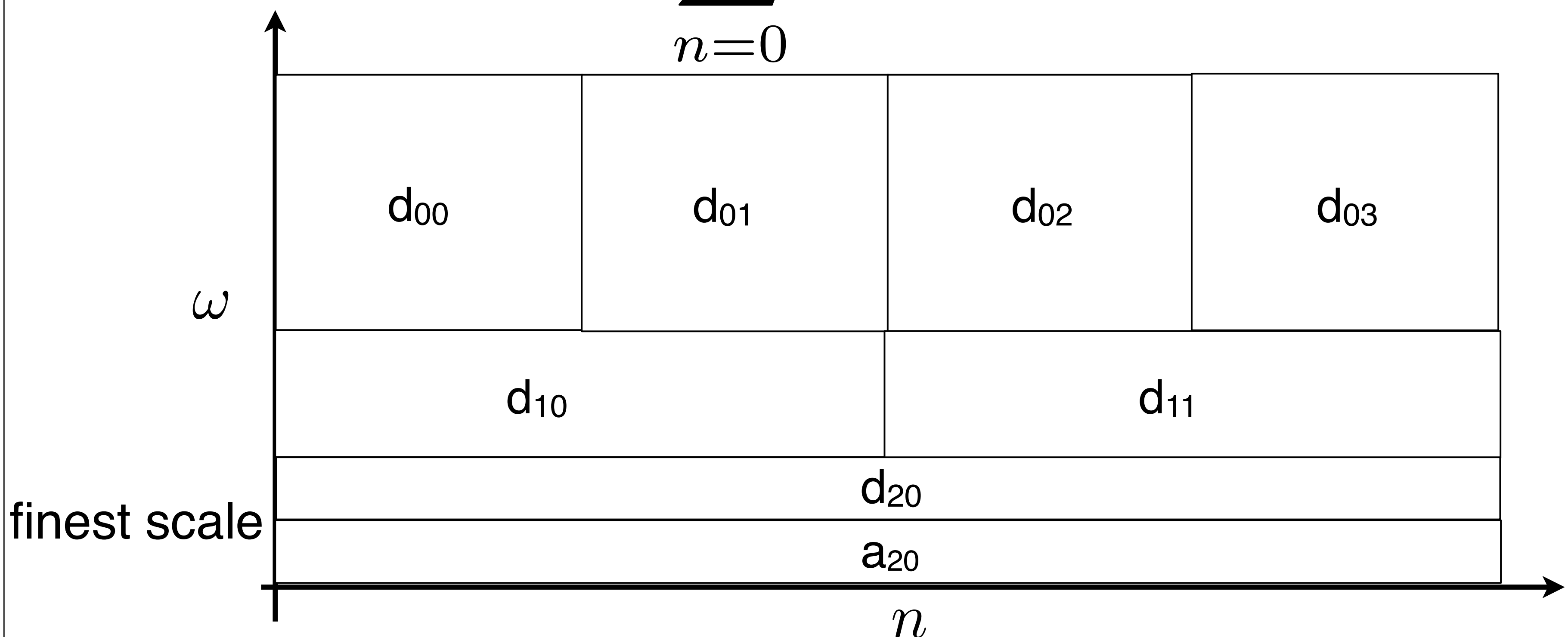
- Early 80’s, theoretical work by Morlett, Grossman and Meyer (math, geophysics)
- Late 80’s link to DSP by Daubechies and Mallat.

- From CWT to DWT not so trivial!
- Must take care to maintain properties

Discrete Wavelet Transform

$$d_{\mathrm{s,u}} = \sum_{n=0}^{N-1} x[n]\Psi_{\mathrm{s,u}}[n]$$

$$a_{\mathrm{s,u}} = \sum_{n=0}^{N-1} x[n]\Phi_{\mathrm{s,u}}[n]$$



Discrete Wavelet Transform

$$d_{\mathrm{s,u}} = \sum_{n=0}^{N-1} x[n]\Psi_{\mathrm{s,u}}[n]$$

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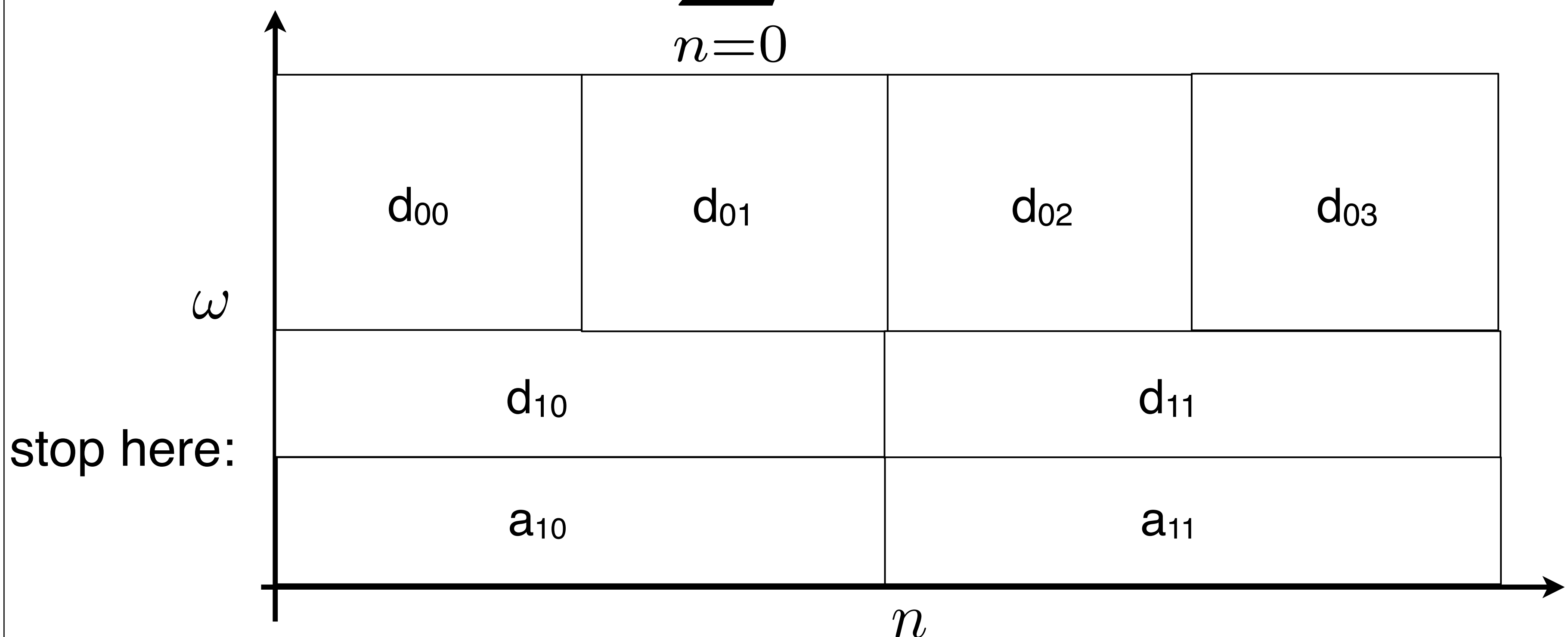
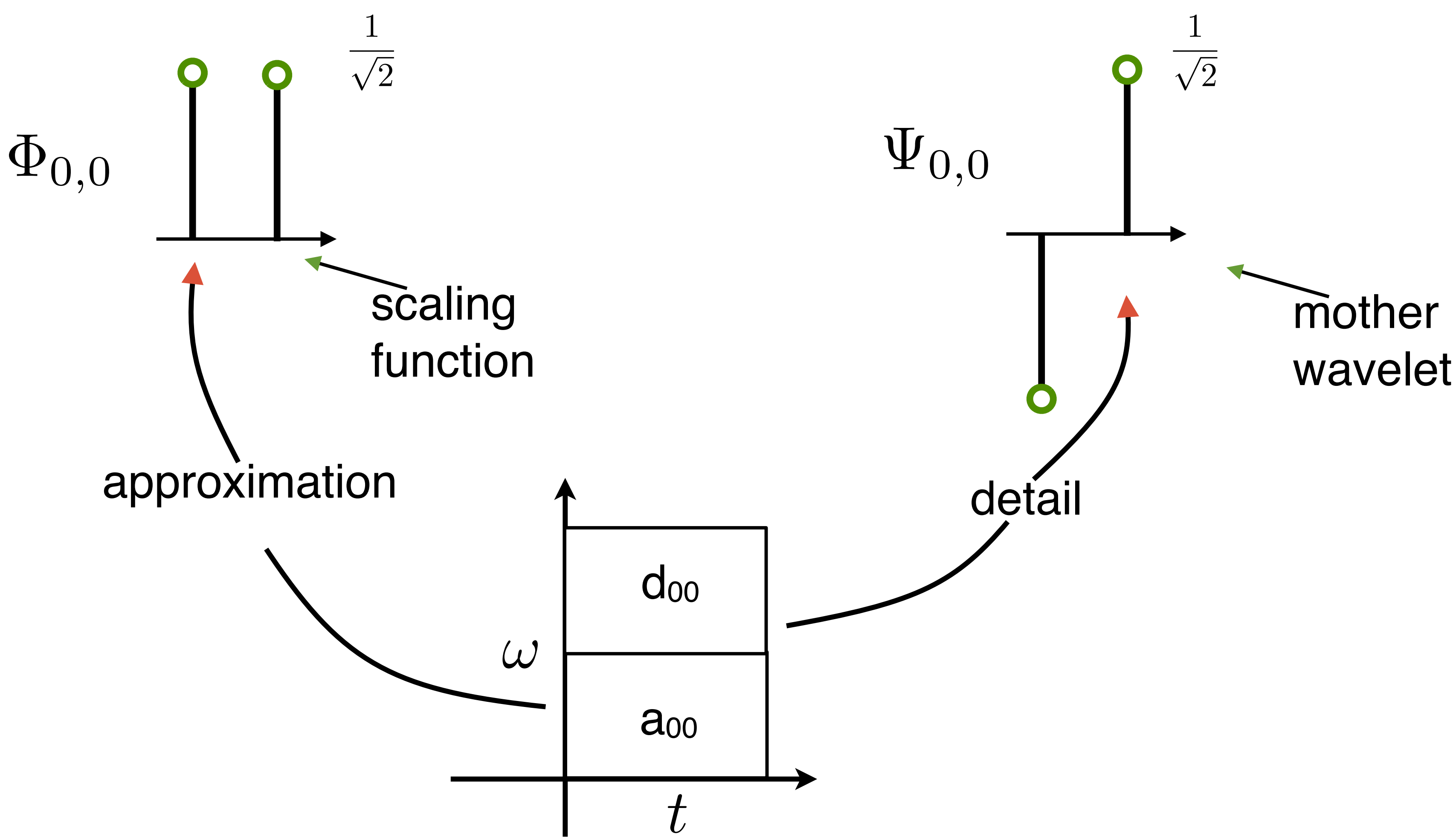


