EE123
Digital Signal Processing

Lecture 14
Sampling

Announcements

- Ham exam Th 3/12 7-10+,The Woz Soda hall
- Lab:
  - –Who is having trouble?

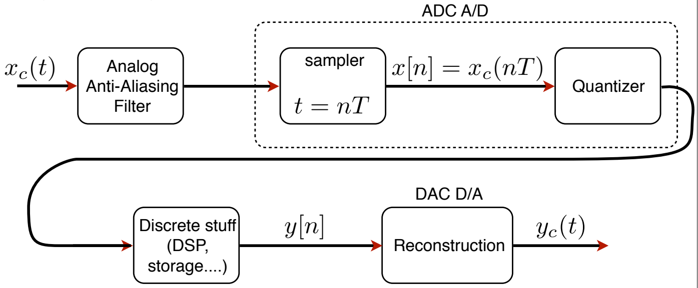
What is this Phenomena?

Sampling of Continuous Time Signals (Ch.4)

- Sampling:
  - –Conversion from C.T (not quantized) into D.T (usually quantized)
- Reconstruction
  - –D.T (quantized) to C.T
- Why?
  - –Digital storage (audio, images, videos)
  - –Digital communications (fiber optics, cellular...)
  - –DSP (compression, correction, restoration)
  - –Digital synthesis (speech, graphics)
  - – Learning

Sampling of C.T. Signals

- Typical System:



ADC A/D

$x_c(t)$

Analog Anti-Aliasing Filter

sampler

$t = nT$

$x[n] = x_c(nT)$

Quantizer

DAC D/A

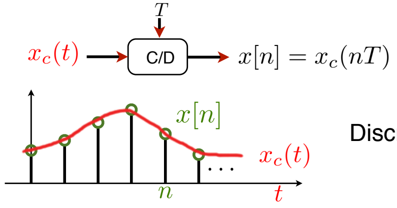
Discrete stuff (DSP, storage....)

$y[n]$

Reconstruction

$y_c(t)$

Ideal Sampling Model

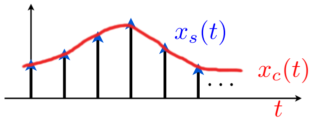


T

$x_c(t)$ → C/D → $x[n] = x_c(nT)$

Discrete and Continuous

define impulsive sampling:



Continuous

$$x_s(t) = \cdots + x_c(0)\delta(t) + x_c(T)\delta(t-T) + \cdots$$

$$x_s(t) = x_c \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

Ideal Sampling Model

$$x_s(t) = x_c \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

- Not physical: used for modeling & derivations

$$x[n] \leftrightarrow x_s(t) \leftrightarrow x_c(t)$$

- How is x[n] related to x_s(t) in freq. domain?

Frequency Domain Analysis

- How is x[n] related to x_s(t) in the Freq. Domain?

$x_s(t)$:C.T $\qquad X_s(j\Omega) = \sum_n x_c(nT) e^{-j\Omega nT}$

$x[n]$:D.T $\qquad X(e^{j\omega}) = \sum_n x[n] e^{-j\omega n}$

$\omega = \Omega T$

$$\boxed{X(e^{j\omega}) = X_s(j\Omega)|_{\Omega = \omega/T}} \qquad \boxed{X_s(j\Omega) = X(e^{j\omega})|_{\omega = \Omega T}}$$

Frequency Domain Analysis

- How is $x_s(t)$ related to $x_c(t)$?

$$x_s(t) = x_c(t) \underbrace{\sum_n \delta(t - nT)}_{\triangleq s(t)}$$

Frequency Domain Analysis

- How is $x_s(t)$ related to $x_c(t)$?

$$x_s(t) = x_c(t) \underbrace{\sum_n \delta(t - nT)}_{\triangleq s(t)}$$

Notation Break

recall $\text{Ш}(t) = \sum_n \delta(t-n)$

$$s(t) = \sum_n \delta(t-nT) = \sum_n \delta\left(T\left(\frac{t}{T} - n\right)\right) =$$

recall $\delta(at) = \frac{1}{|a|}\delta(t)$

$$= \frac{1}{T} \sum_n \delta\left(\frac{t}{T} - n\right) = \frac{1}{T} \text{Ш}\left(\frac{t}{T}\right)$$

