EE123
Digital Signal Processing

Lecture 19
Practical ADC/DAC

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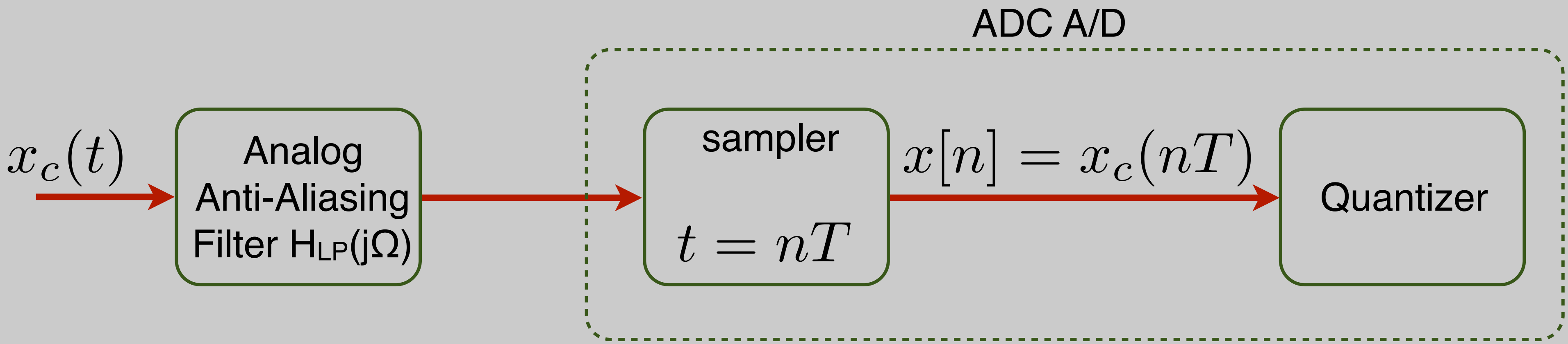
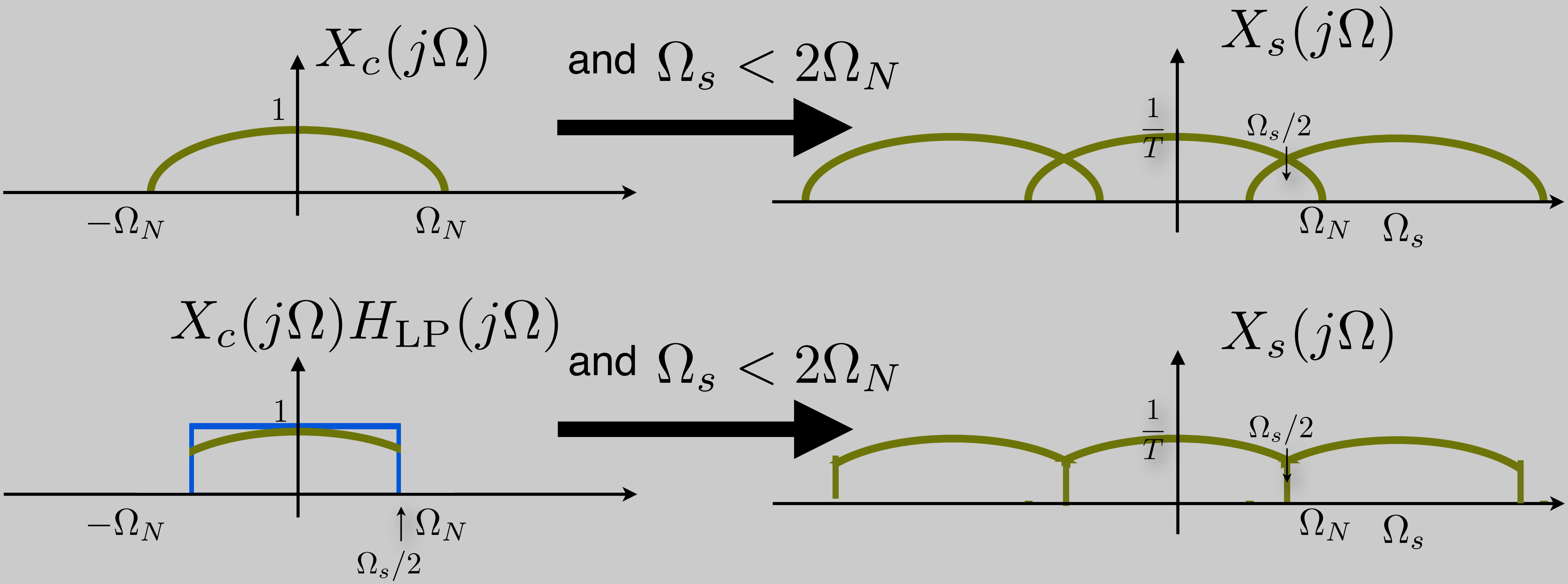
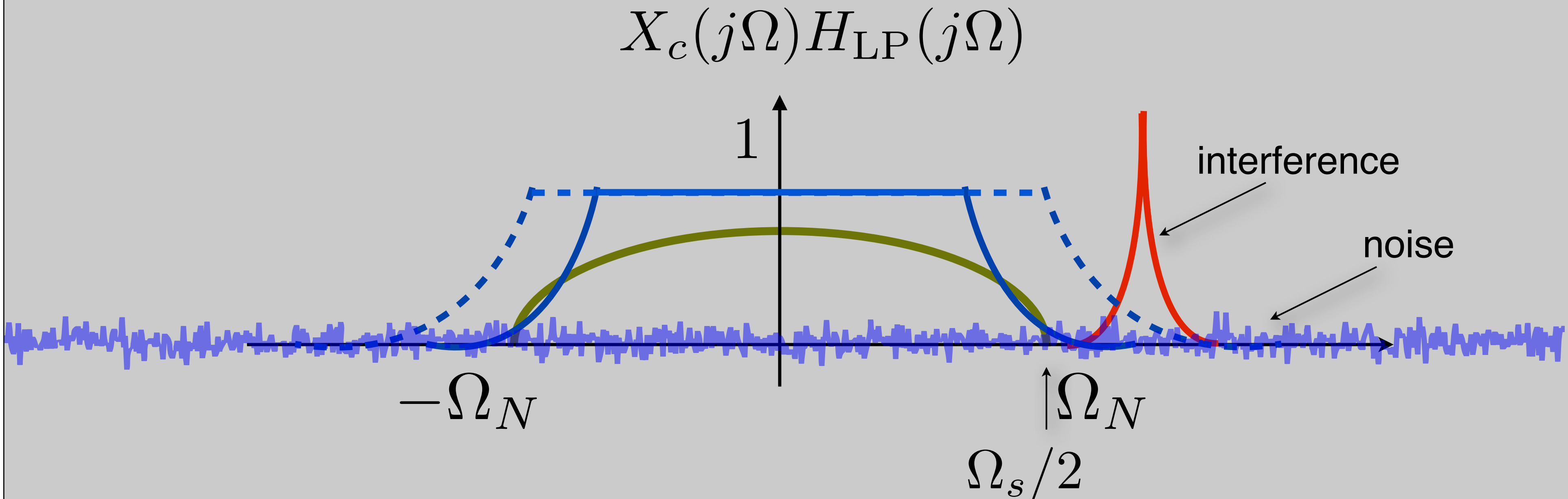
Ideal Anti-Aliasing