Example: Discrete Haar Wavelet

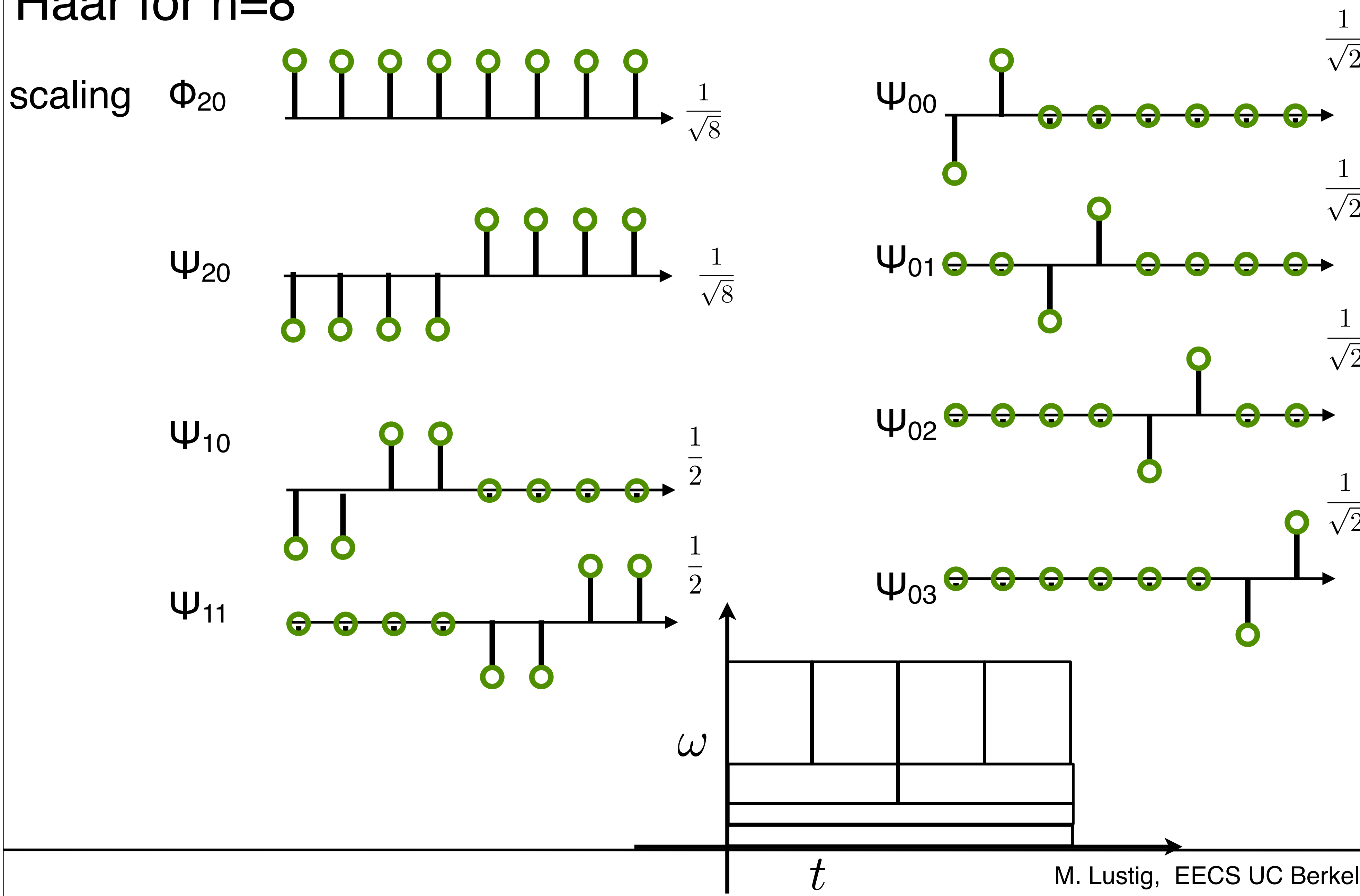
Haar for n=2



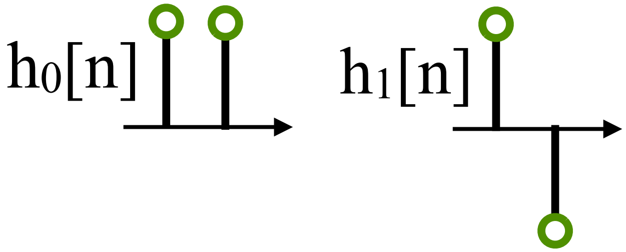
Equivalent to DFT_2!

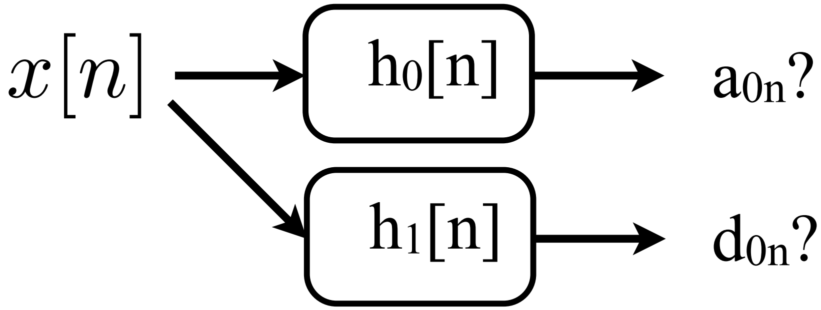
Discrete Orthogonal Haar Wavelet

Haar for n=8

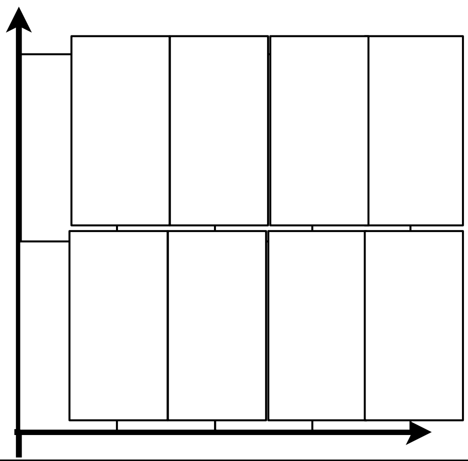


Fast DWT with Filter Banks (more Later!)

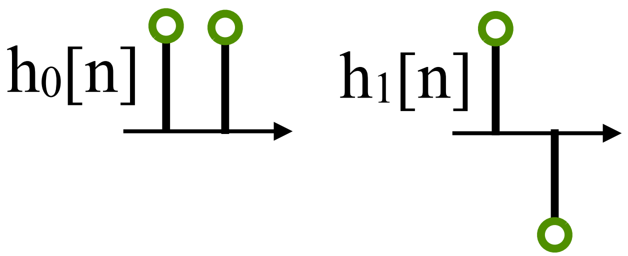


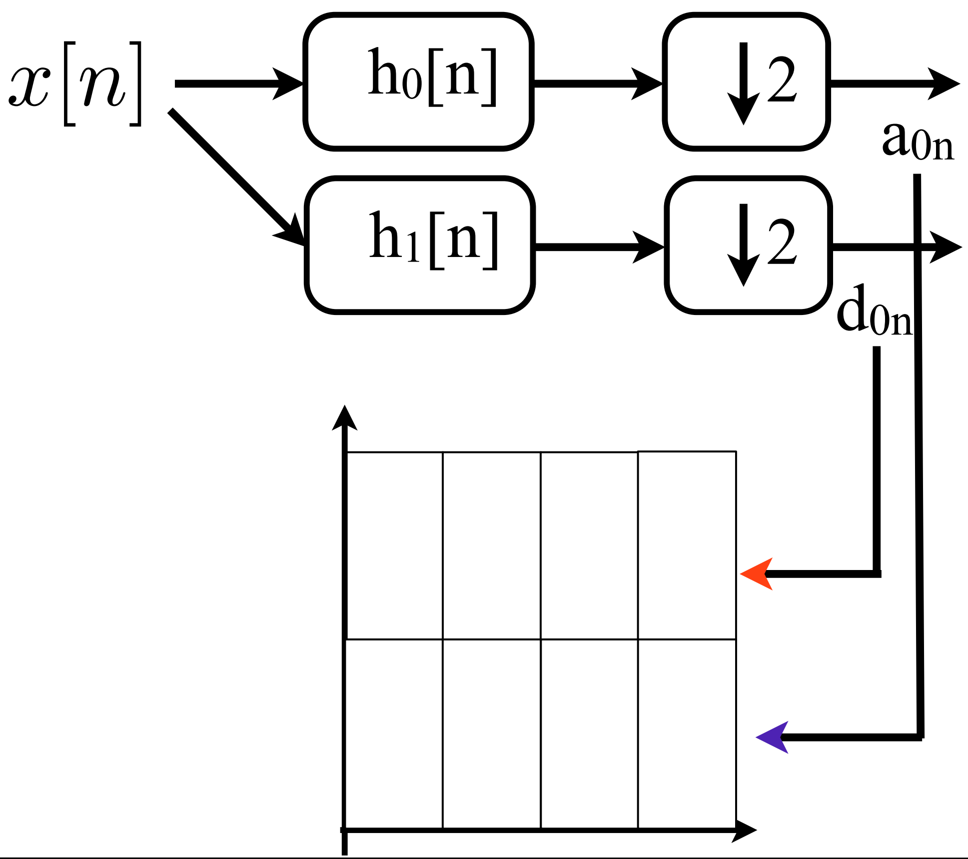


not quite...
too many
coefficients

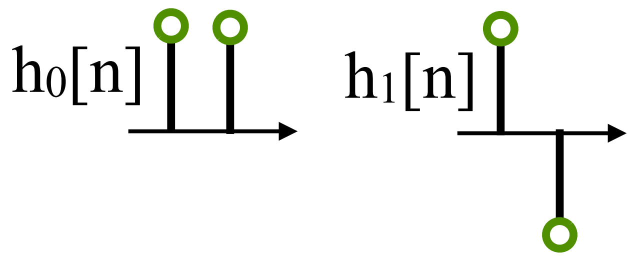


Fast DWT with Filter Banks





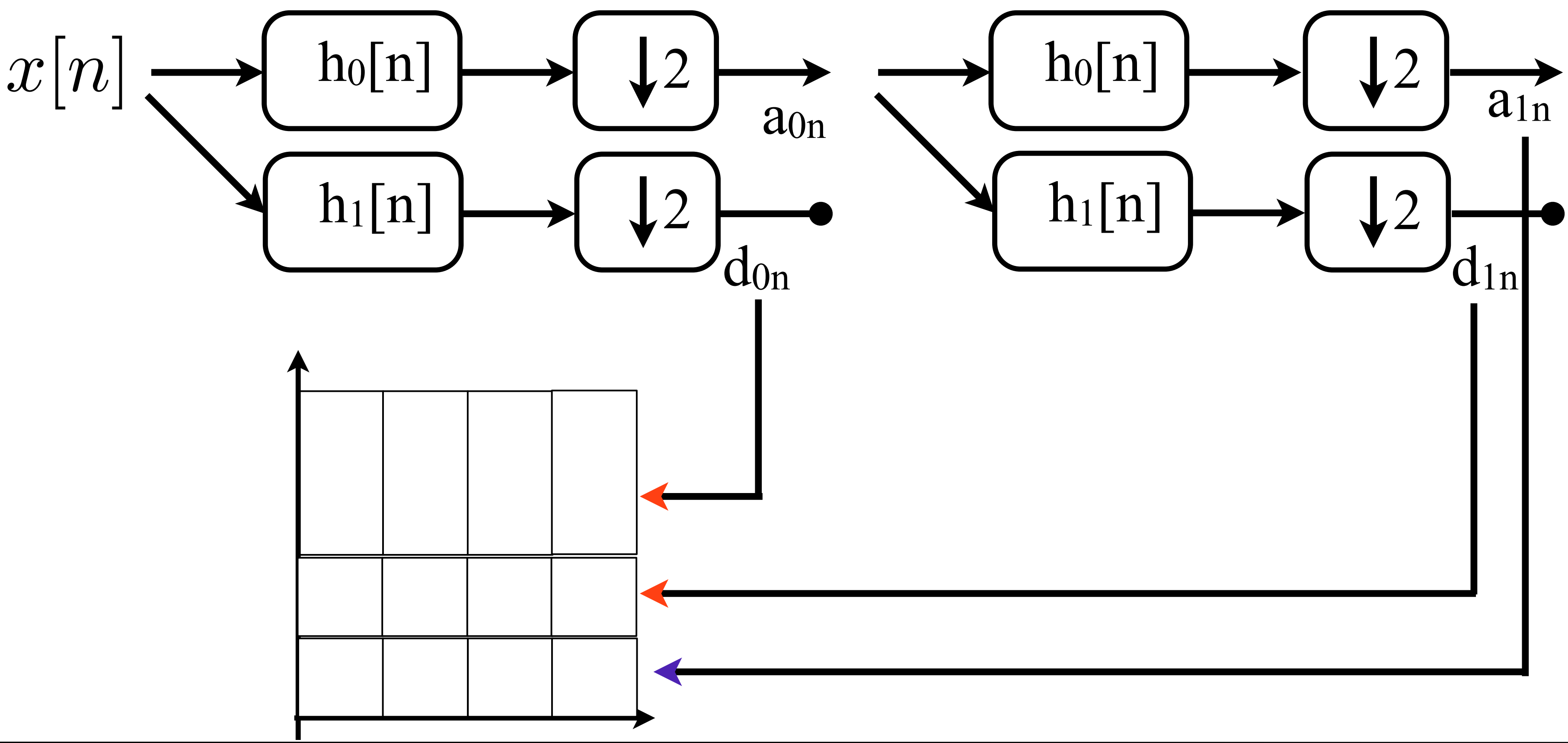
Fast DWT with Filter Banks



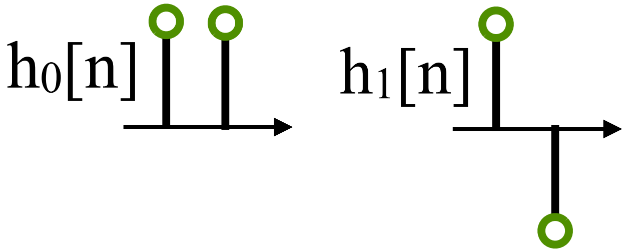
complexity:

