EE123
Digital Signal Processing

Lecture 17
Lab III
Polyphase Filters

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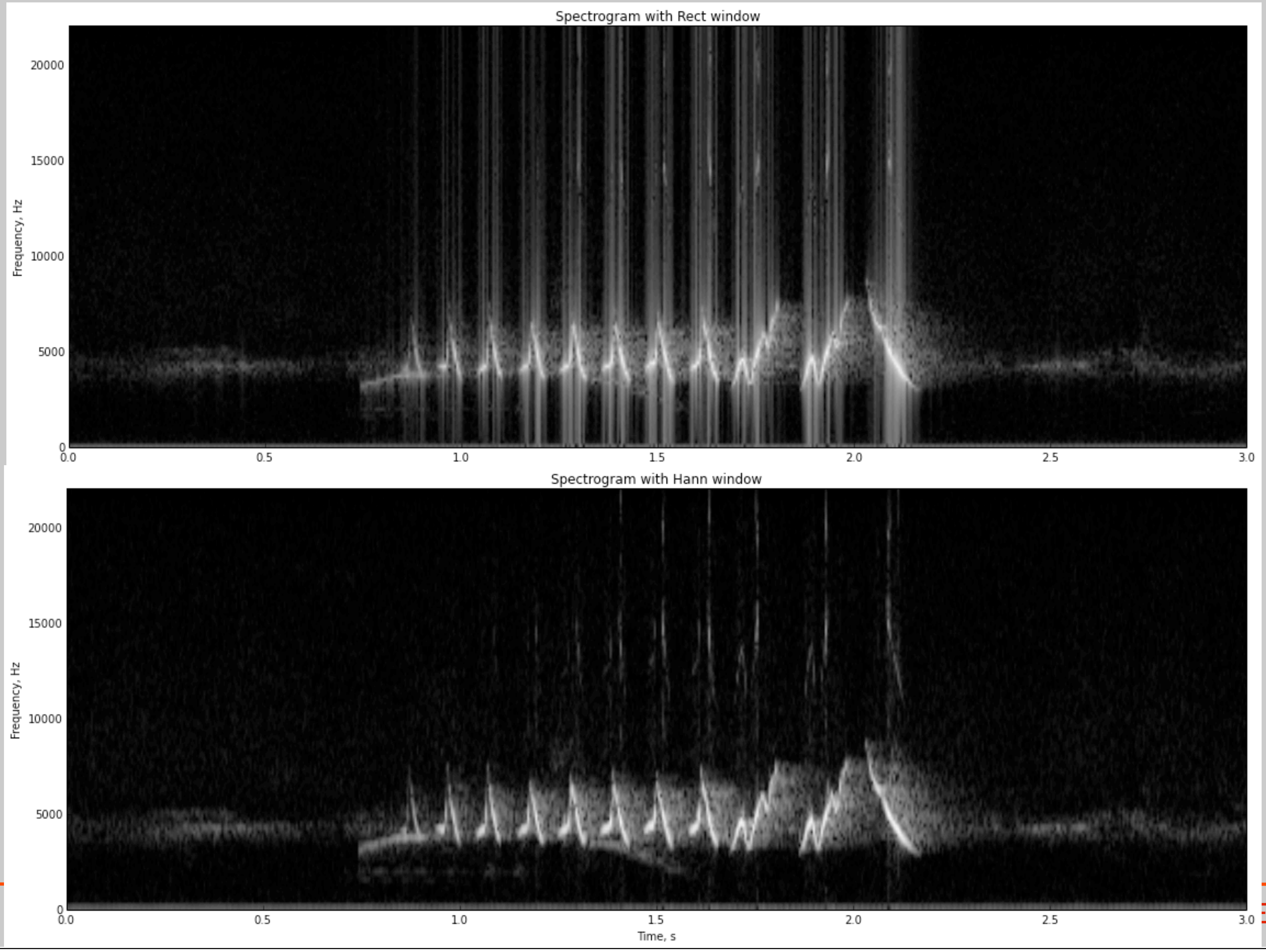
Topics

- Last time
  - Changing Sampling Rate via DSP
  - Upsampling
  - Rational resampling
- Today
  - Lab III
  - Interchanging Compressors/Expanders and filtering
  - Polyphase decomposition
  - Multi-rate processing