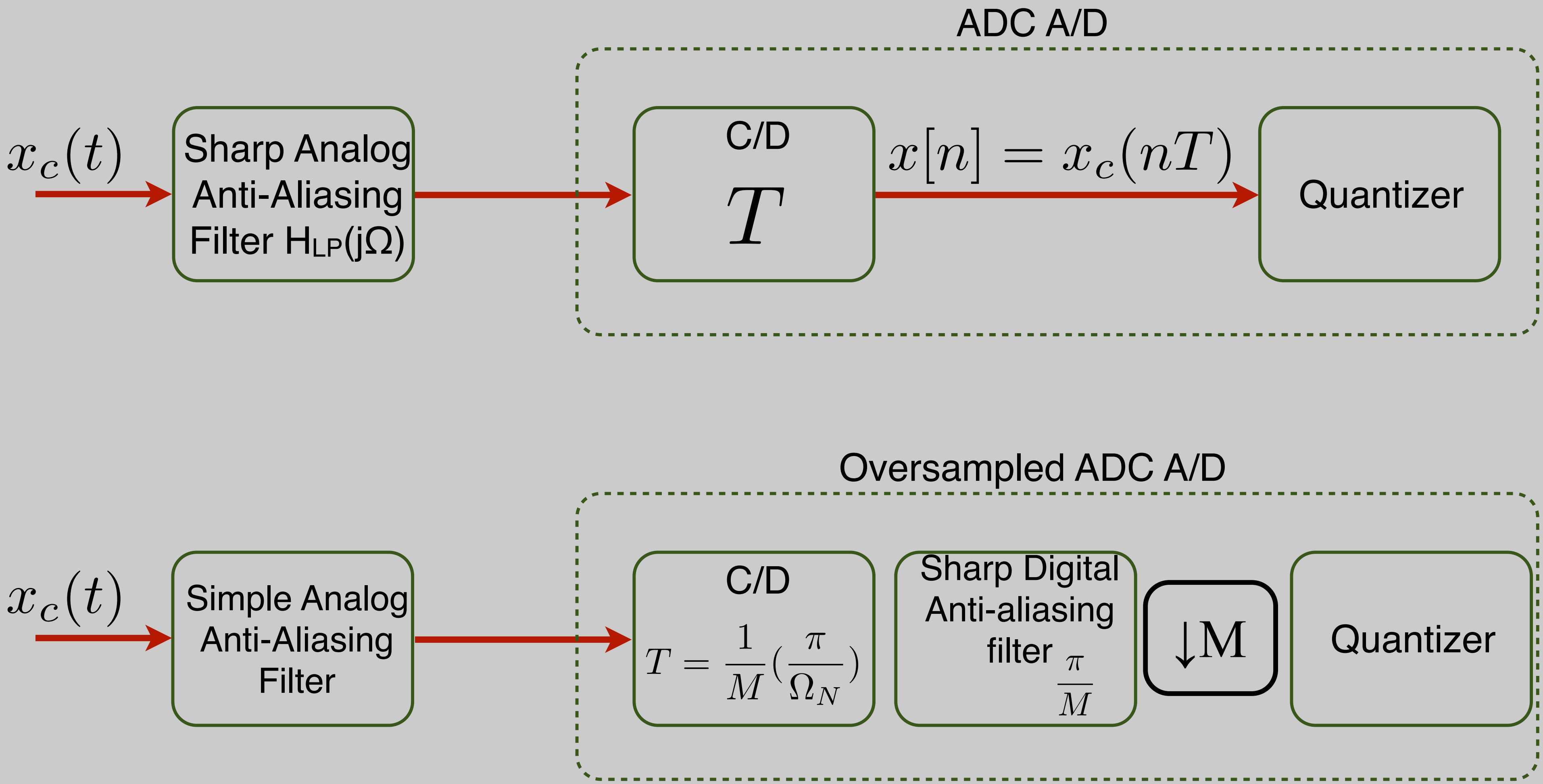


Non Ideal Anti-Aliasing

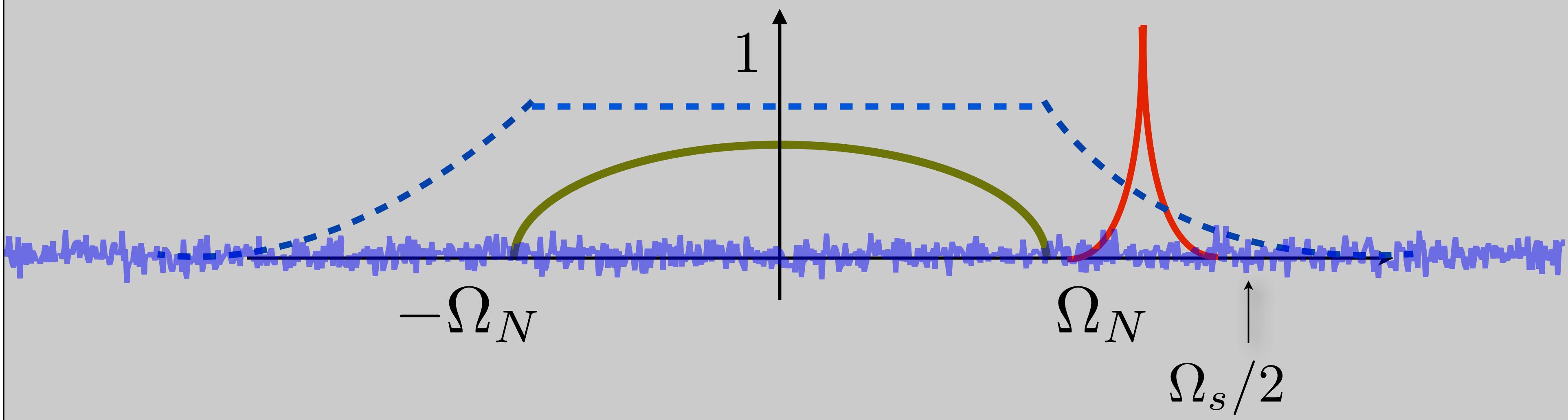


- Problem: Hard to implement sharp analog filter
- Tradeoff:
  - –Crop part of the signal
  - –Suffer from noise and interference (See lab II !)