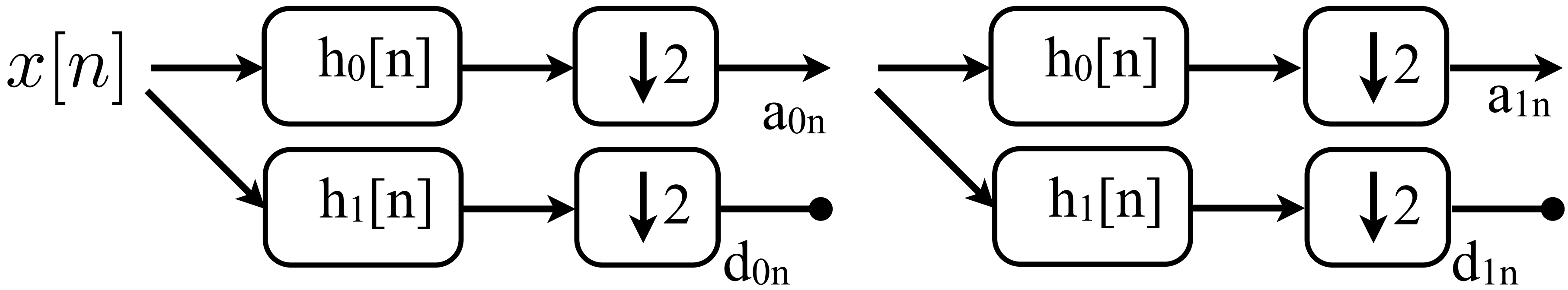
N + N/2 + N/4 + N/8 +...+ = 2N

=O(N)