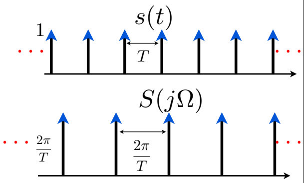
Frequency Domain Analysis

- How is $x_s(t)$ related to $x_c(t)$?

$$x_s(t) = x_c(t) \underbrace{\sum_n \delta(t - nT)}_{\triangleq s(t)}$$

$$s(t) \leftrightarrow S(j\Omega)$$

$$S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\Big(\Omega - \underbrace{\frac{2\pi}{T}}_{=\Omega_s} k\Big)$$

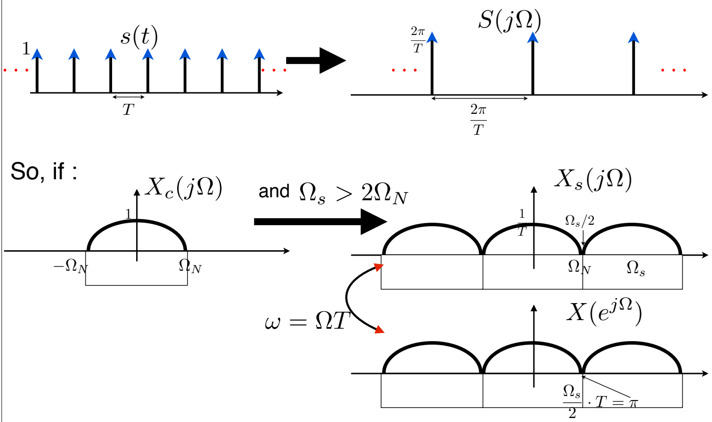


Frequency Domain Analysis

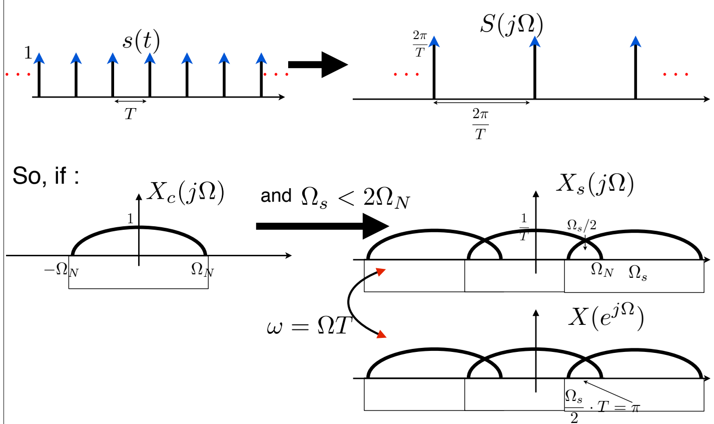
$$\boxed{\begin{aligned} X_s(j\Omega) &= \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega) \\ &= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - \Omega_s)) \quad | \quad \Omega_s = \frac{2\pi}{T} \end{aligned}}$$

- X_s is replication of X_c !