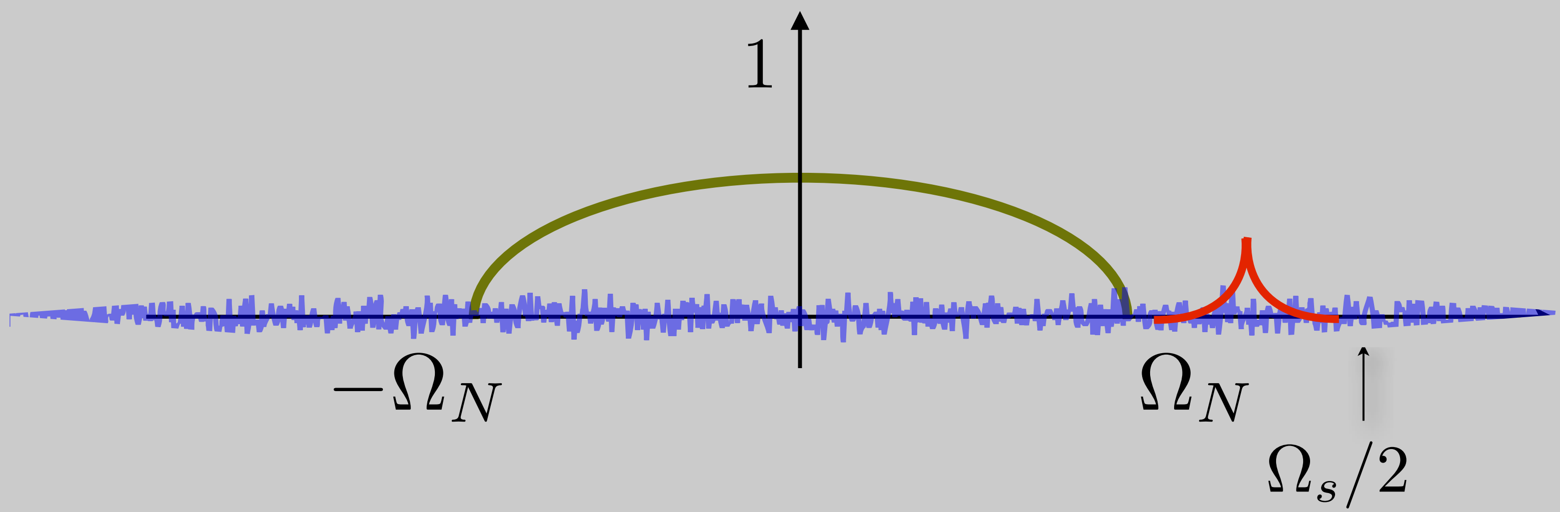
Oversampled ADC



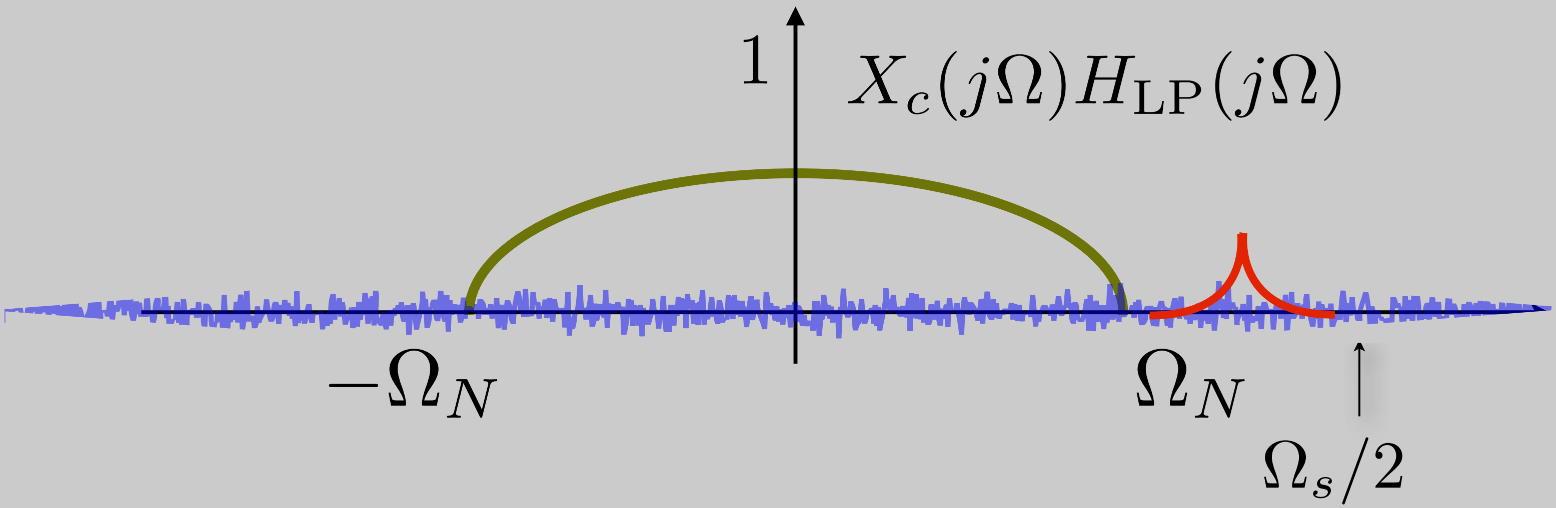
Oversampled ADC



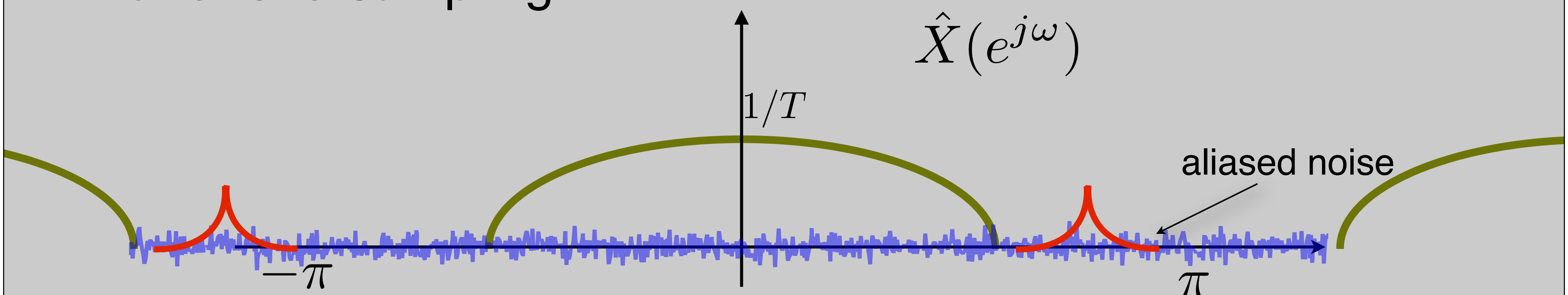