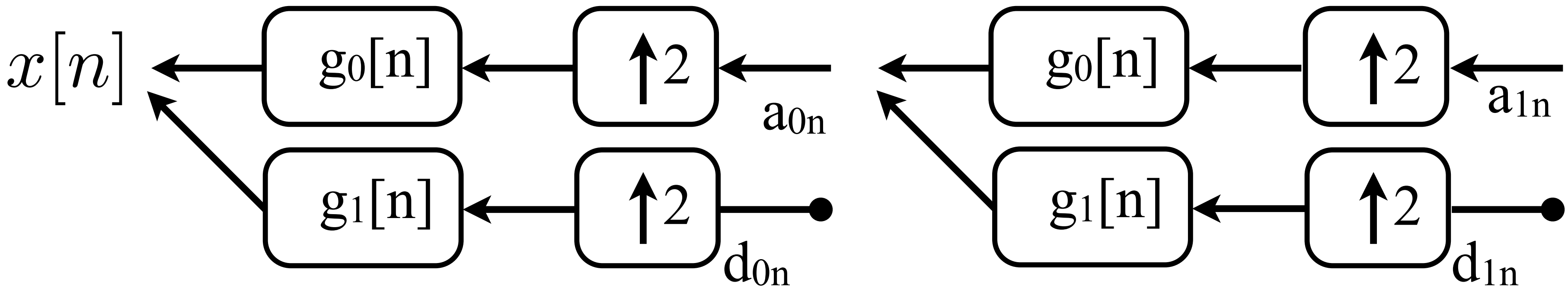


Decomposition



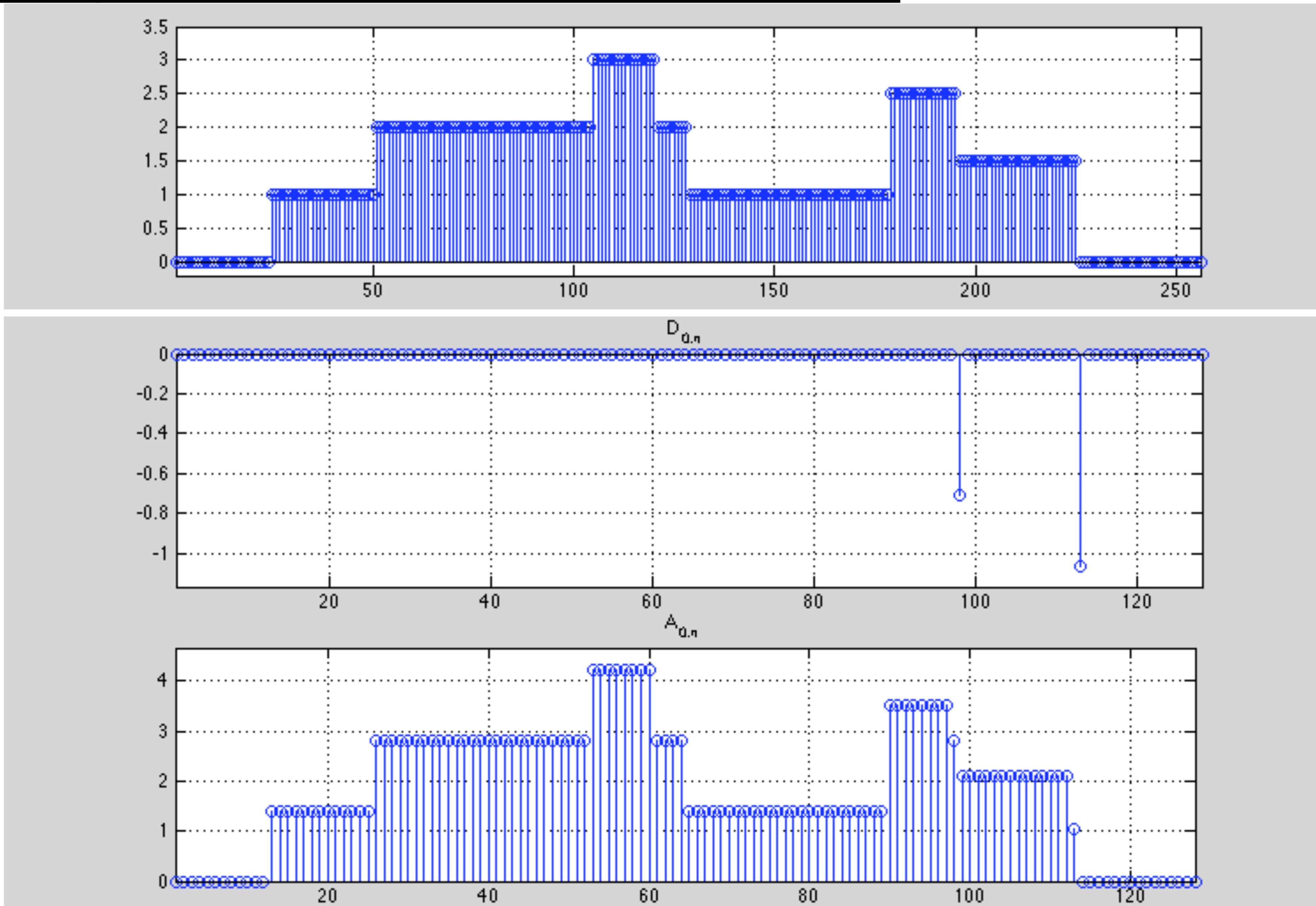


Reconstruction

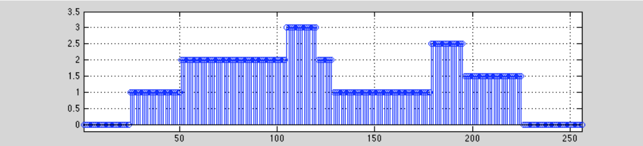


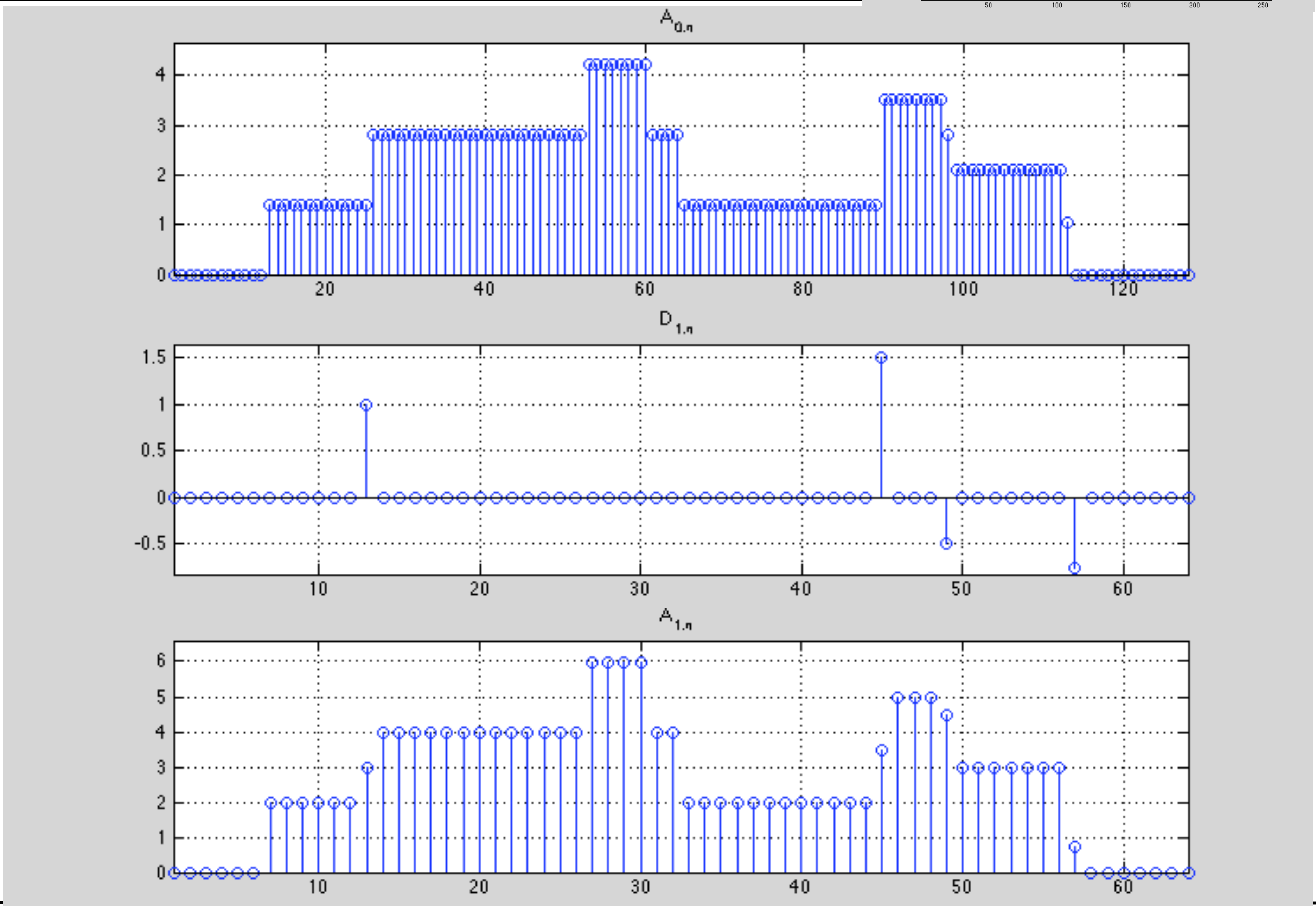
Just flip arrows,
replace h with g

Example, Haar DWT - Level 0

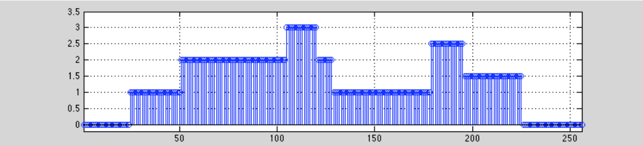


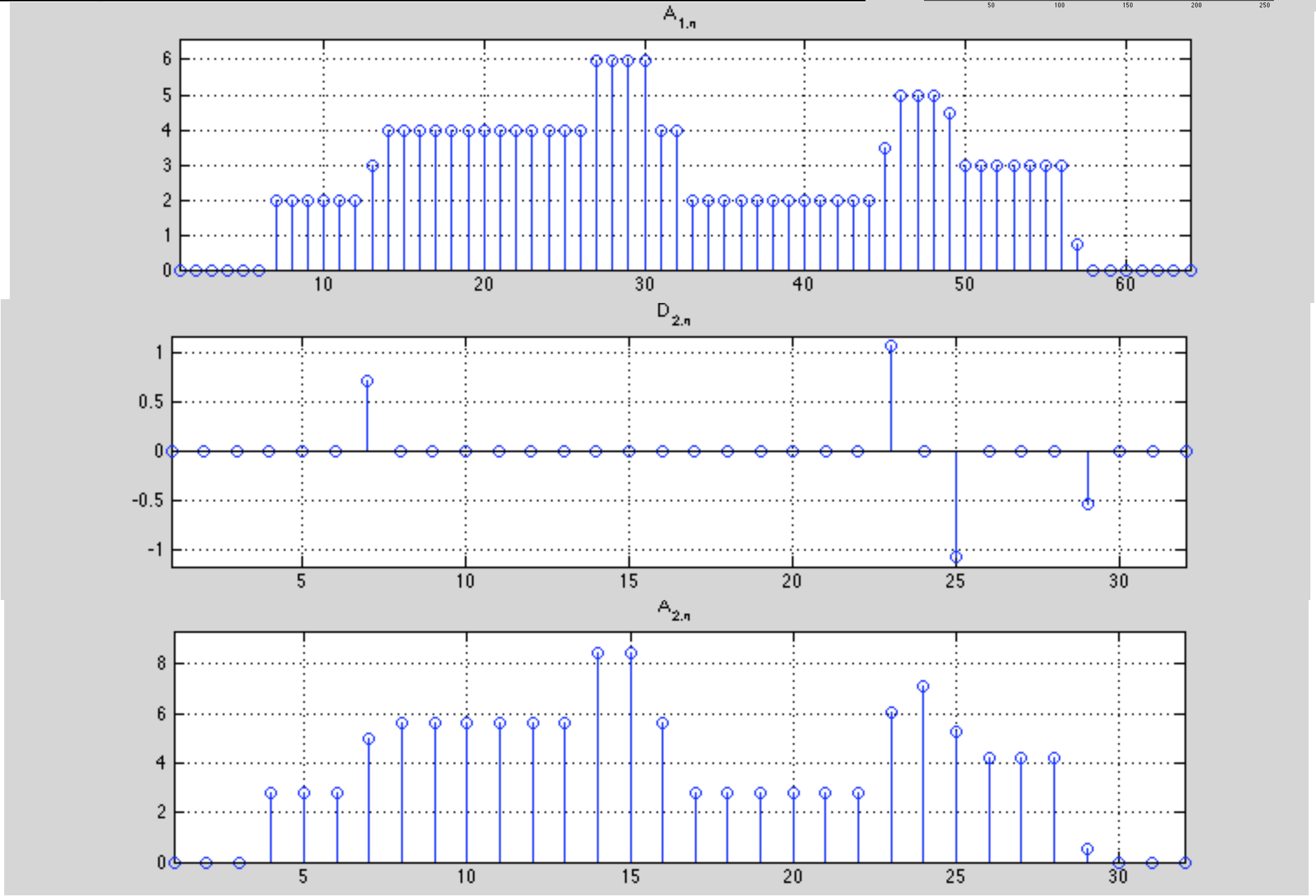
Example, Haar DWT - Level 1



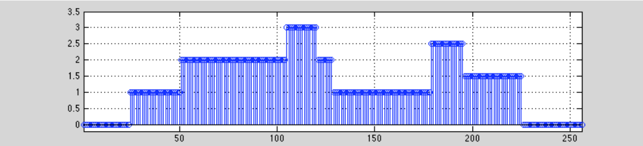