Frequency Domain Analysis

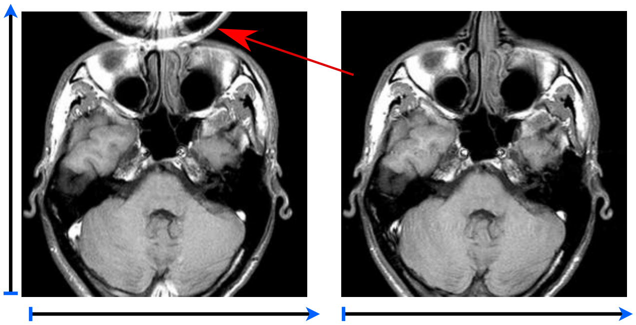


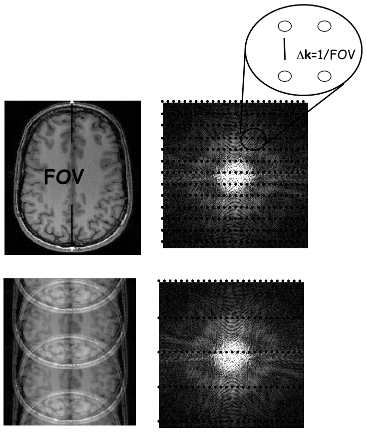
Aliasing

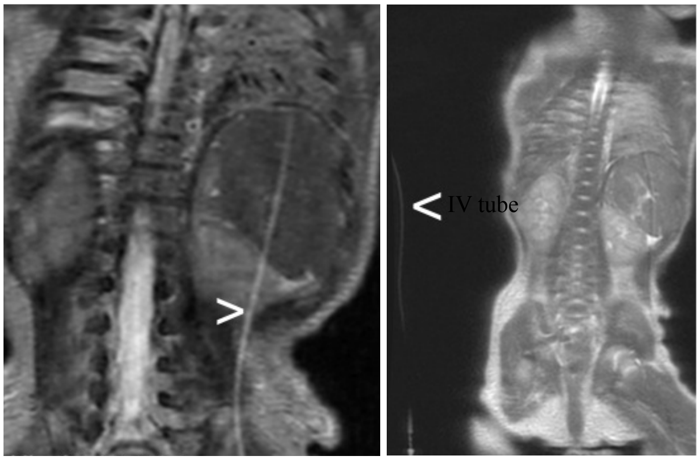


Aliasing

Q: What is the difference in acquisition between the two images ?







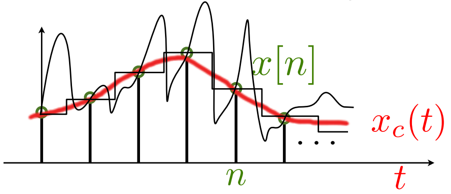
Reconstruction of Bandlimited Signals

- Nyquist Sampling Thm: suppose $x_c(t)$ is bandlimited

$$X_c(j\Omega) = 0 \ \ \forall \ \ |\Omega| \geq \Omega_N$$

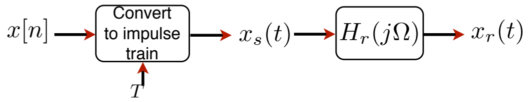
if $\Omega_s \geq 2\Omega_N$, then $x_c(t)$ can be uniquely determined from its samples $x[n] = x_c(nT)$

- Bandlimitedness is the key to uniqueness

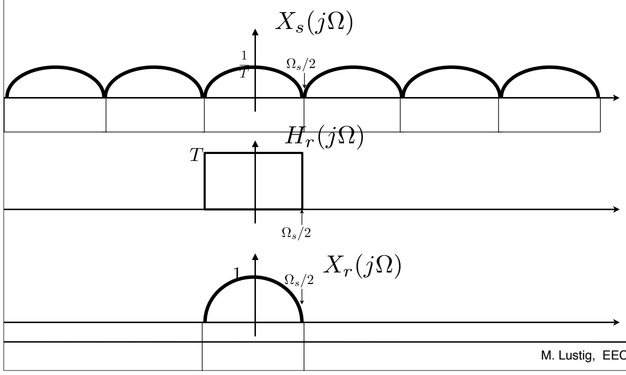


multiple signals go through the samples, but only one is bandlimited!

Reconstruction in Frequency Domain


$x[n]$ → Convert to impulse train → $x_s(t)$ → $H_r(j\Omega)$ → $x_r(t)$

T



Reconstruction in Time Domain

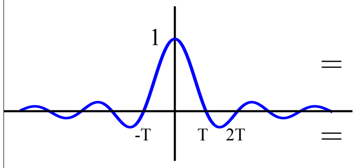
$$h_r(t) = \frac{1}{2\pi}\int_{-\Omega_s/2}^{\Omega_s/2} T e^{j\Omega t} d\Omega$$