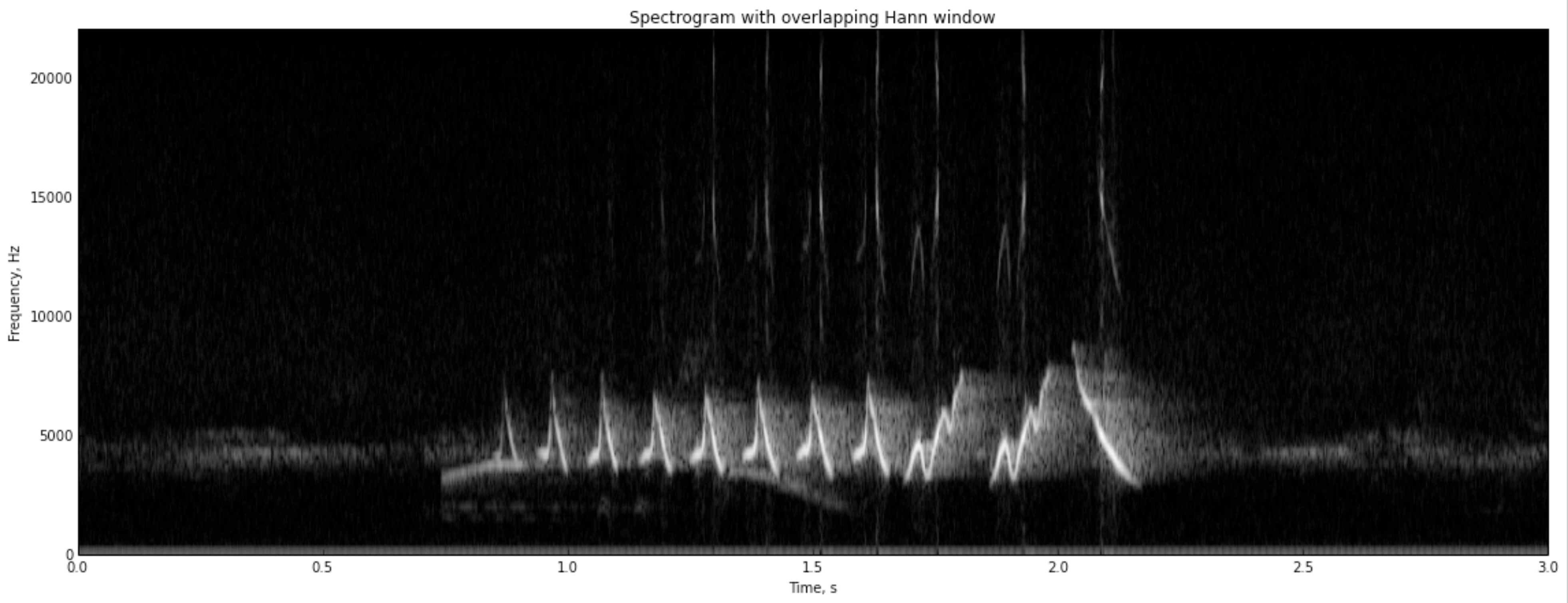
Lab III - Time-Frequency

- compute spectrograms with w/o windowing



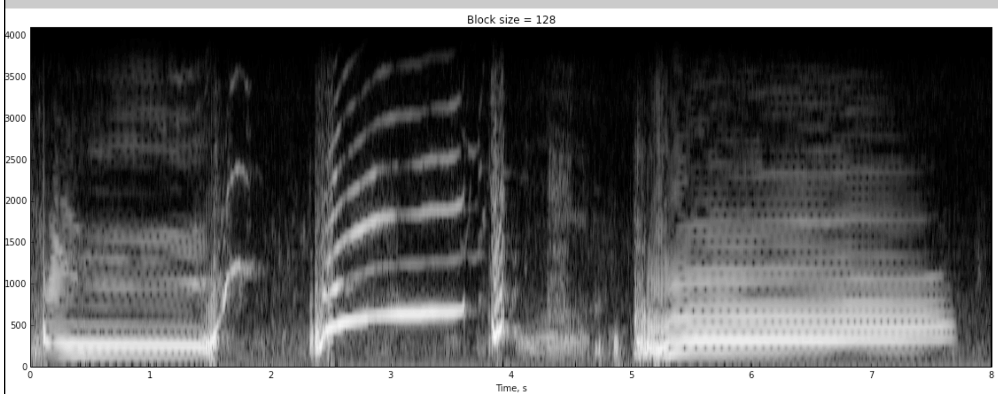
Lab III - Time-Frequency

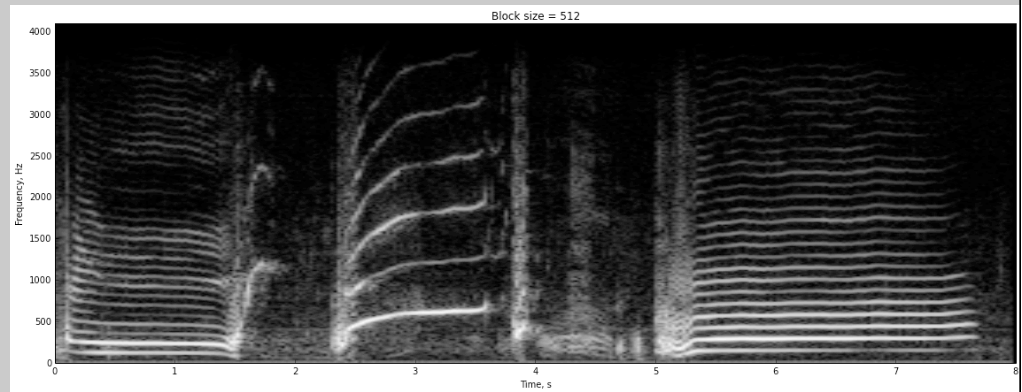
- Compute with overlapping window

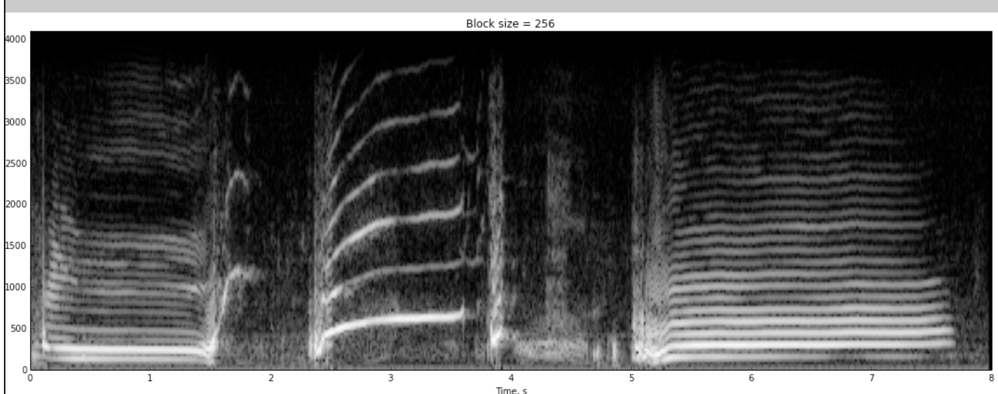


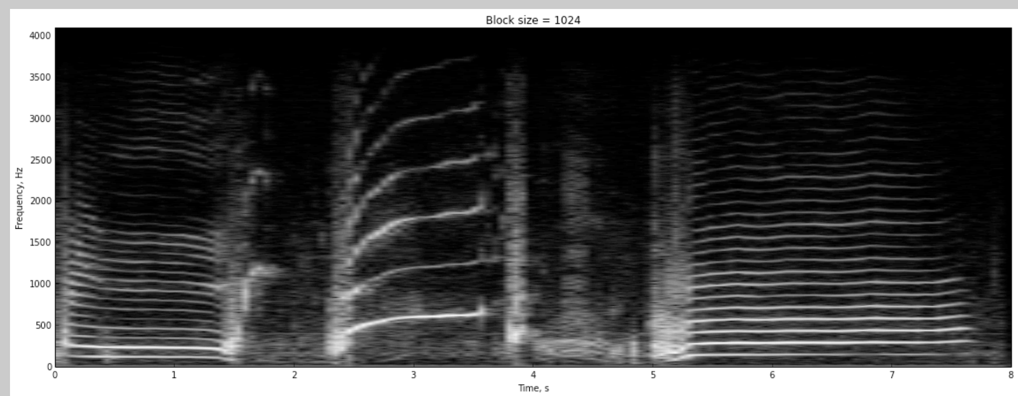
Lab III - Time-Frequency

- Look at temporal/frequency resolution tradeoffs:

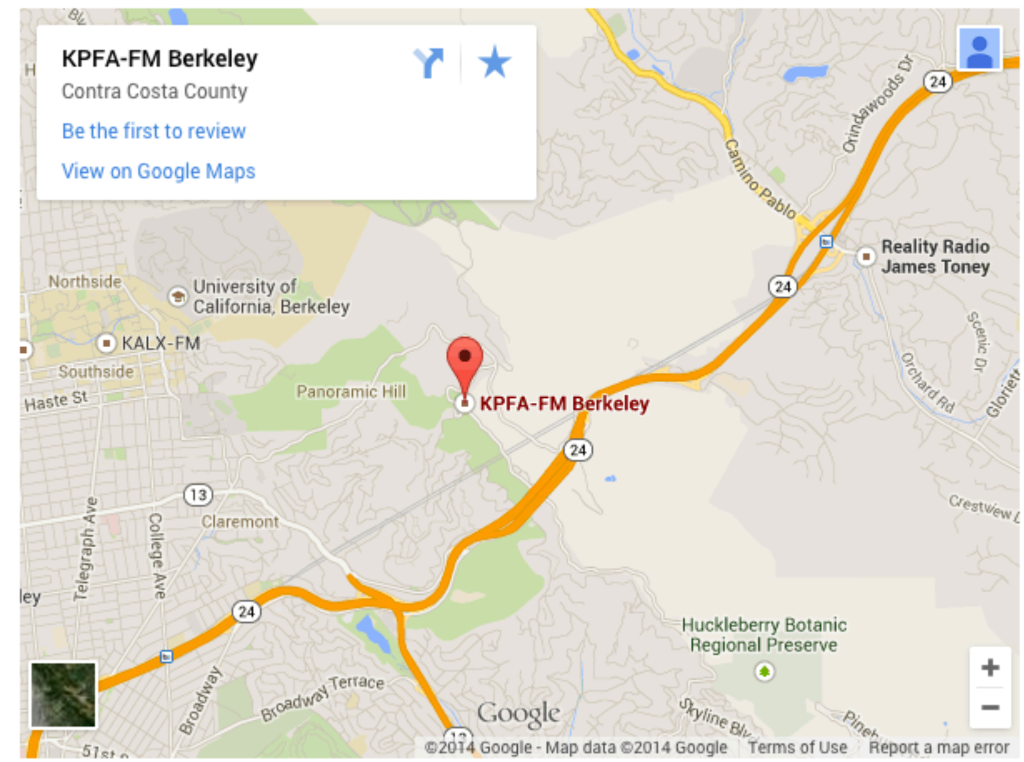




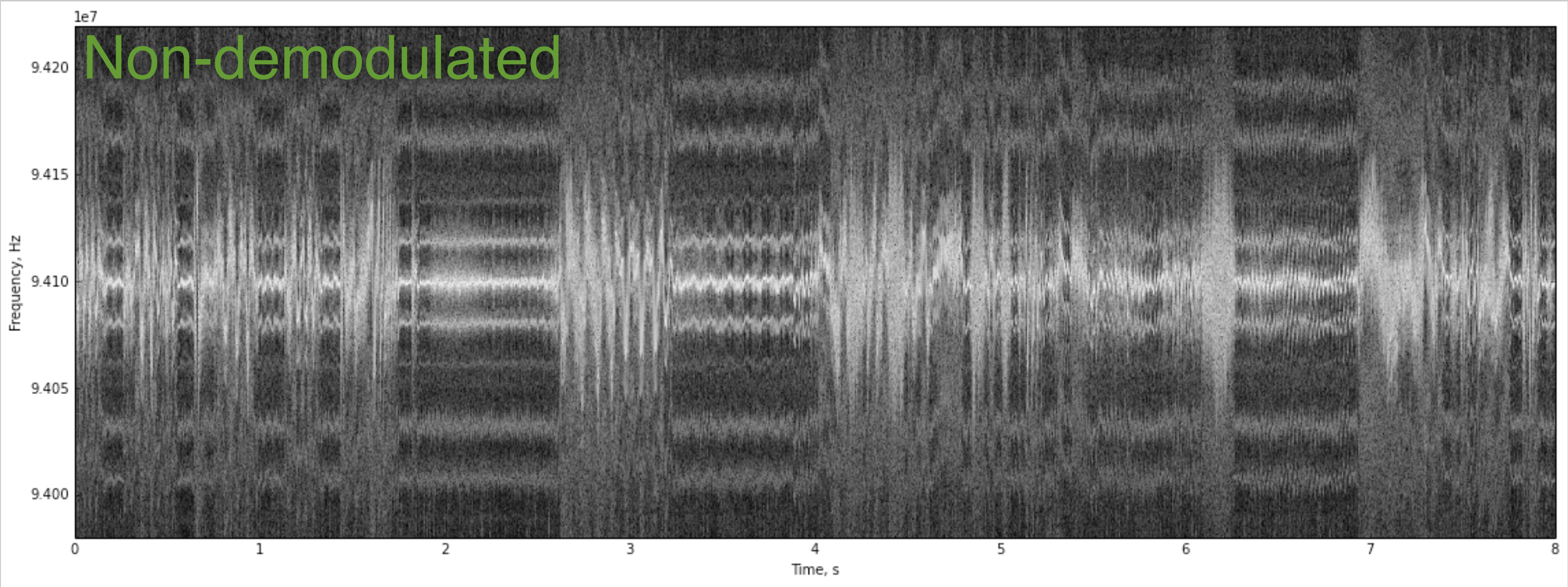


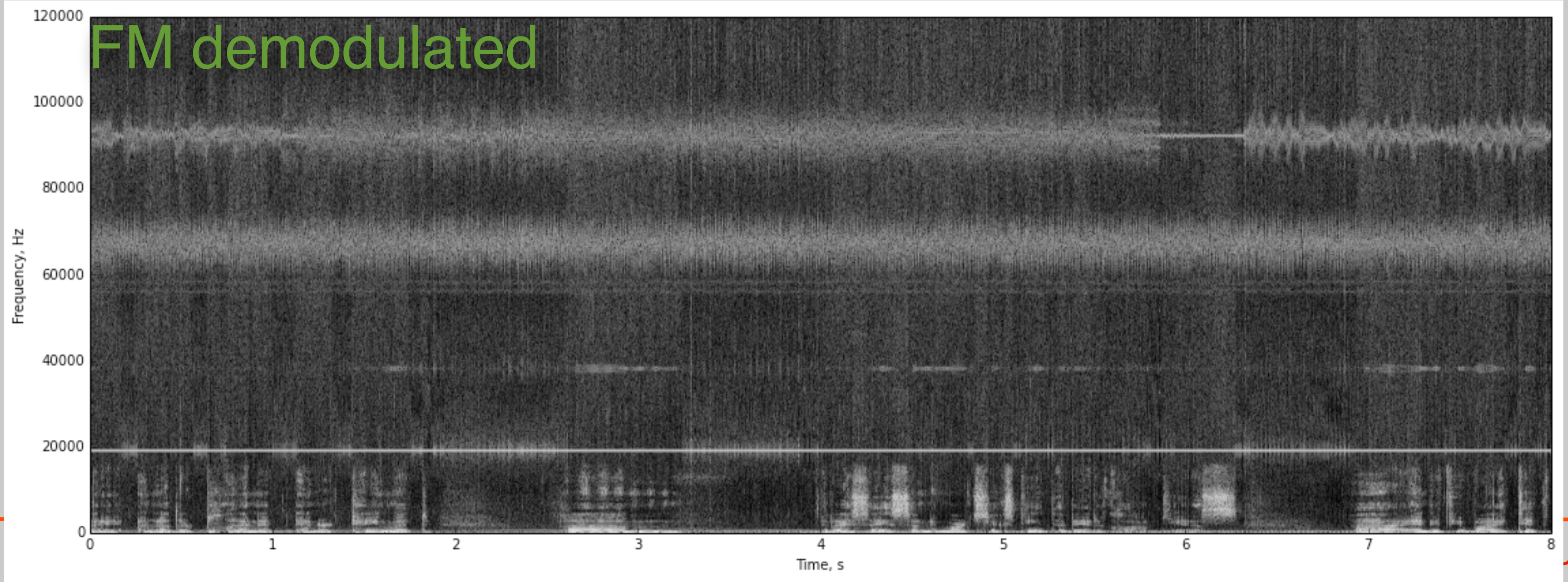


FM Broadcast Radio - KPFA 94.1MHz

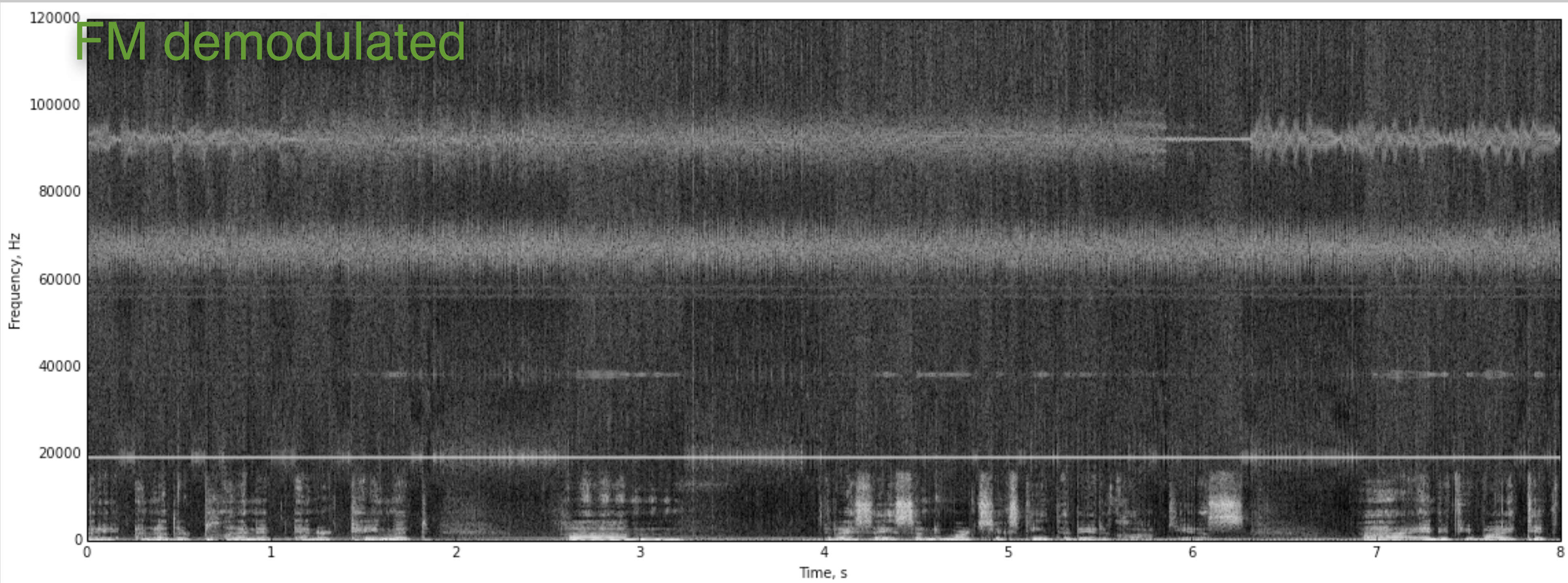


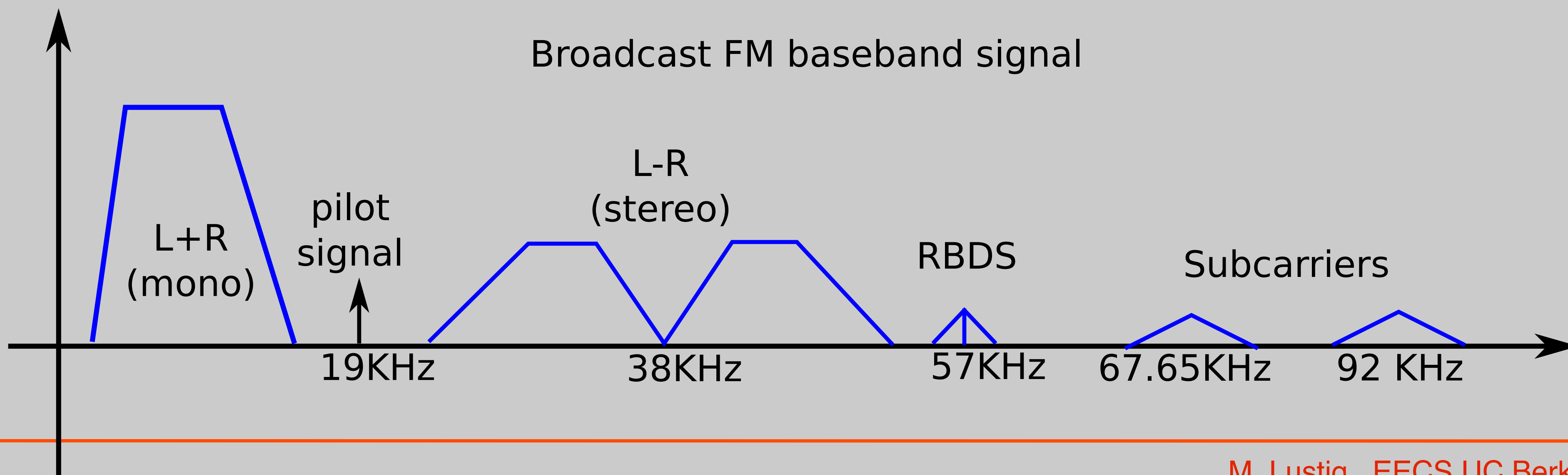