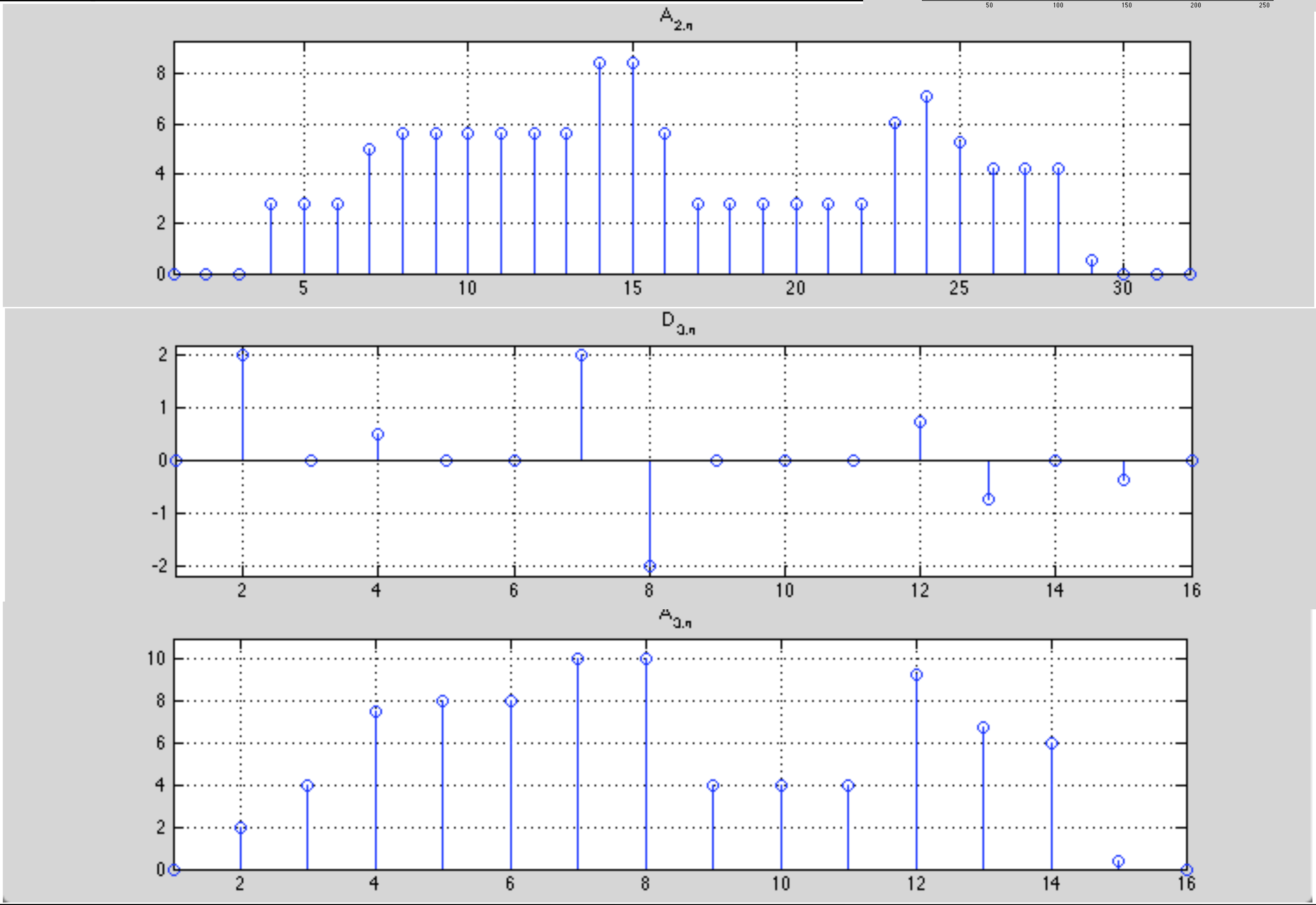
Example, Haar DWT - Level 2



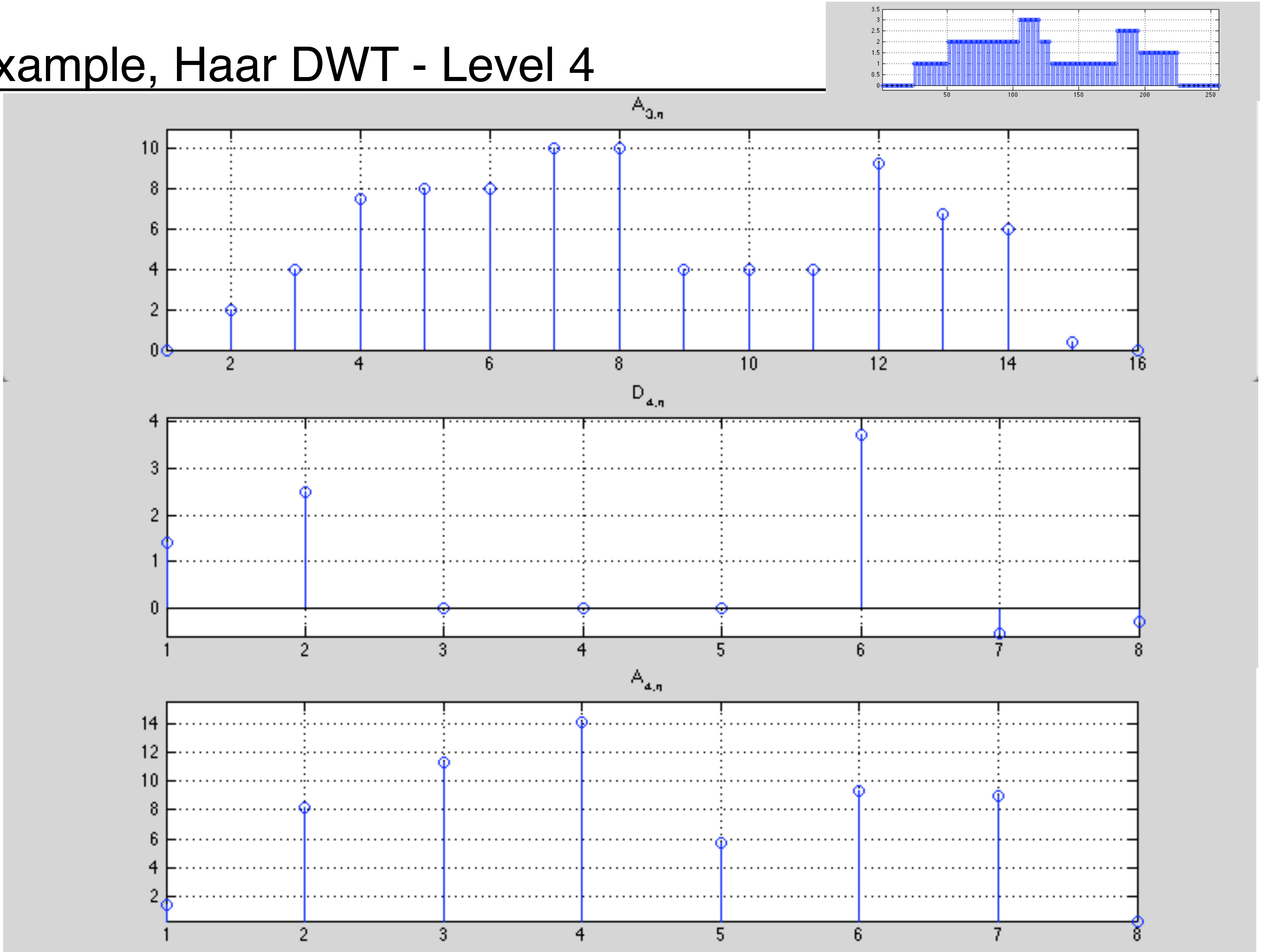


Example, Haar DWT - Level 3

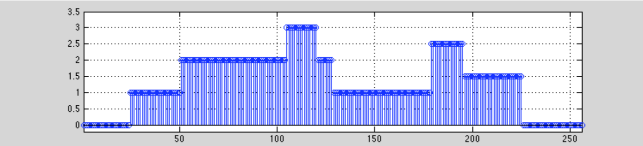


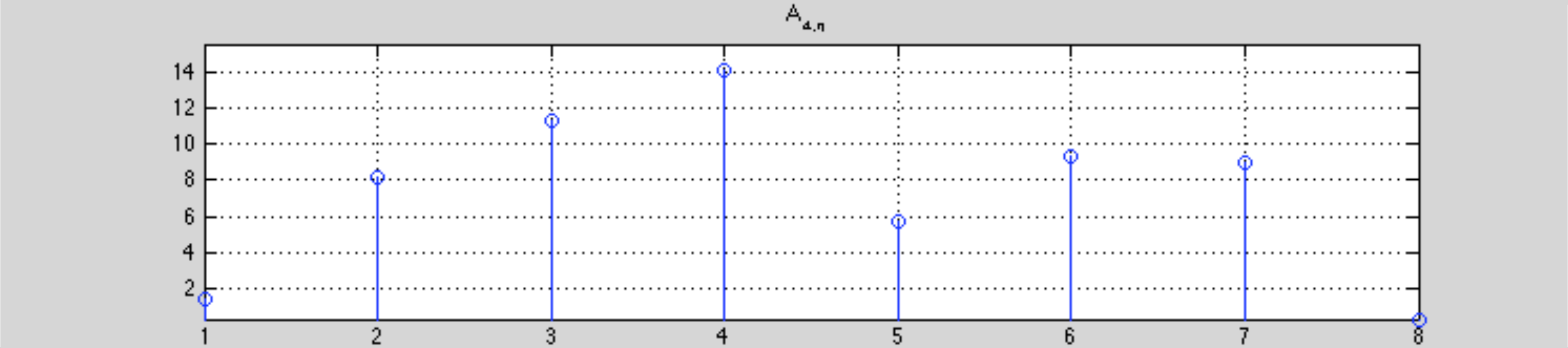


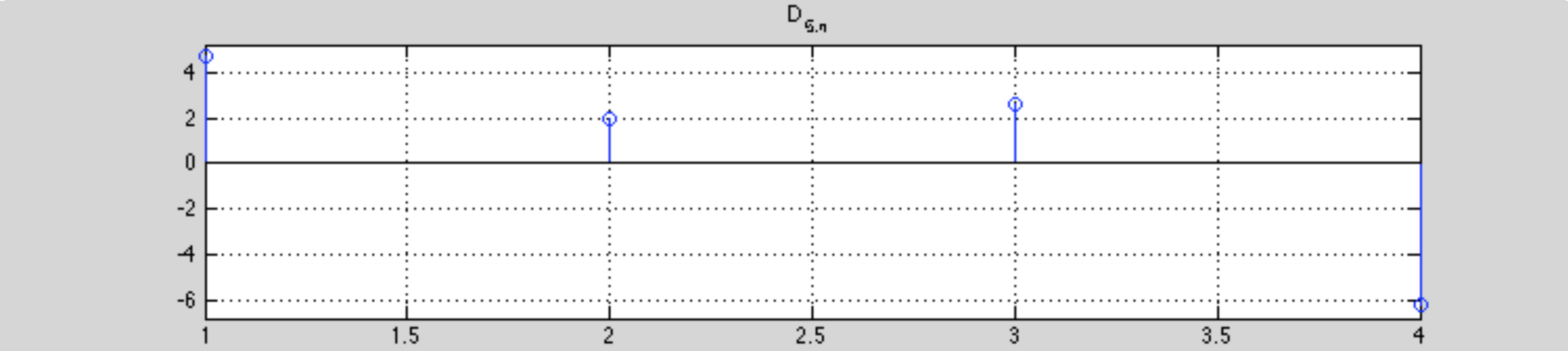
Example, Haar DWT - Level 4

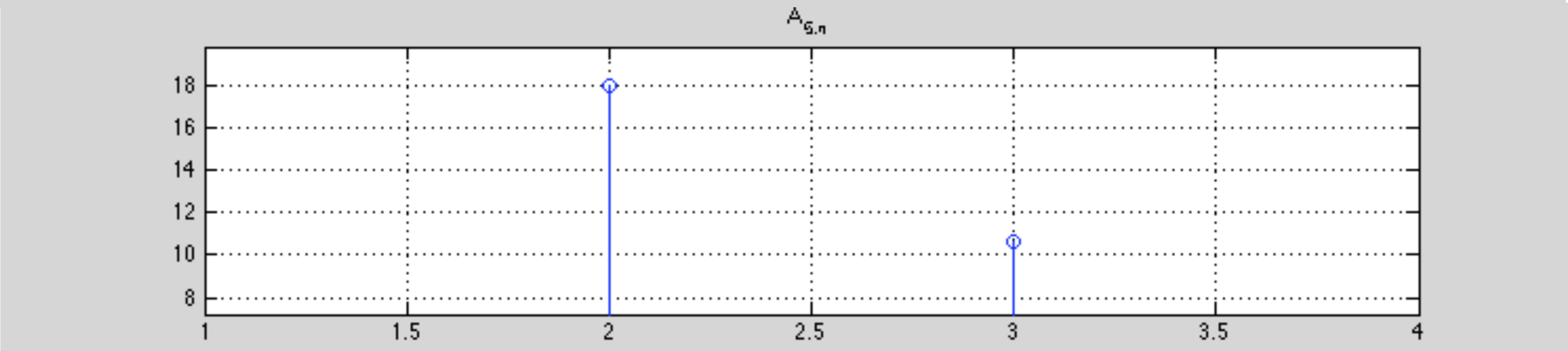


Example, Haar DWT - Level 5

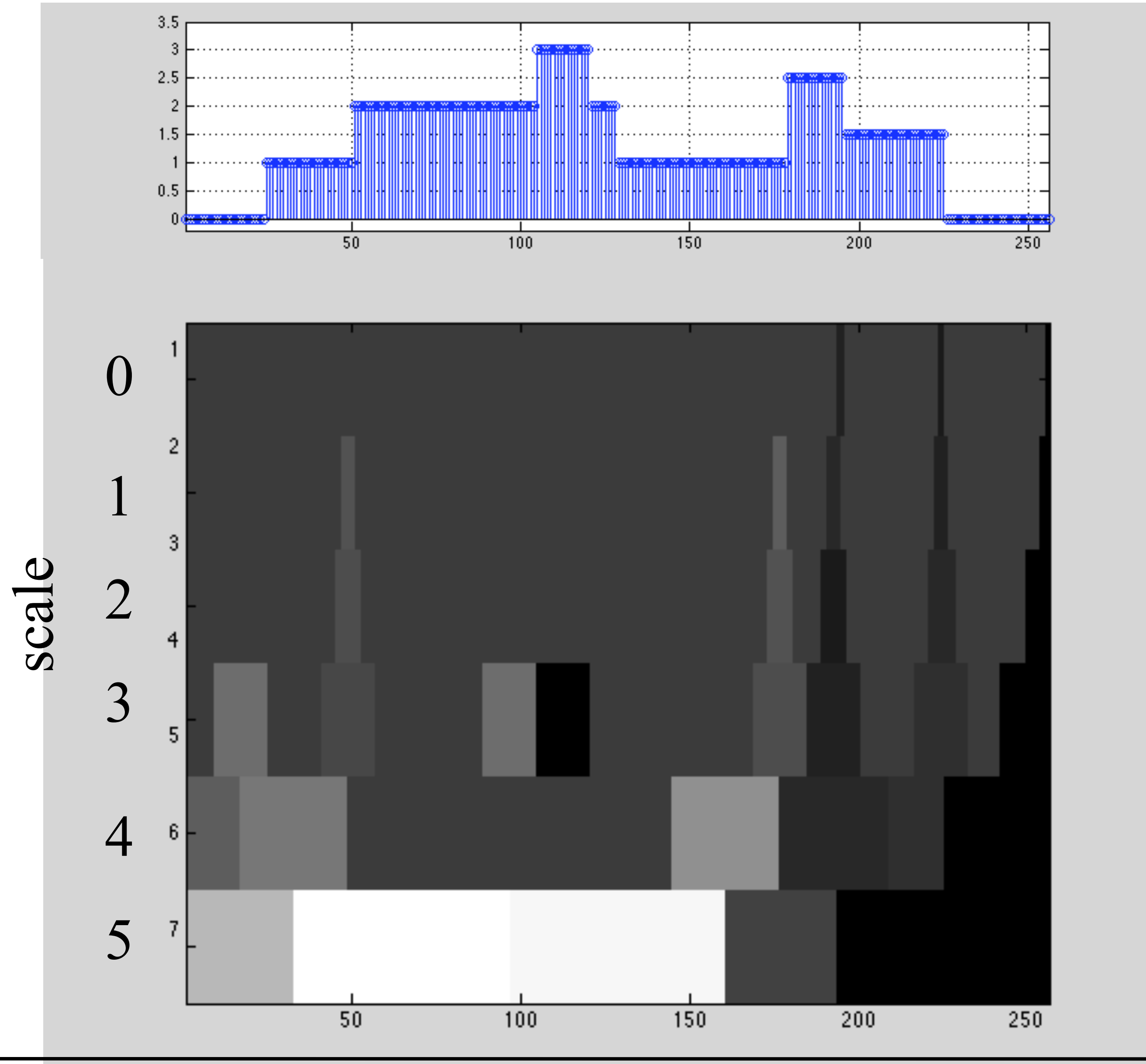




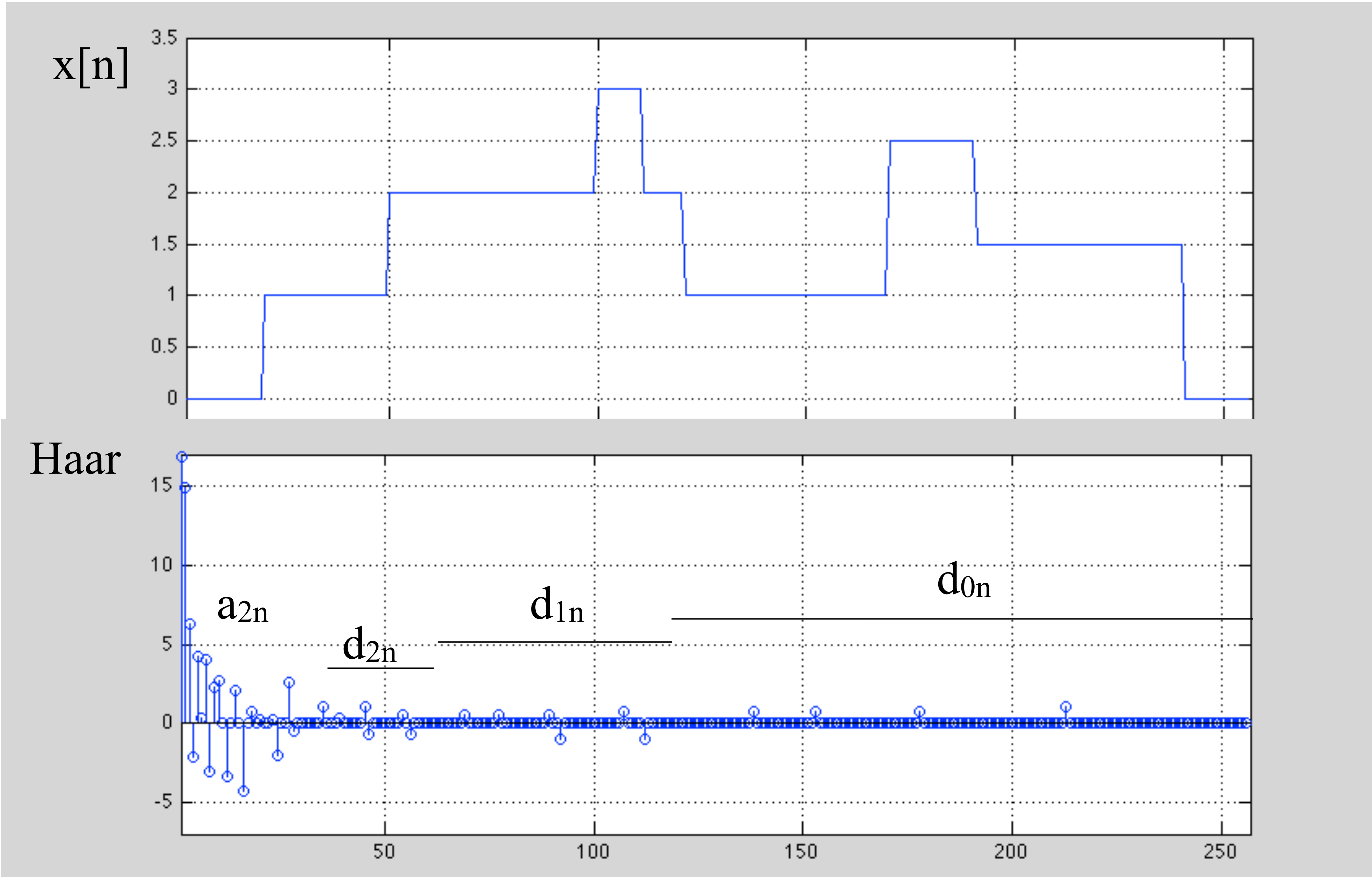