$$= \frac{T}{2\pi}\frac{1}{jt} s^{j\Omega t}\Big|_{-\Omega_s/2}^{\Omega_s/2}$$

$$= \frac{T}{\pi t}\frac{e^{j\frac{\Omega_s}{2}t} - e^{-j\frac{\Omega_s}{2}t}}{2j}$$

$$= \frac{T}{\pi t}\sin(\frac{\Omega_s}{2}t) = \frac{T}{\pi t}\sin(\frac{\pi}{T}t)$$

$$= \mathrm{sinc}(\frac{t}{T})$$

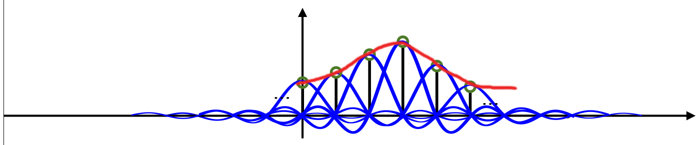


Reconstruction in Time Domain

$$x_r(t) = x_s(t) * h_r(t) = \left(\sum_n x[n]\delta(t-nT)\right) * h_r(t)$$

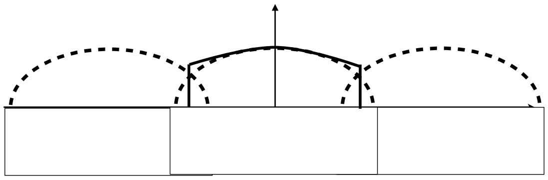
$$= \sum_n x[n] h_r(t-nT)$$

The sum of “sincs gives $x_r(t)$ ⇒ Unique signal bandlimited by Ω_s



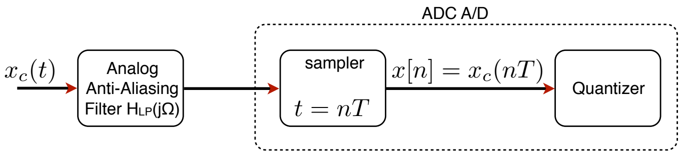
Aliasing

- If $\Omega_N > \Omega_s/2$, $x_r(t)$ an aliased version of $x_c(t)$

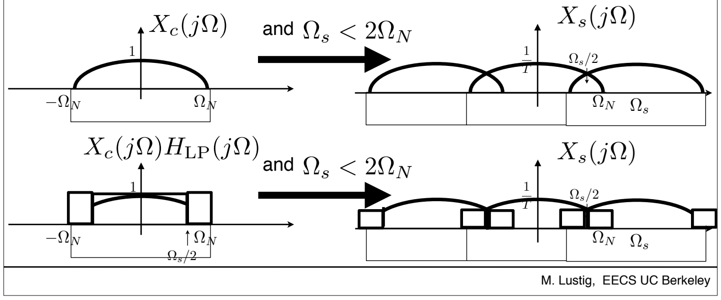


$$X_r(j\Omega) = \begin{cases} TX_s(j\Omega) & \text{if} \quad |\Omega| \le \Omega_s/2 \\ 0 & \text{otherwise} \end{cases}$$

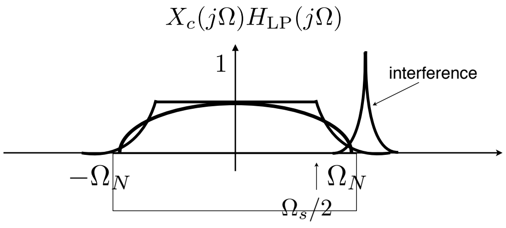
Anti-Aliasing

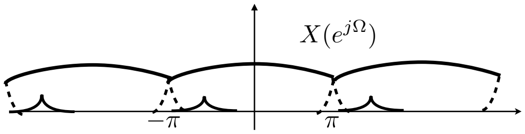

ADC A/D

$x_c(t)$ → Analog Anti-Aliasing Filter $H_{LP}(j\Omega)$ → sampler $t = nT$ → $x[n] = x_c(nT)$ → Quantizer



Non Ideal Anti-Aliasing





SDR non-perfect anti-Aliasing Demo