Spectrogram of Broadcast FM





Spectrogram of Broadcast FM

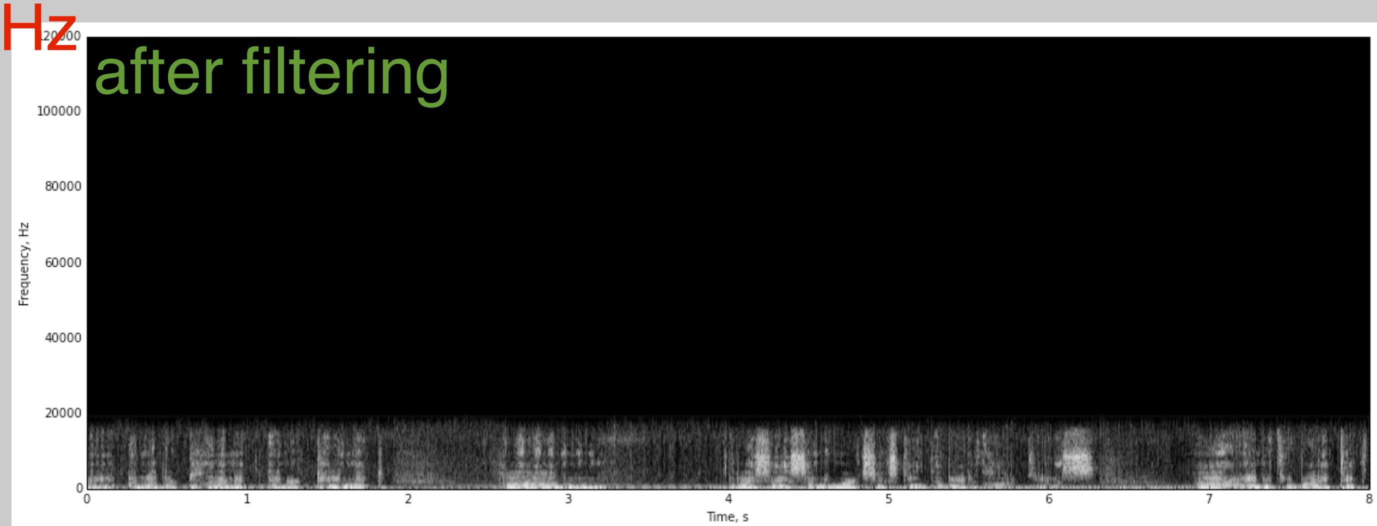




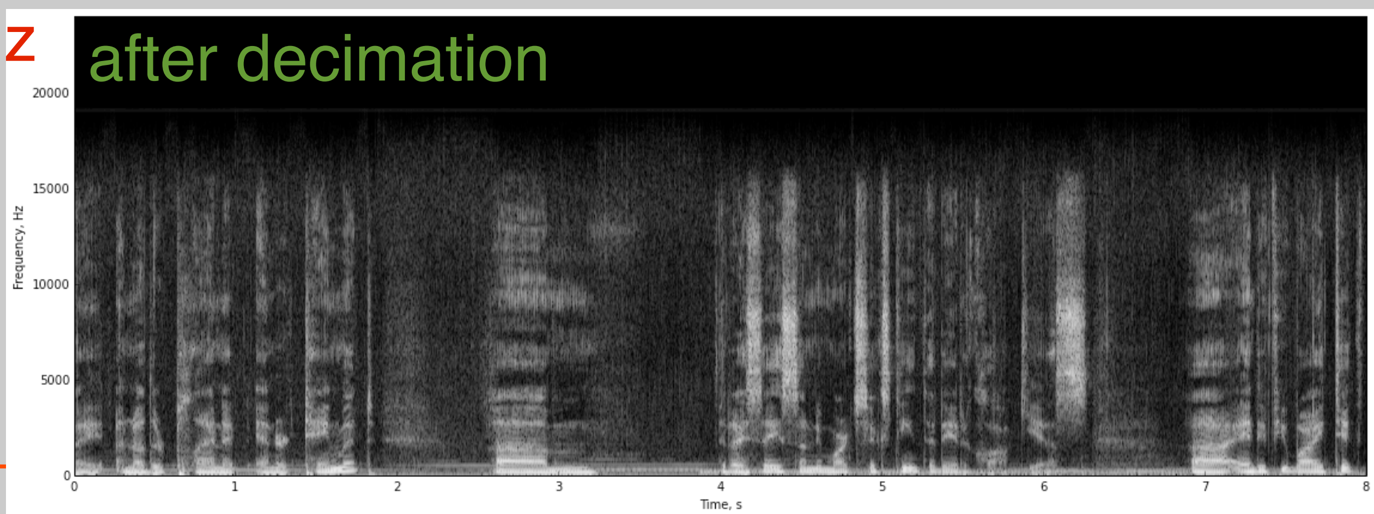
Filter Mono and down

- To play we need to filter the right signal
- Downsample to 48KHz so we can play on the computer