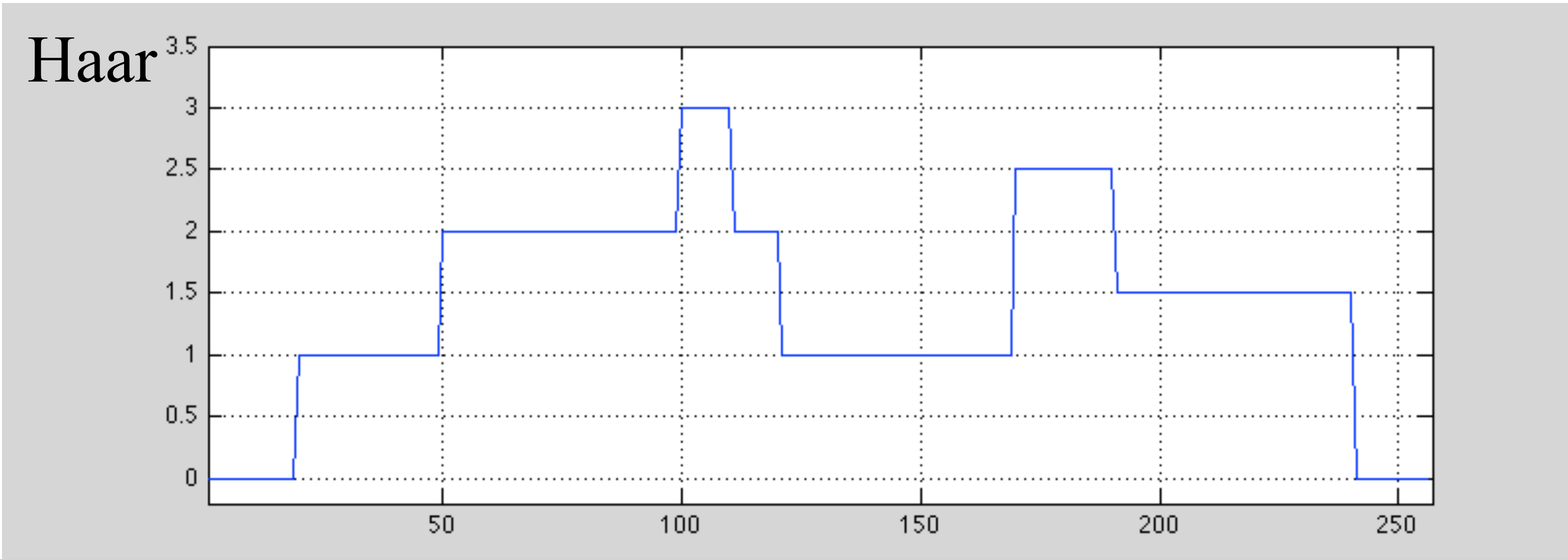
DWT Another view

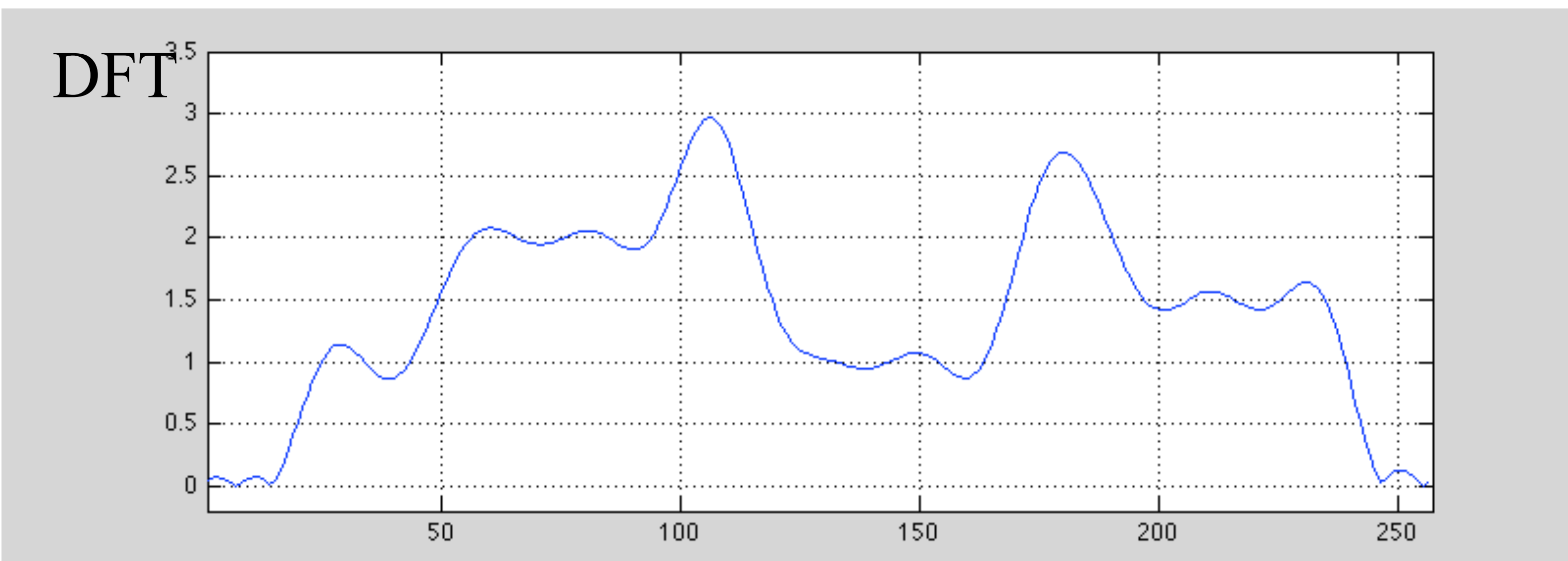


Haar DWT Example

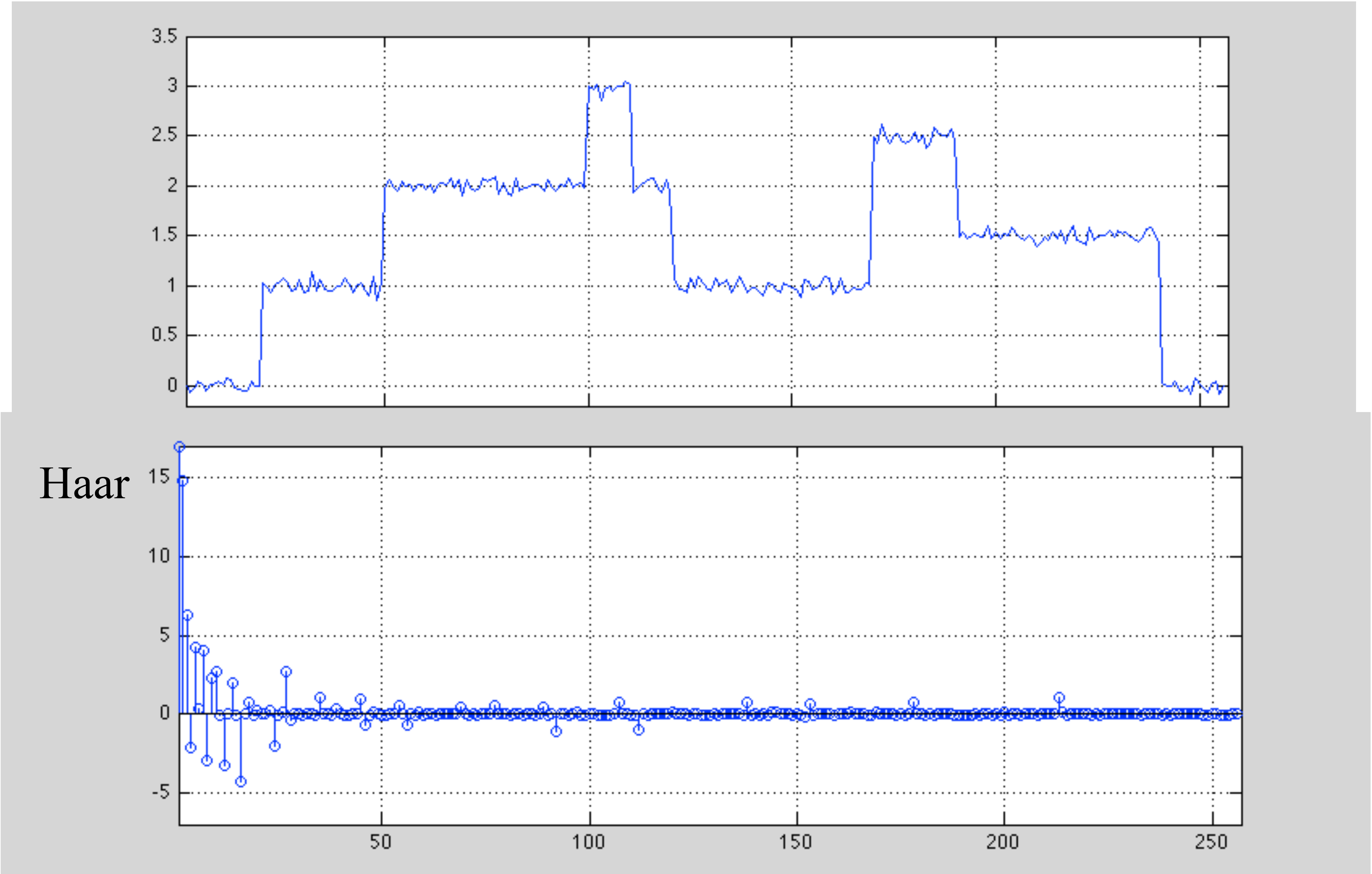


Approximation from 25/256 coefficients

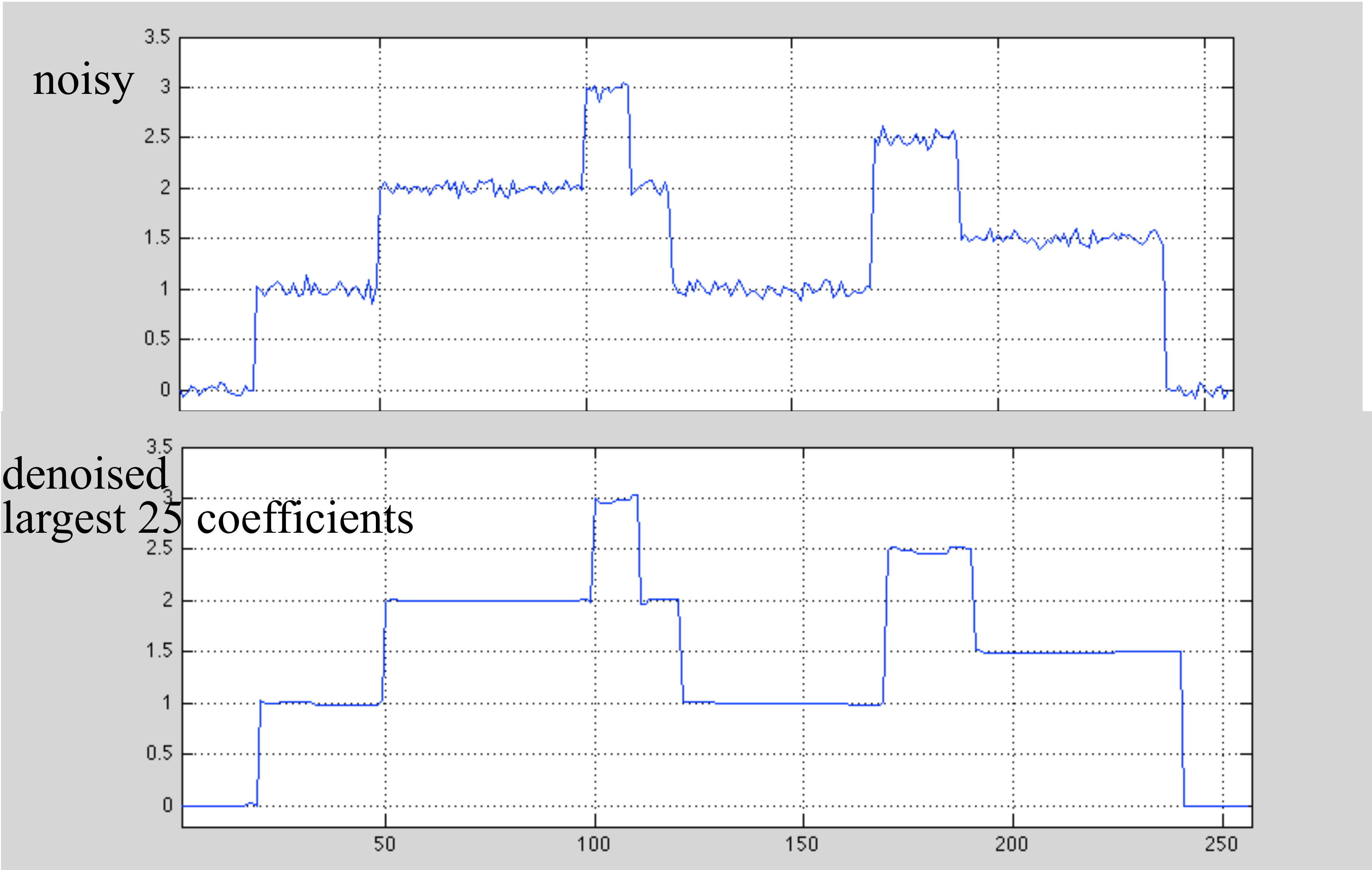




Example: Denoising Noisy Signals



Example: Denoising by Thresholding



Compression - JPEG2000 vs JPEG

Jpeg2000 - Wavelet

Jpeg - DCT

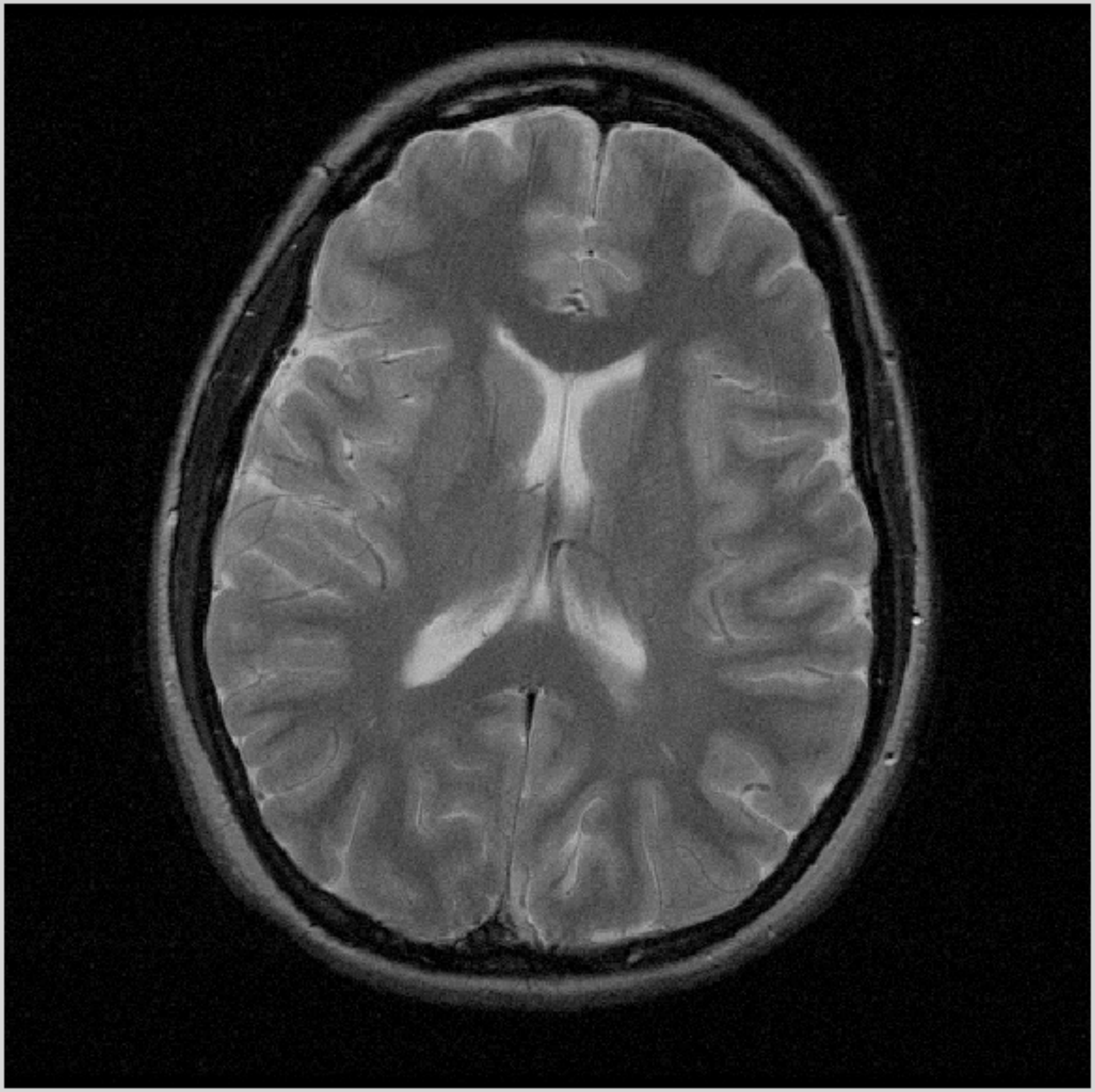