$X_c(j\Omega)H_{\mathrm{LP}}(j\Omega)$



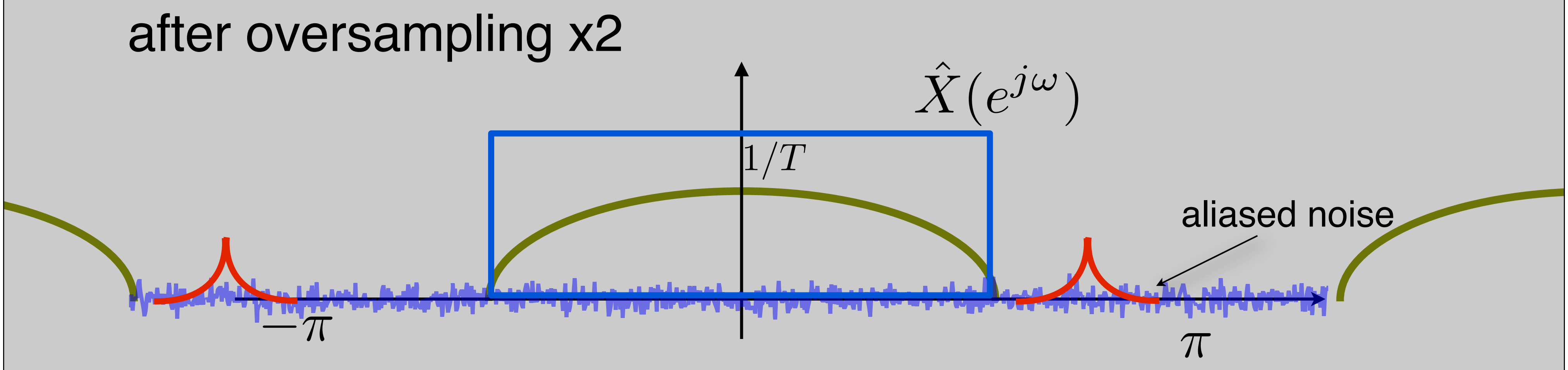
Oversampled ADC

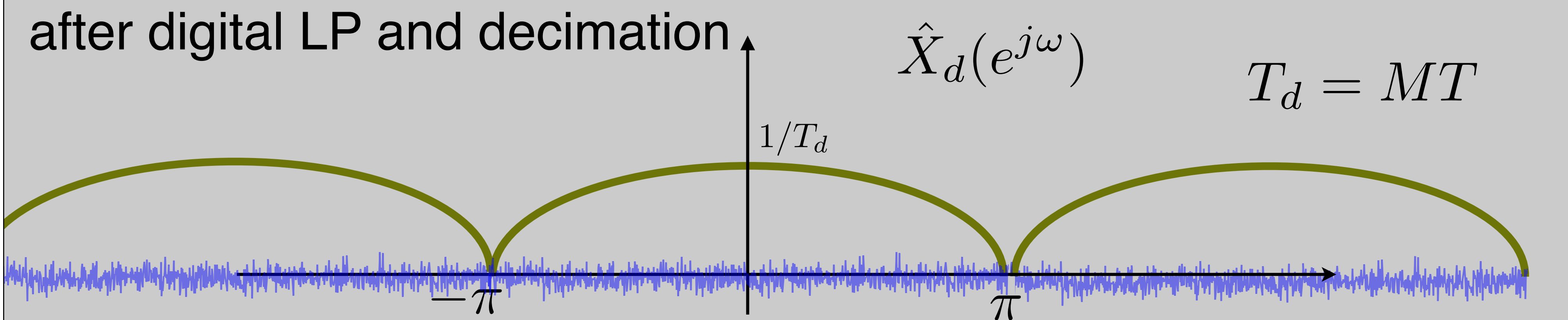


after oversampling x2

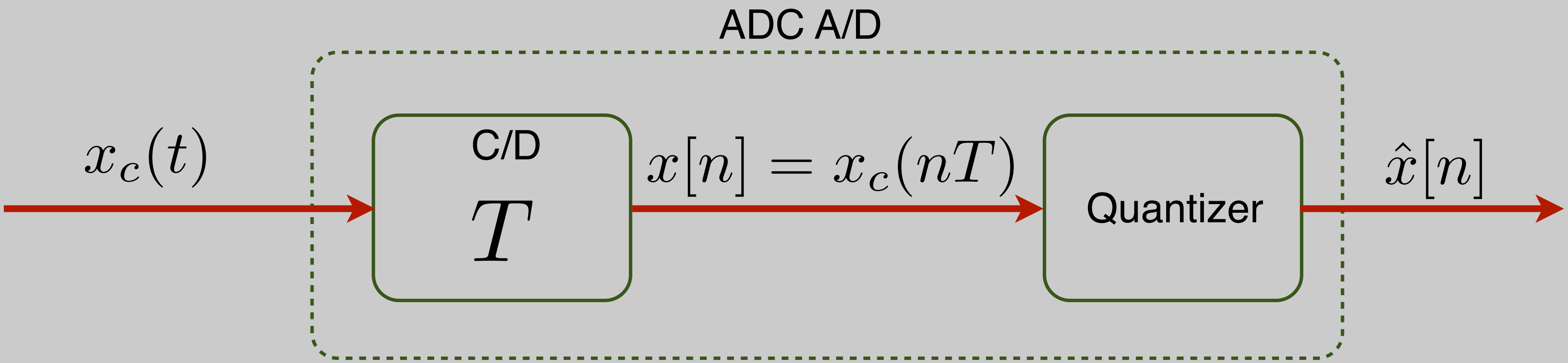