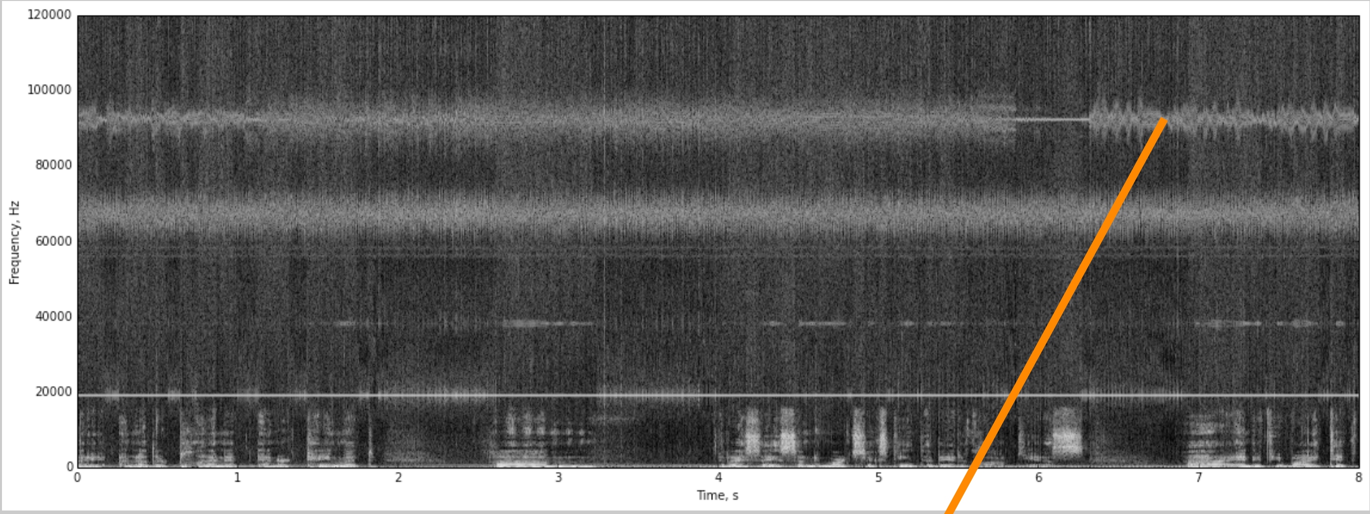
120KHz



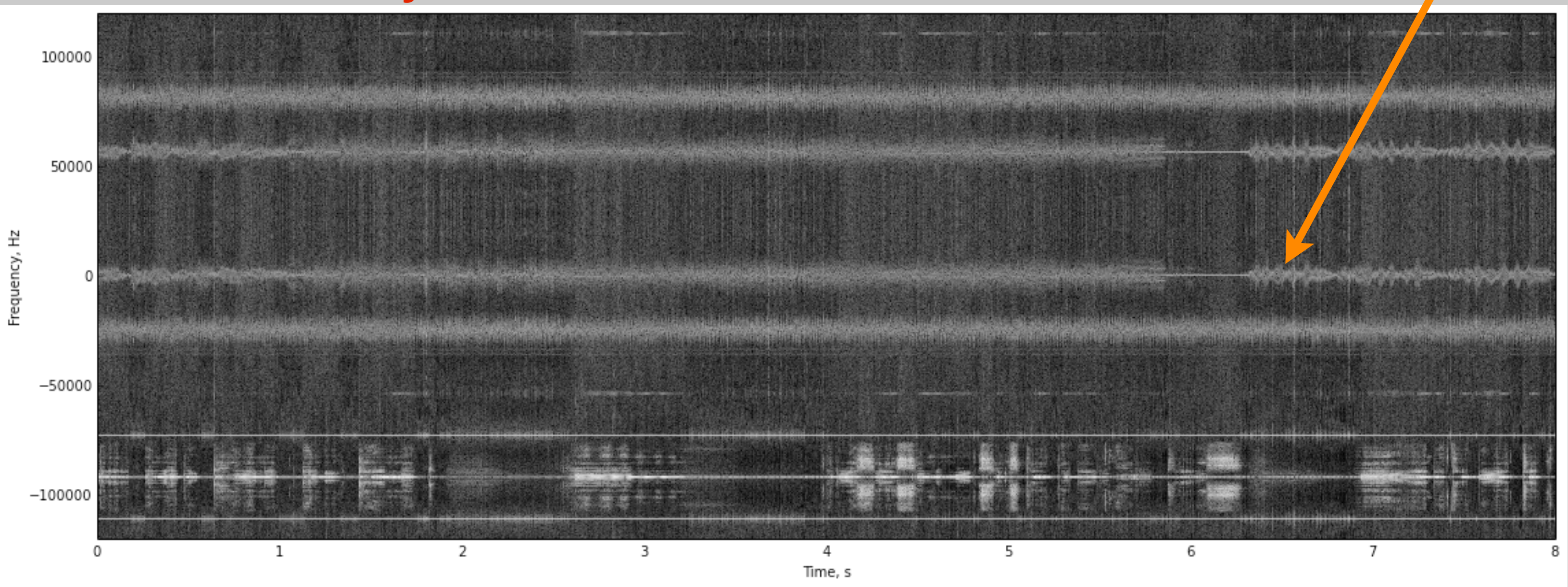
24KHz



Demodulate subcarriers: Example 92KHz

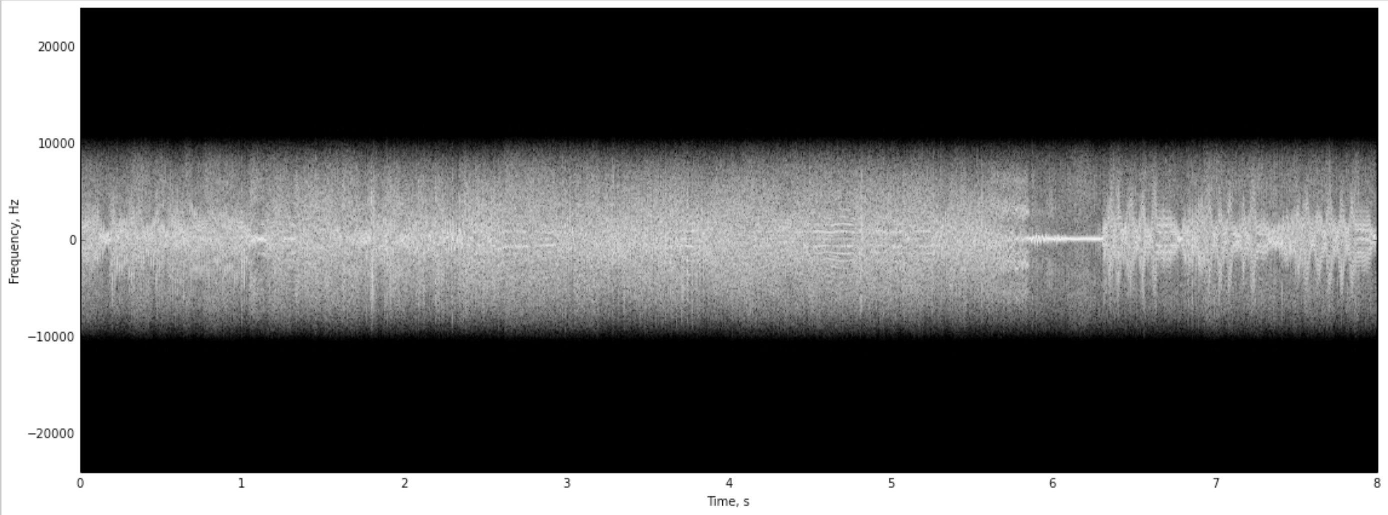


demodulate by 92KHz

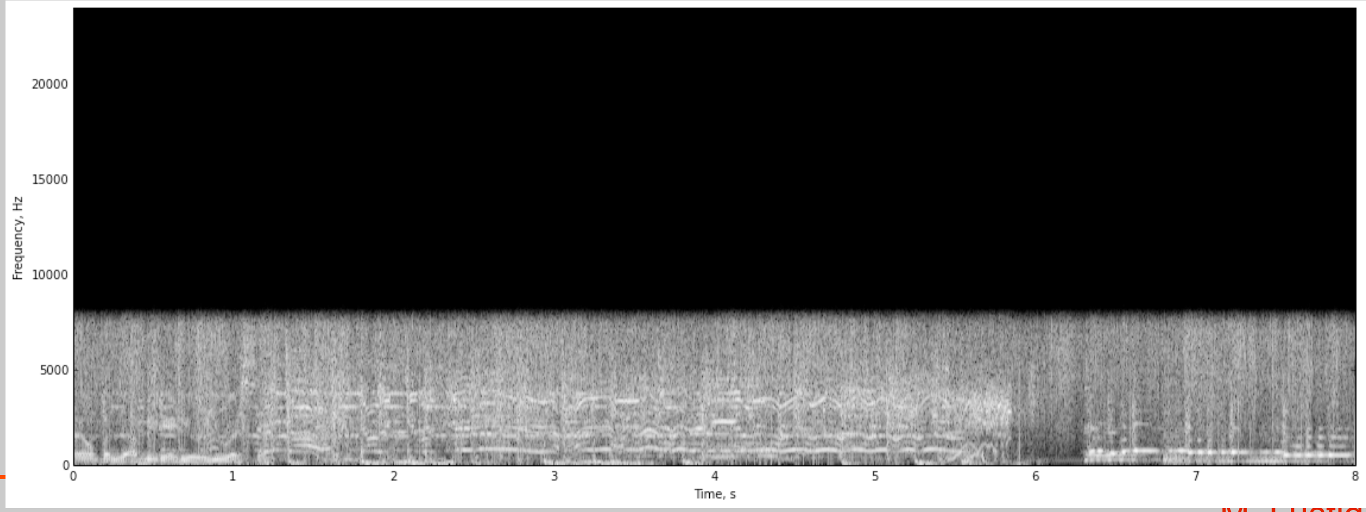


Demodulate subcarriers: Example 92KHz

- Filter and decimate

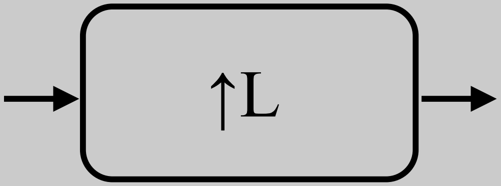


- FM demodulate and filter

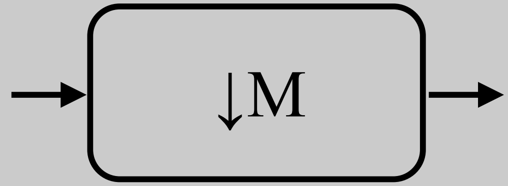


- What if we want to resample by 1.01T?
  - Expand by L=100
  - Filter π/101 ($$$$$)
  - Downsample by M=101
- Fortunately there are ways around it!
  - Called multi-rate
  - Uses compressors, expanders and filtering

Interchanging Operations



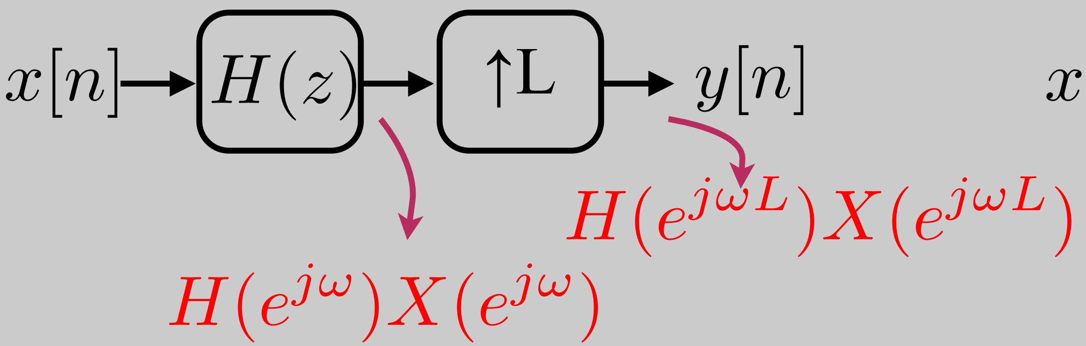
“expander”

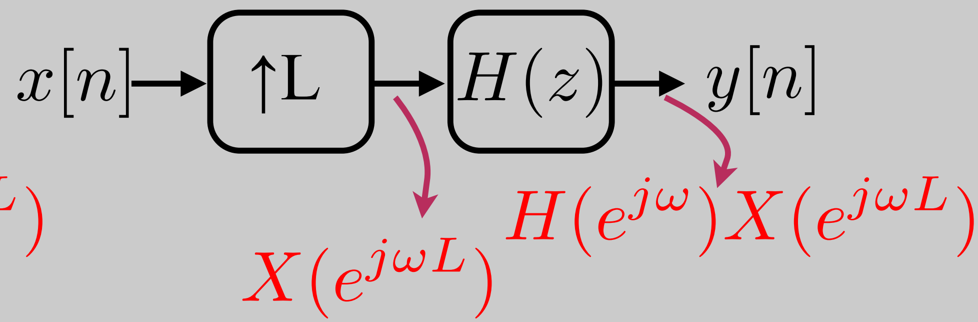


“compressor”

not LTI!