@ 66 fold compression ratio

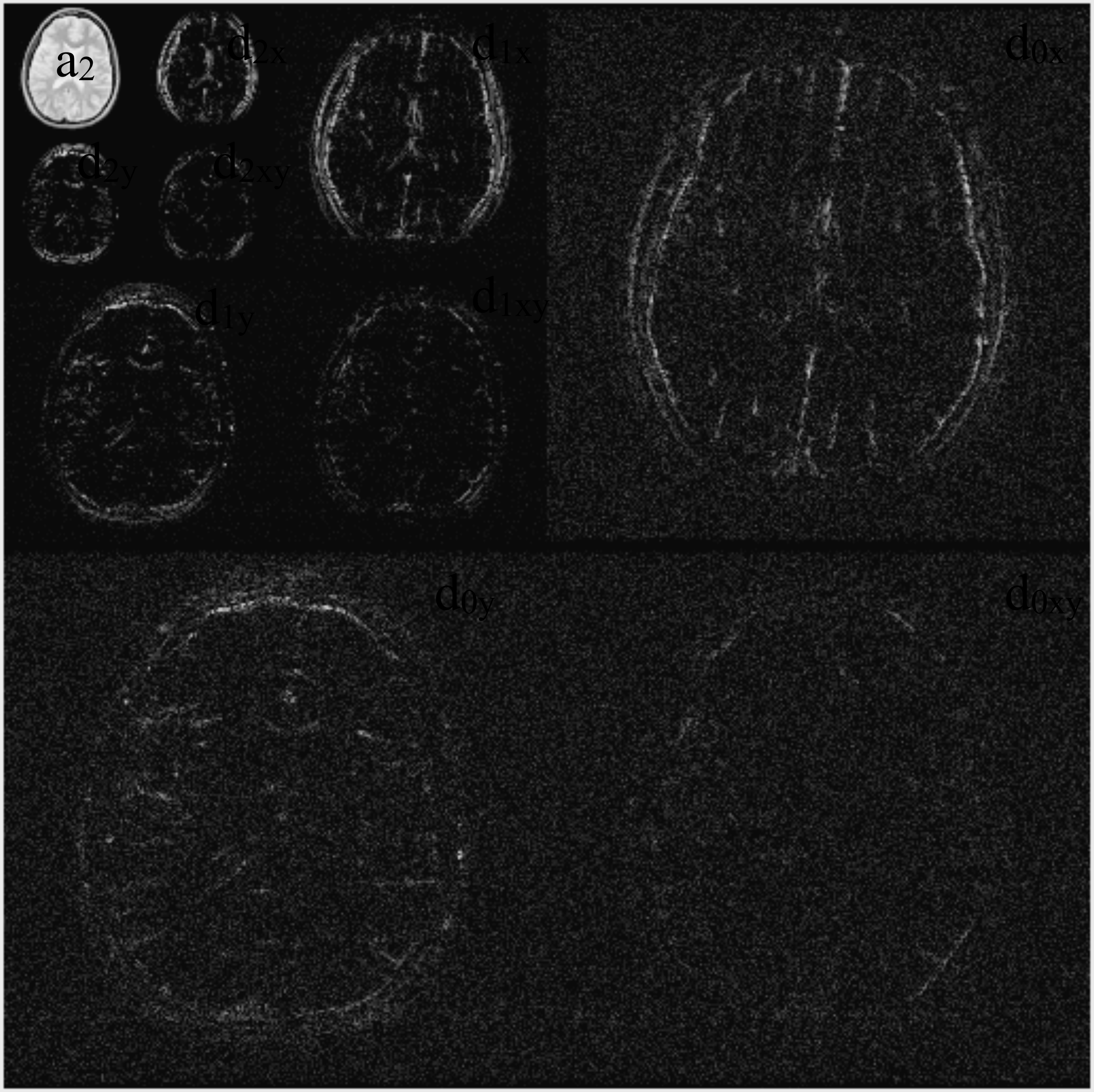
Compression - JPEG2000 vs JPEG

Jpeg2000 - Wavelet

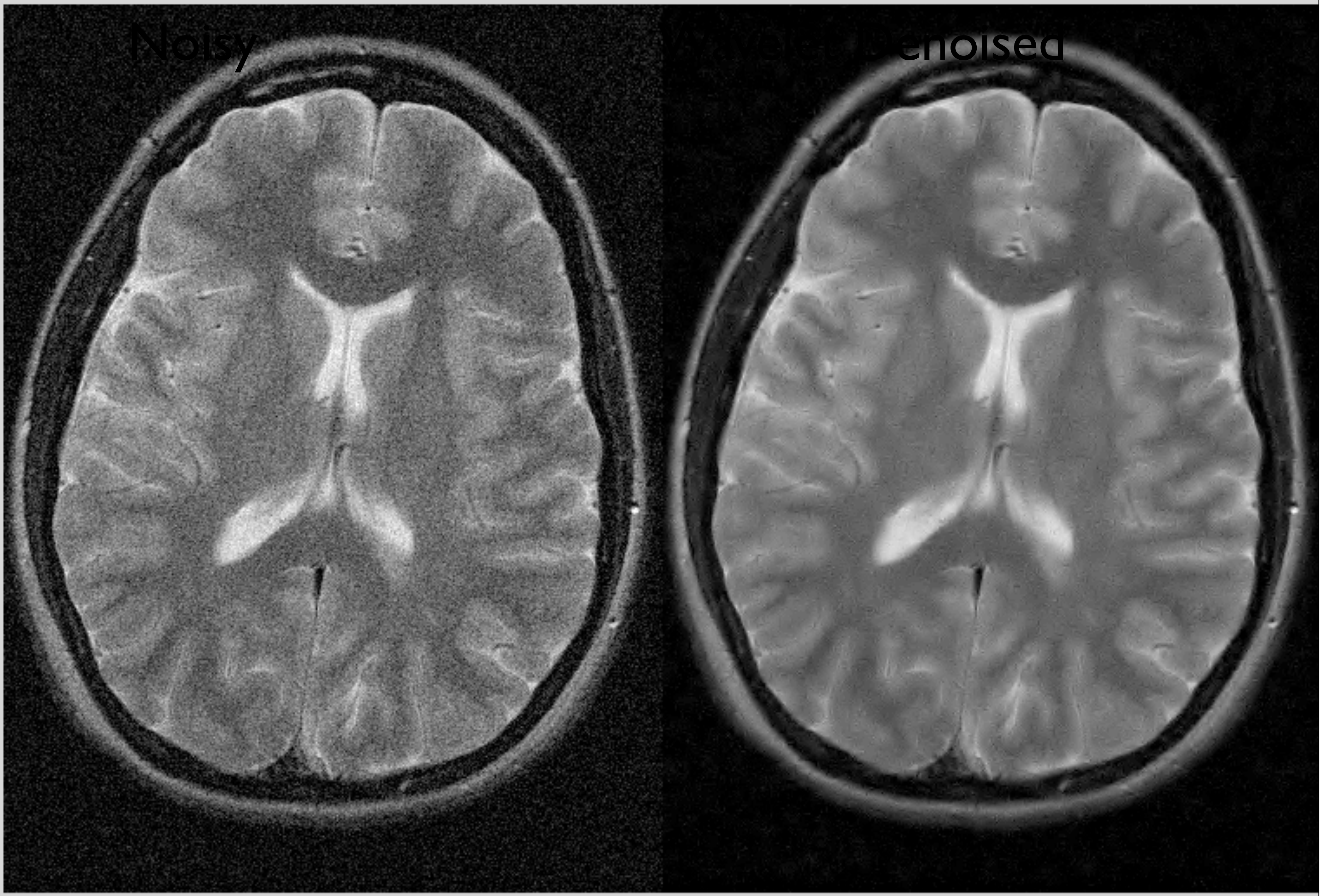
Jpeg - DCT

@ 66 fold compression ratio

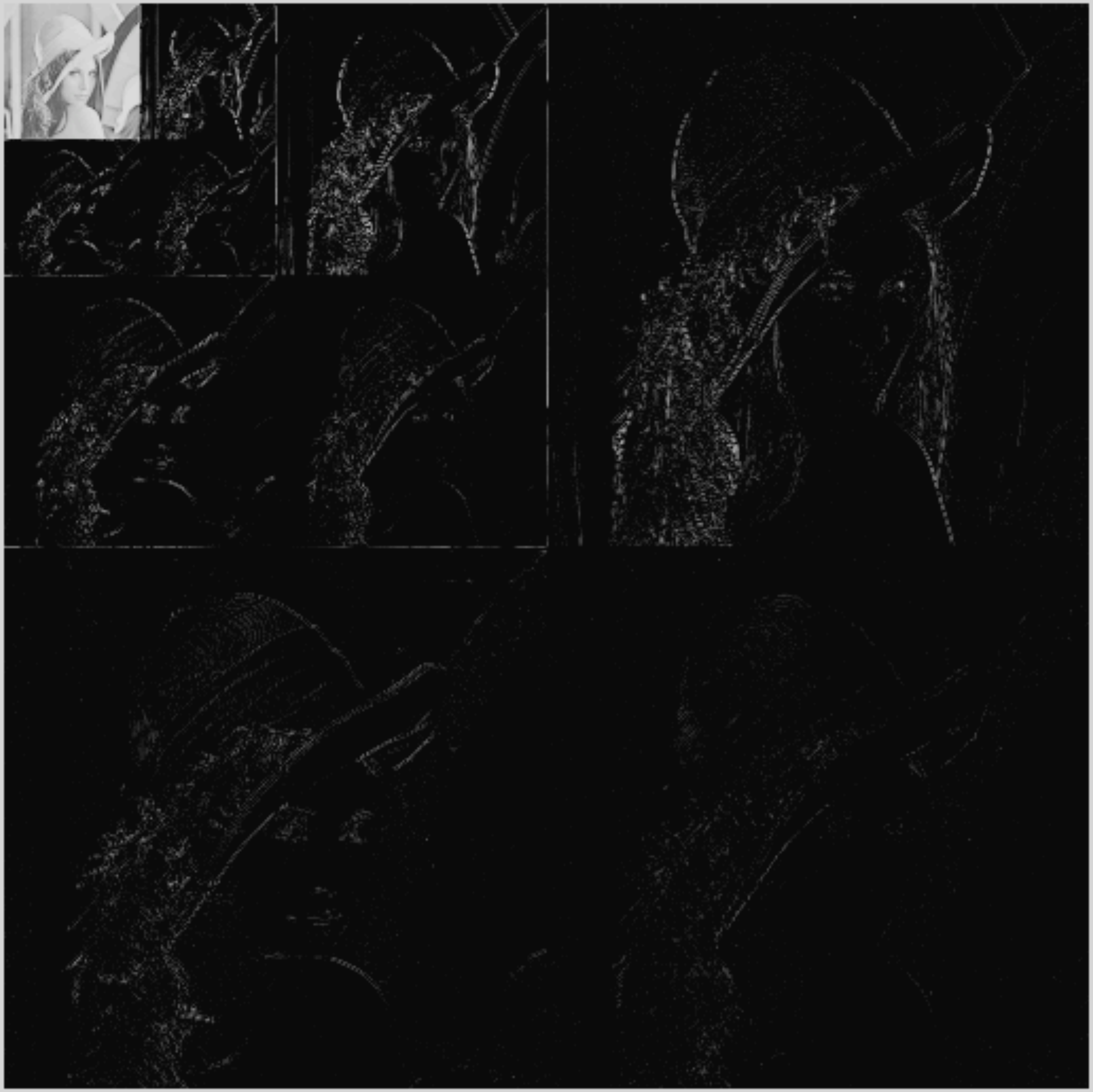




a_2
d_{2x}
d_{1x}
d_{0x}
d_{2y}
d_{2xy}
d_{1y}
d_{1xy}
d_{0y}
d_{0xy}


Noisy
Denoised

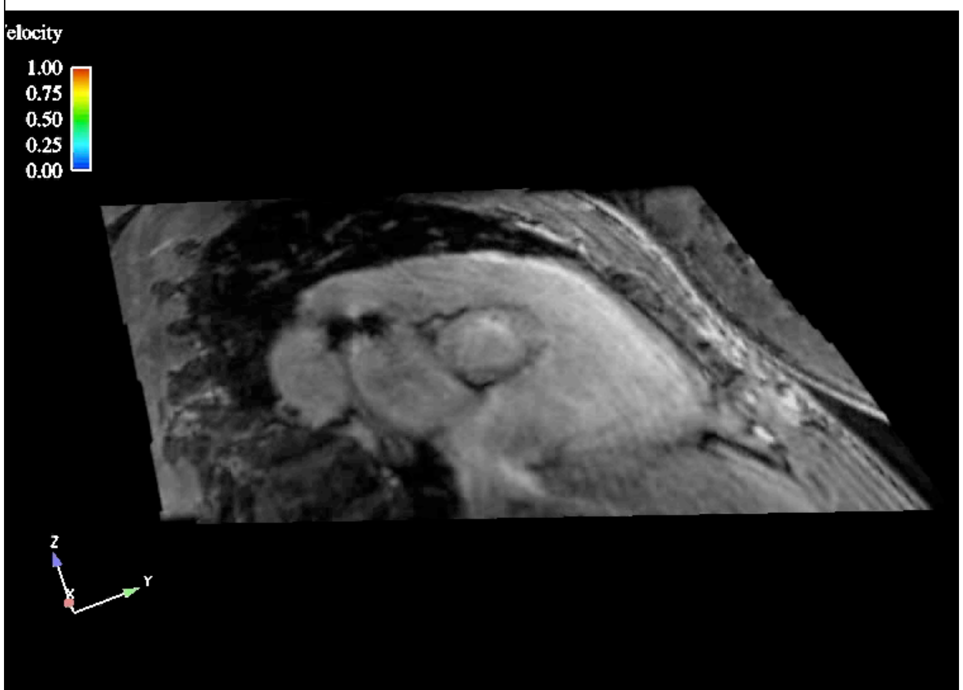
Approximation/Compression

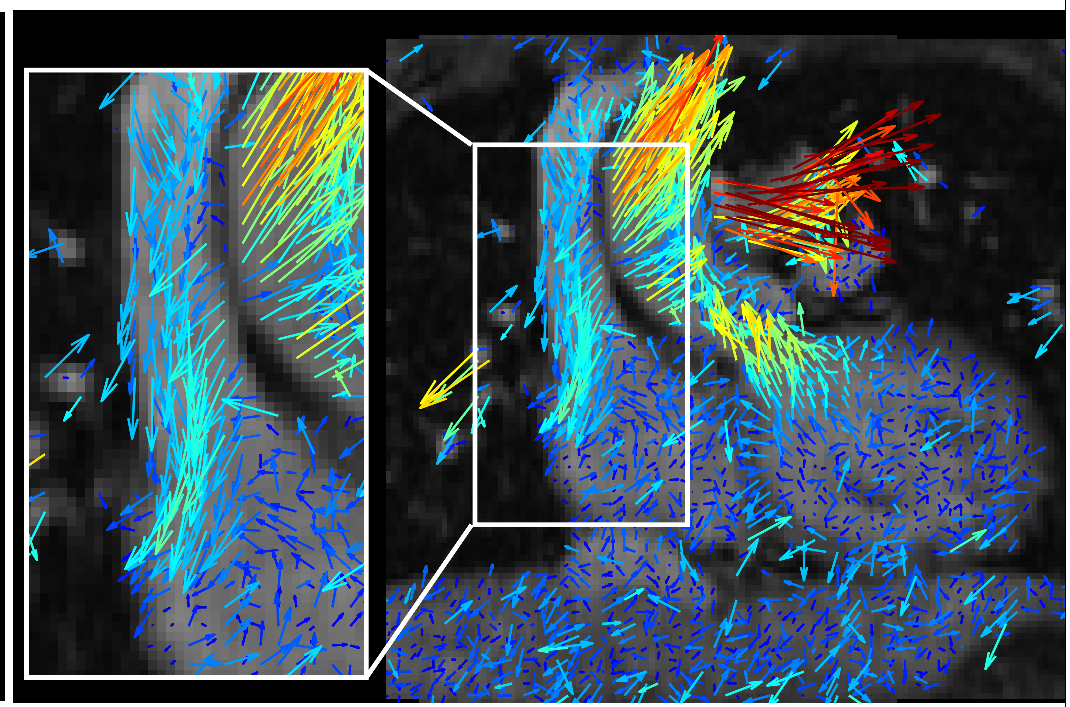