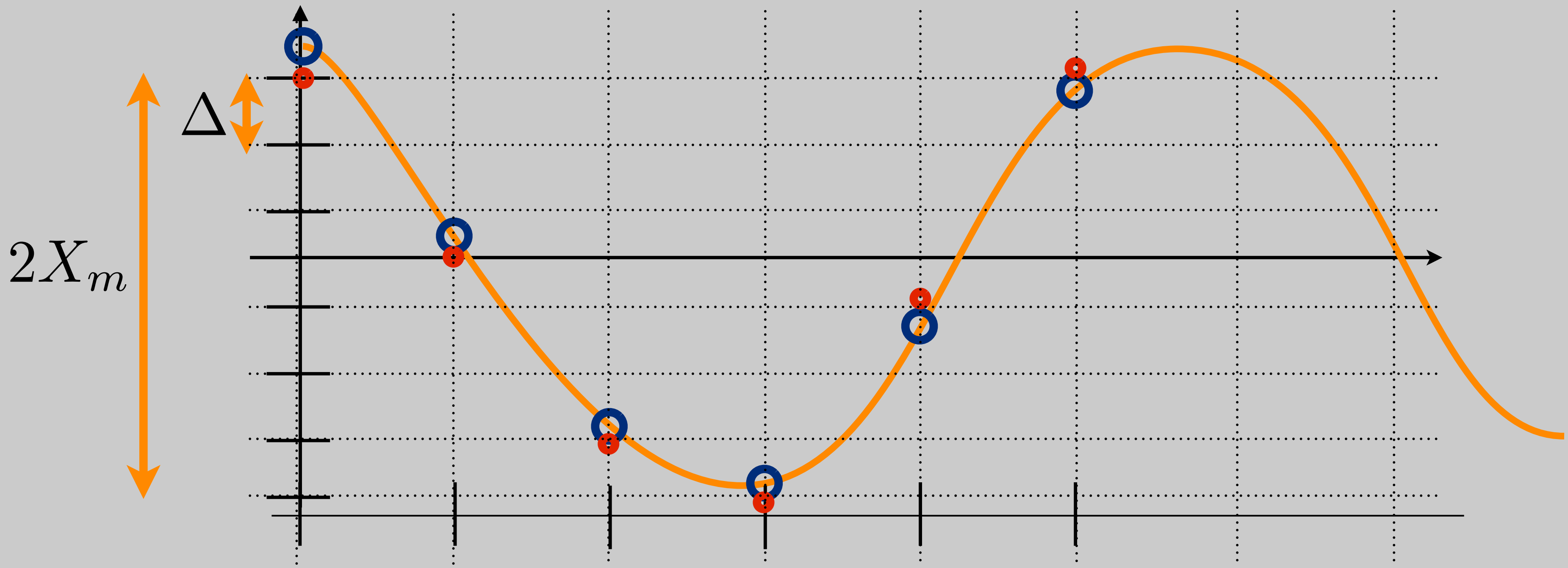
Oversampled ADC





Sampling and Quantization



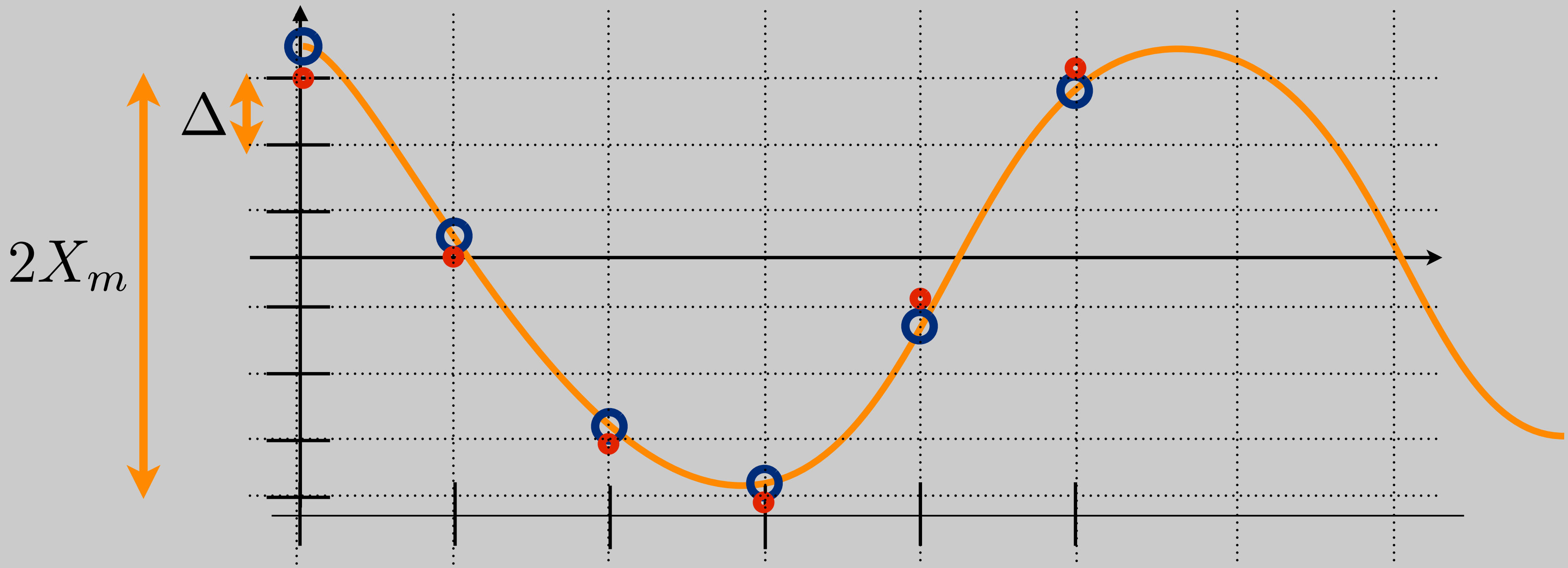


Sampling and Quantization