Note:





Interchanging Operations

$$\rightarrow \boxed{\uparrow \mathrm{L}} \rightarrow$$

"expander"

$$\rightarrow \boxed{\downarrow \mathrm{M}} \rightarrow$$

"compressor"

<u>not LTI!</u>

<u>Note:</u>

$$x[n] \rightarrow \boxed{H(z)} \xrightarrow[H(e^{j\omega})X(e^{j\omega})]{} \boxed{\uparrow \mathrm{L}} \xrightarrow[H(e^{j\omega L})X(e^{j\omega L})]{} y[n] \quad \neq \quad x[n] \rightarrow \boxed{\uparrow \mathrm{L}} \xrightarrow[X(e^{j\omega L})]{} \boxed{H(z)} \xrightarrow[H(e^{j\omega})X(e^{j\omega L})]{} y[n]$$

$$\equiv \quad x[n] \rightarrow \boxed{\uparrow \mathrm{L}} \xrightarrow[X(e^{j\omega L})]{} \boxed{H(z^L)} \xrightarrow[H(e^{j\omega L})X(e^{j\omega L})]{} y[n]$$

Interchanging Filter Expander

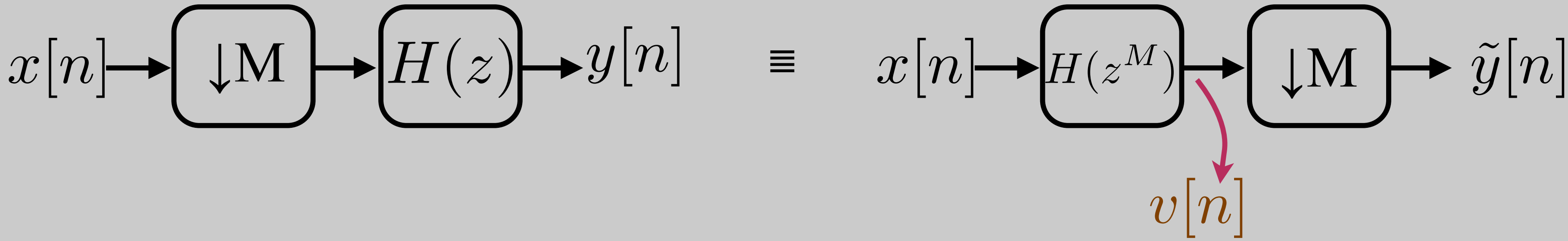
- Q: Can we move expander from Left to Right (with xform)?

$$\rightarrow \boxed{\uparrow \mathrm{L}} \rightarrow \boxed{H(z)} \rightarrow \quad \equiv ? \quad \rightarrow \boxed{H(z^{\frac{1}{L}})} \rightarrow \boxed{\uparrow \mathrm{L}} \rightarrow$$

- A: Yes, if H(z) is rational
  No, otherwise

Compressor

Claim:



Proof:

Compressor

Proof:

$$Y(e^{i\omega}) = H(e^{j\omega})\left(\frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j(\frac{\omega}{M}-\frac{2\pi i}{M})}\right)\right)$$

$$= \frac{1}{M}\sum_{i=0}^{M-1} \underbrace{H\left(e^{j(\omega-2\pi i)}\right)}_{H(e^{j\omega})} X\left(e^{j(\frac{\omega}{M}-\frac{2\pi i}{M})}\right)$$

$$= \frac{1}{M}\sum_{i=0}^{M-1} H\left(e^{jM(\frac{\omega}{M}-\frac{2\pi i}{M})}\right) X\left(e^{j(\frac{\omega}{M}-\frac{2\pi i}{M})}\right)$$

$$V(e^{j\omega}) = H(e^{j\omega M})X(e^{j\omega})$$

after compressor

Compressor

Claim:

$$x[n] \rightarrow \boxed{\downarrow \mathrm{M}} \rightarrow \boxed{H(z)} \rightarrow y[n] \quad \equiv \quad x[n] \rightarrow \boxed{H(z^M)} \xrightarrow{v[n]} \boxed{\downarrow \mathrm{M}} \rightarrow \tilde{y}[n]$$

Proof:

$$\begin{aligned} Y(e^{i\omega}) &= H(e^{j\omega})\left(\frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right)\right) \\ &= \frac{1}{M}\sum_{i=0}^{M-1} \underbrace{H\left(e^{j(\omega-2\pi i)}\right)}_{H(e^{j\omega})} X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right) \\ &= \frac{1}{M}\sum_{i=0}^{M-1} H\left(e^{jM\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right) X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right) \end{aligned}$$

after compressor

$$V(e^{j\omega}) = H(e^{j\omega M})X(e^{j\omega})$$

Q: Move Compressor from right to left?

A: Only if $H(z^{1/M})$ is rational!

Interchanging Operations

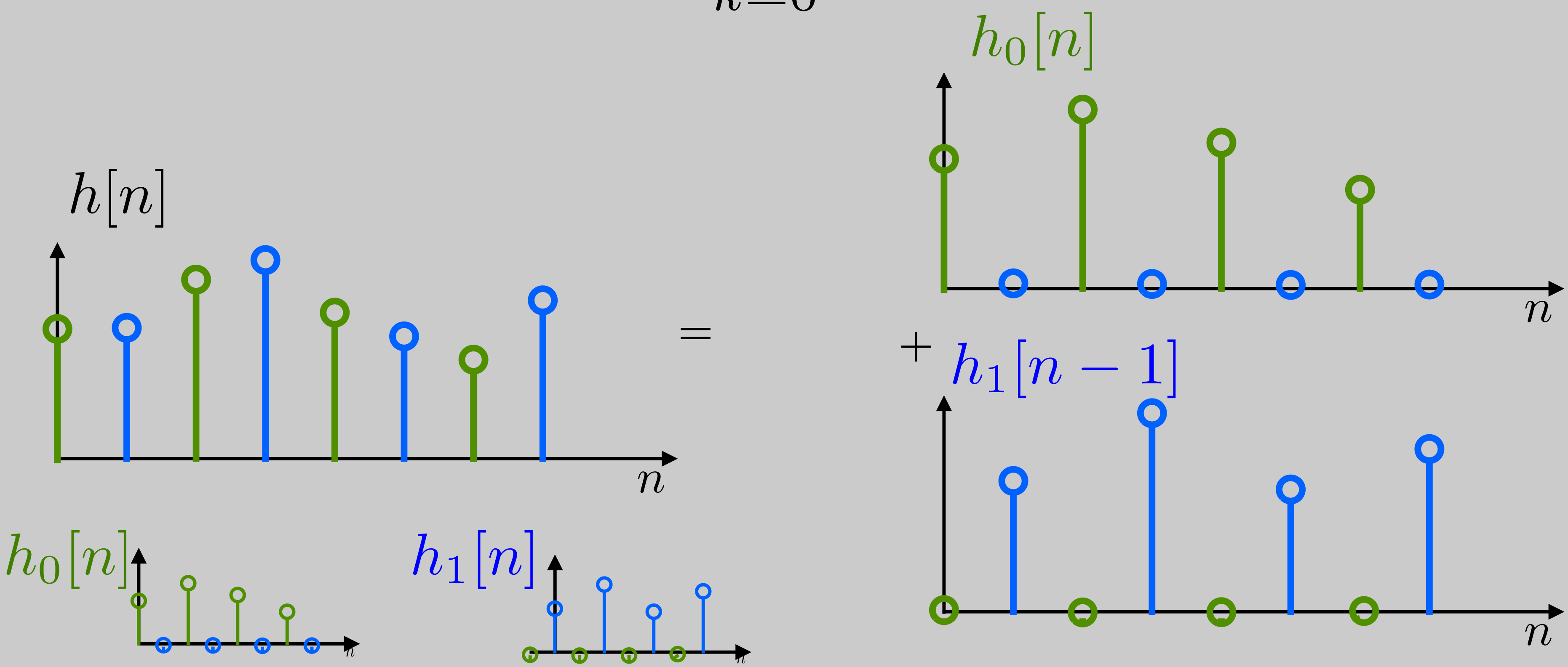
$$x[n] \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow \mathrm{L}} \rightarrow y[n] \quad \equiv \quad x[n] \rightarrow \boxed{\uparrow \mathrm{L}} \rightarrow \boxed{H(z^L)} \rightarrow y[n]$$

$$x[n] \rightarrow \boxed{\downarrow \mathrm{M}} \rightarrow \boxed{H(z)} \rightarrow y[n] \quad \equiv \quad x[n] \rightarrow \boxed{H(z^M)} \rightarrow \boxed{\downarrow \mathrm{M}} \rightarrow y[n]$$