Example in Research

Robust 4D Flow Denoising using Divergence-free Wavelet Transform

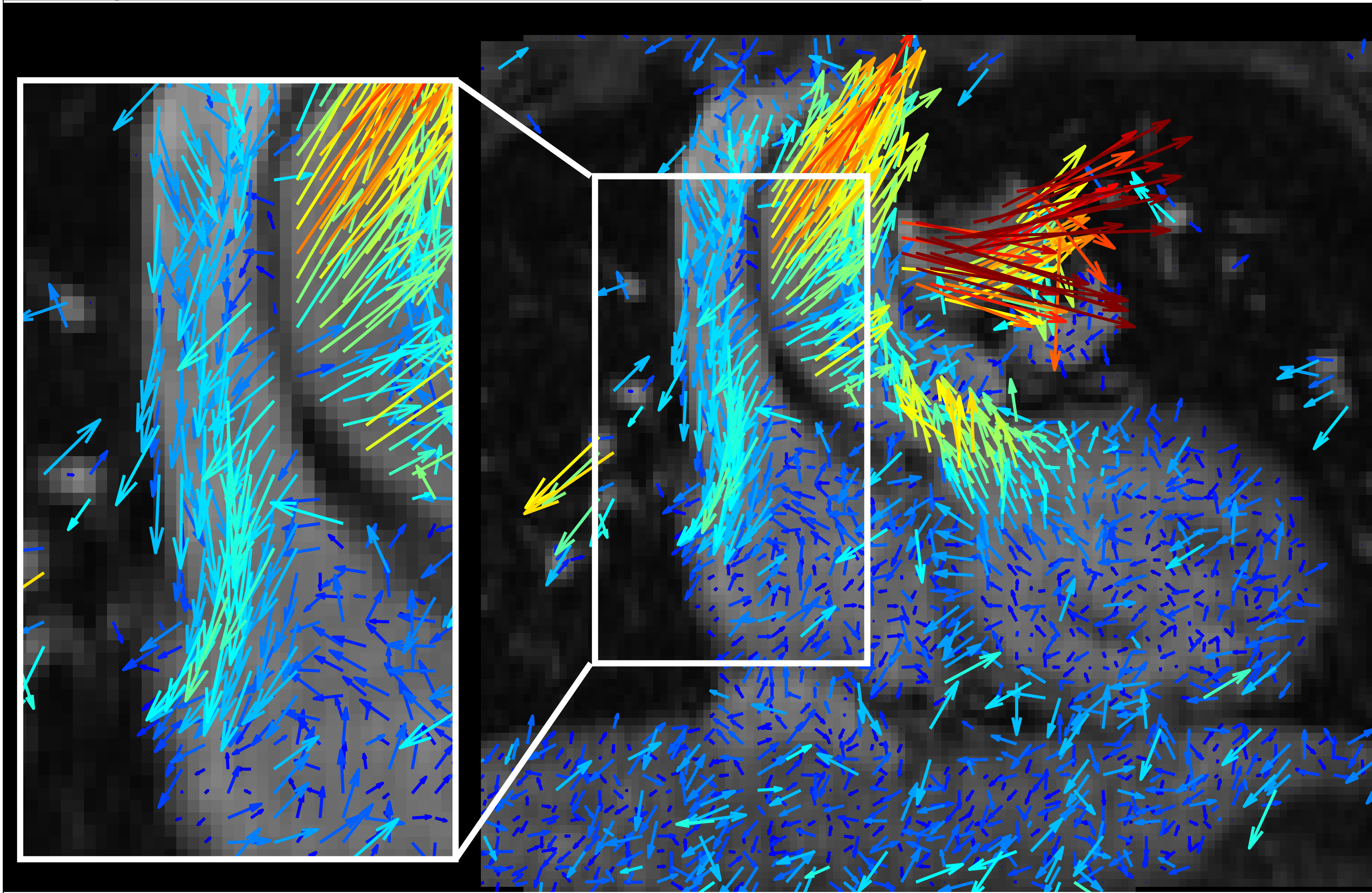
Frank Ong[1], Martin Uecker[1], Umar Tariq[2], Albert Hsiao[2], Marcus T Alley[2],
Shreyas S Vasanawala[2], Michael Lustig[1]



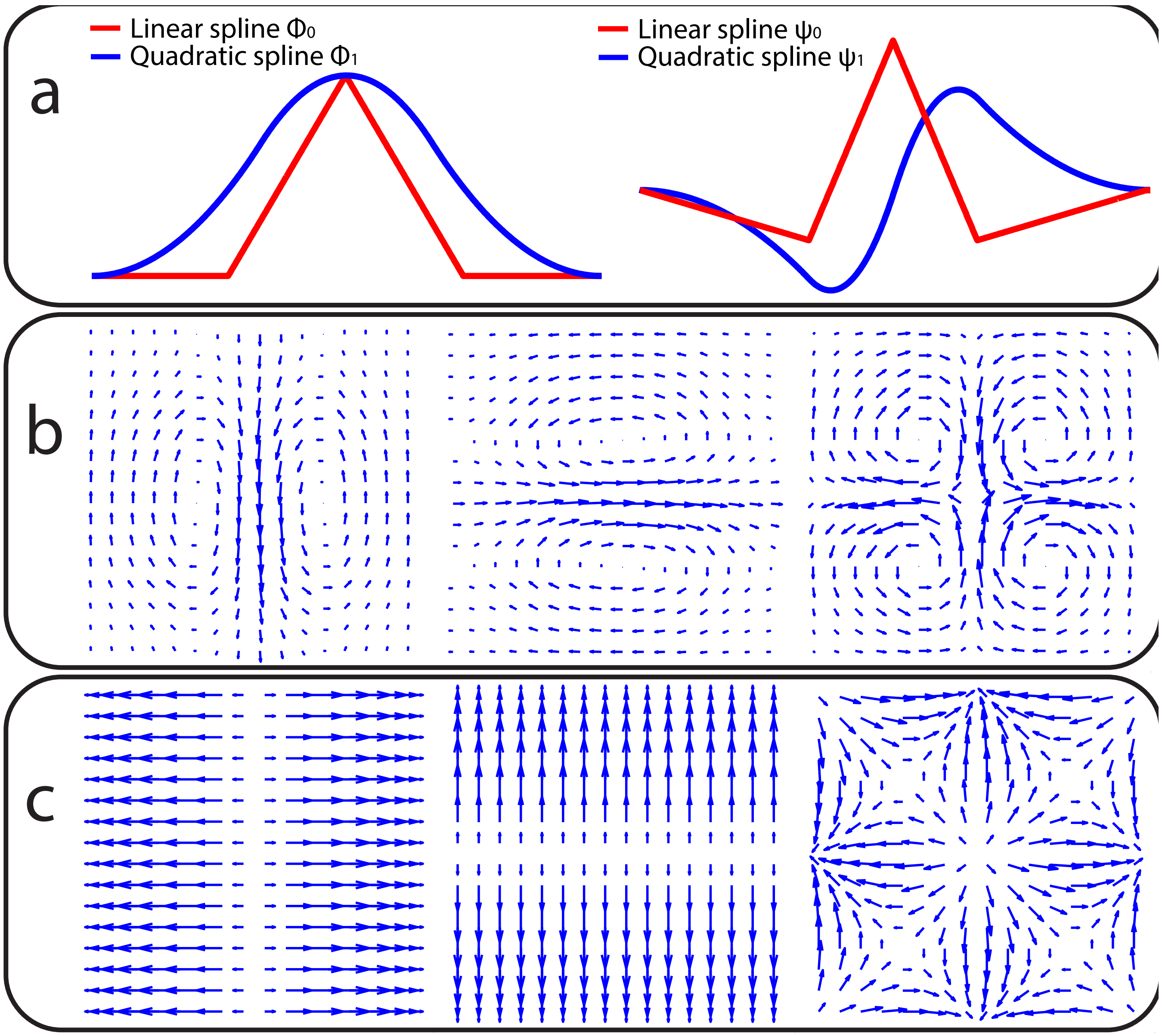


courtesy, Frank Ong and Marcus Alley

Noisy Flow Data



Divergence Free Wavelets



Divergence-Free Wavelet Denoising