Polyphase Decomposition

- We can decomposed an impulse response to:

$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

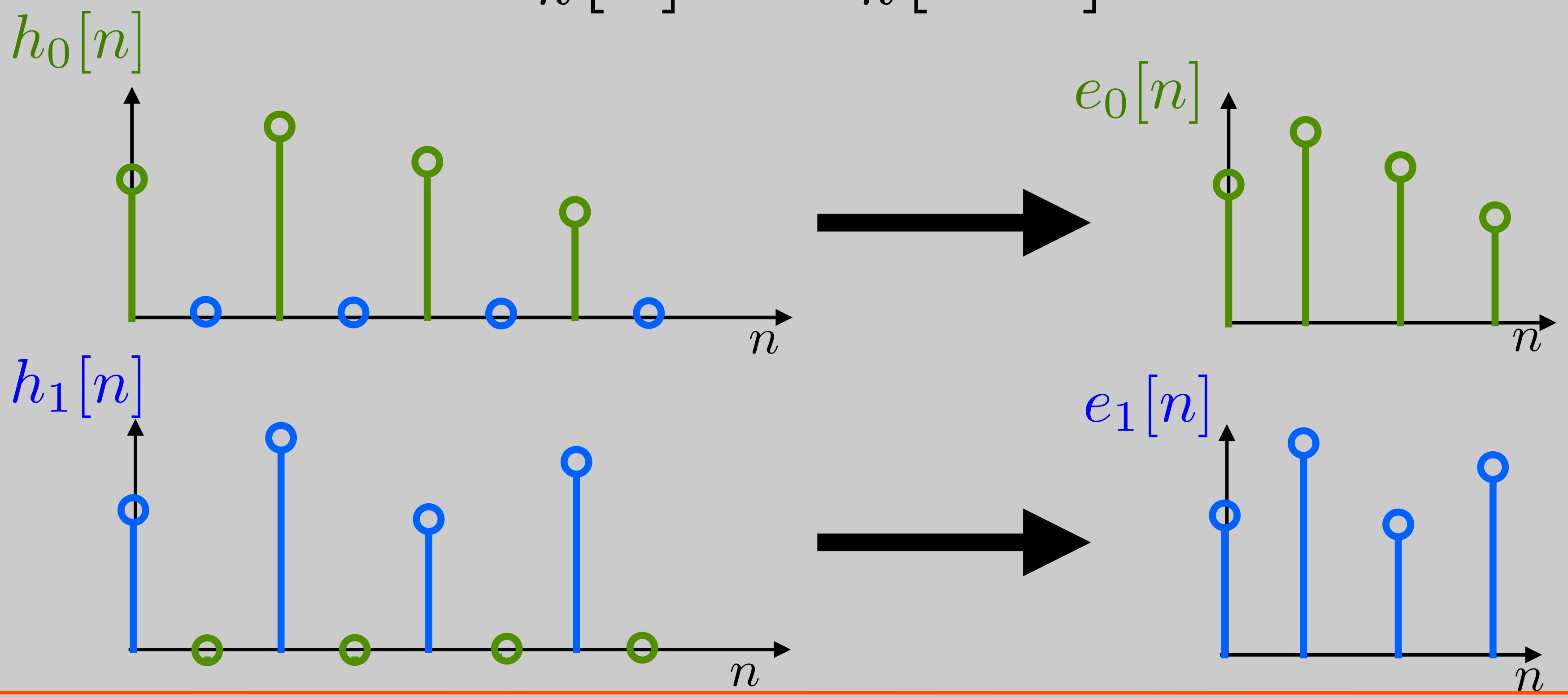


Polyphase Decomposition

- Define:

$$h_k[n] \rightarrow \boxed{\downarrow \mathrm{M}} \rightarrow e_k[n]$$

$$e_k[n] = h_k[nM]$$



Polyphase Decomposition

$$e_k[n] \longrightarrow \boxed{\uparrow \mathrm{M}} \longrightarrow h_k[n]$$

recall upsampling ⇒ scaling

$$H_k(z) = E_k(z^M)$$

Also, recall:

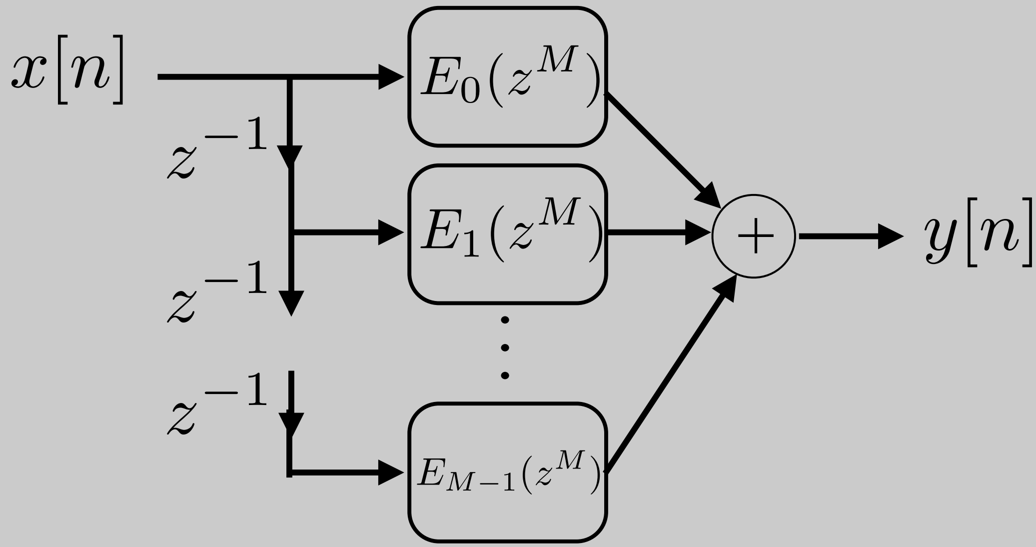
$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

So,

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$

Polyphase Decomposition

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$



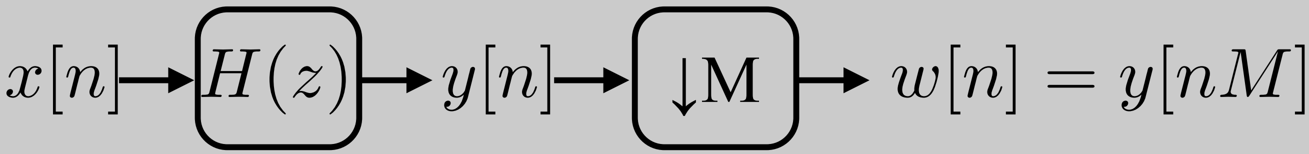
Why should you care?

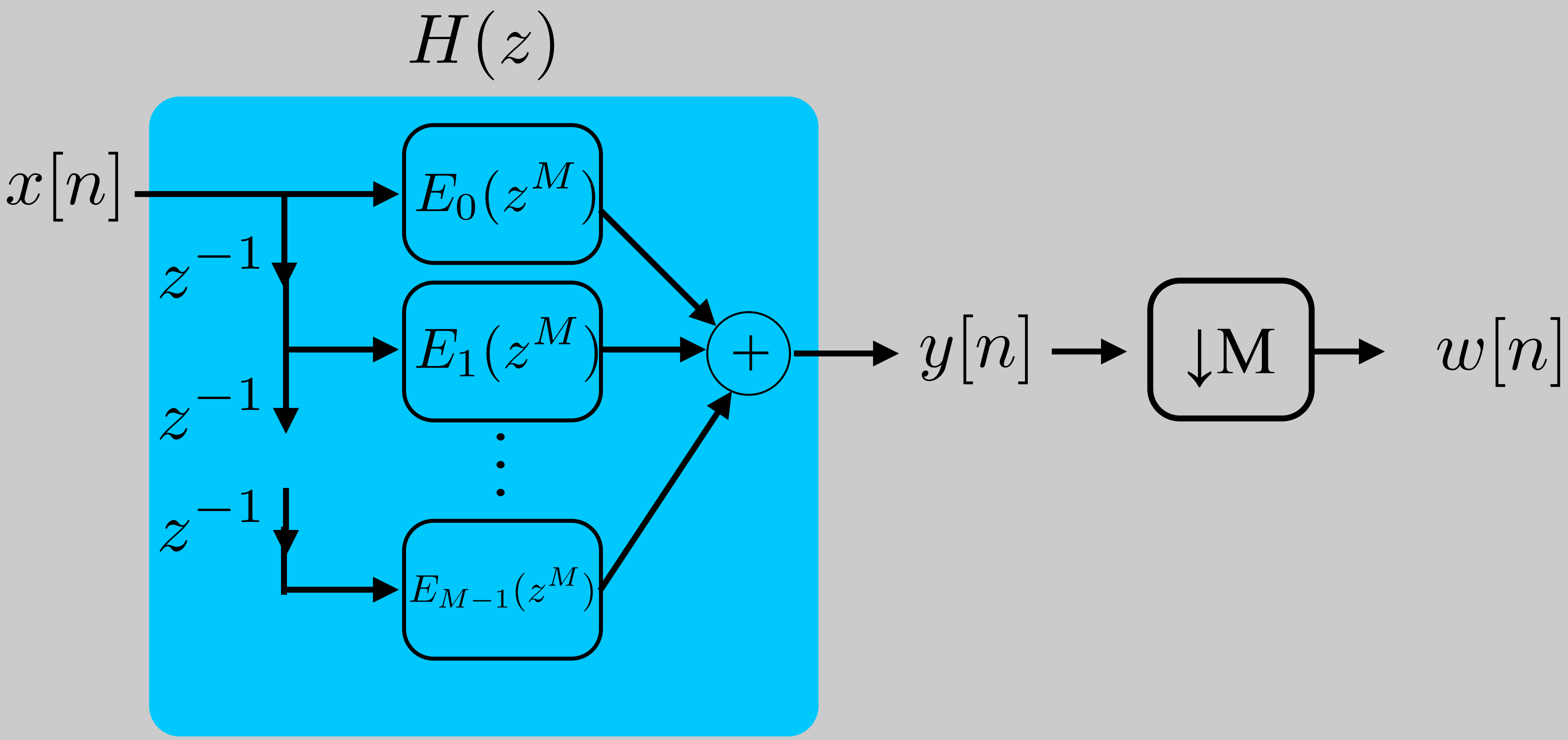
Polyphase Implementation of Decimation

$$x[n] \rightarrow \boxed{H(z)} \rightarrow y[n] \rightarrow \boxed{\downarrow \mathrm{M}} \rightarrow w[n] = y[nM]$$

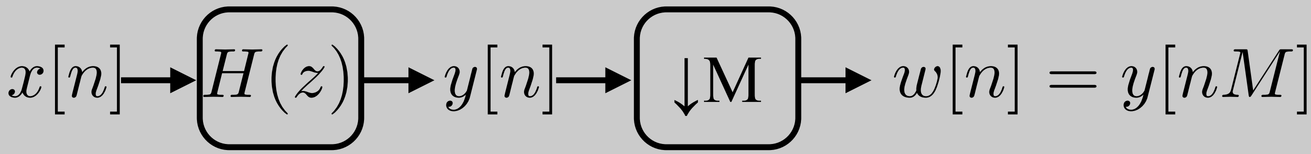
- Problem:
  - –Compute all y[n] and then throw away -- wasted computation!
    - For FIR length N ⇒ N mults/unit time
  - –Can interchange Filter with compressor?
    - Not in general!

Polyphase Implementation of Decimation

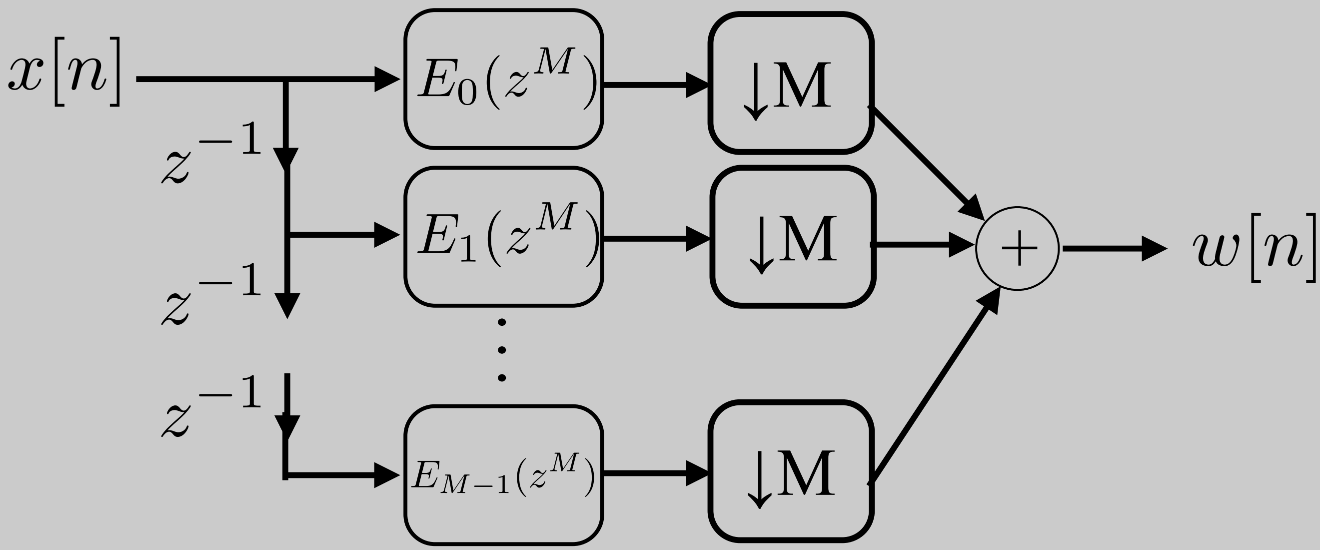




Polyphase Implementation of Decimation

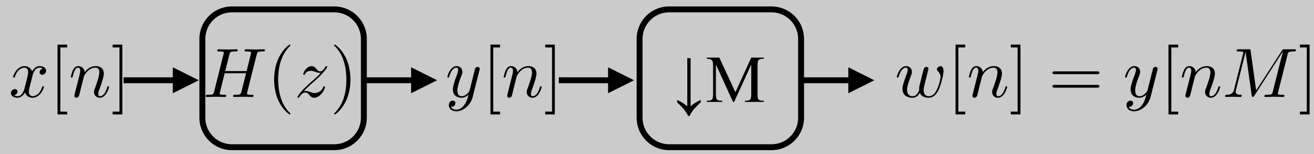


Interchange sum with decimation

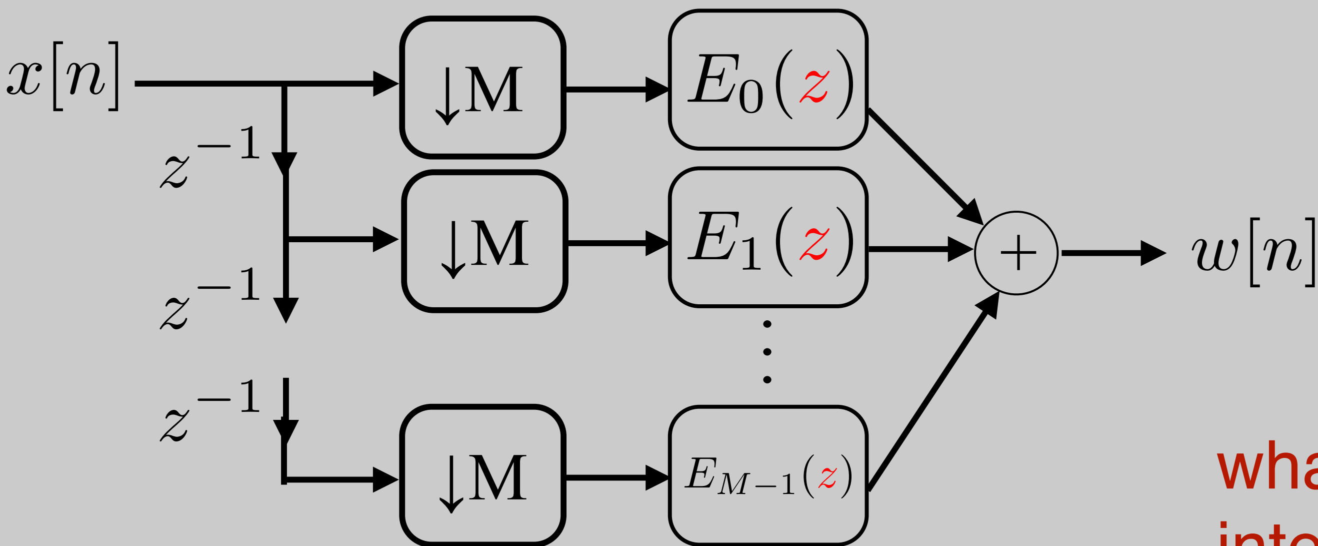


now, what can we do?

Polyphase Implementation of Decimation



Interchange filter with decimation



what about interpolation?

Computation:

Each Filter: N/M *(1/M) mult/unit time

Total: N/M mult/unit time