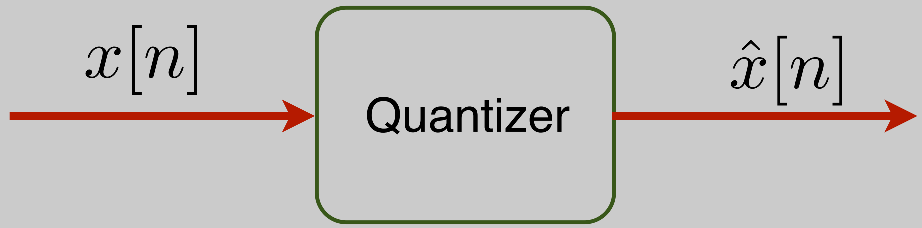
- for 2's complement with B+1 bits $-1 \leq \hat{x}_B[n] < 1$

$$\Delta = \frac{2X_m}{2^{B+1}} = \frac{X_m}{2^B}$$

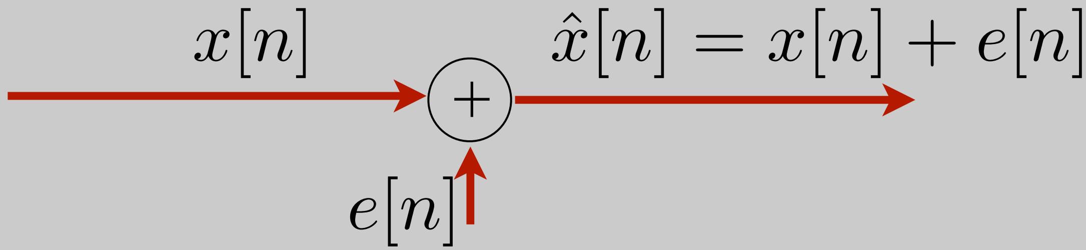
$$\hat{x}[n] = X_m \hat{x}_B[n]$$



Quantization Error



- Model quantization error as noise



- In that case:

$$-\Delta/2 \leq e[n] < \Delta/2$$

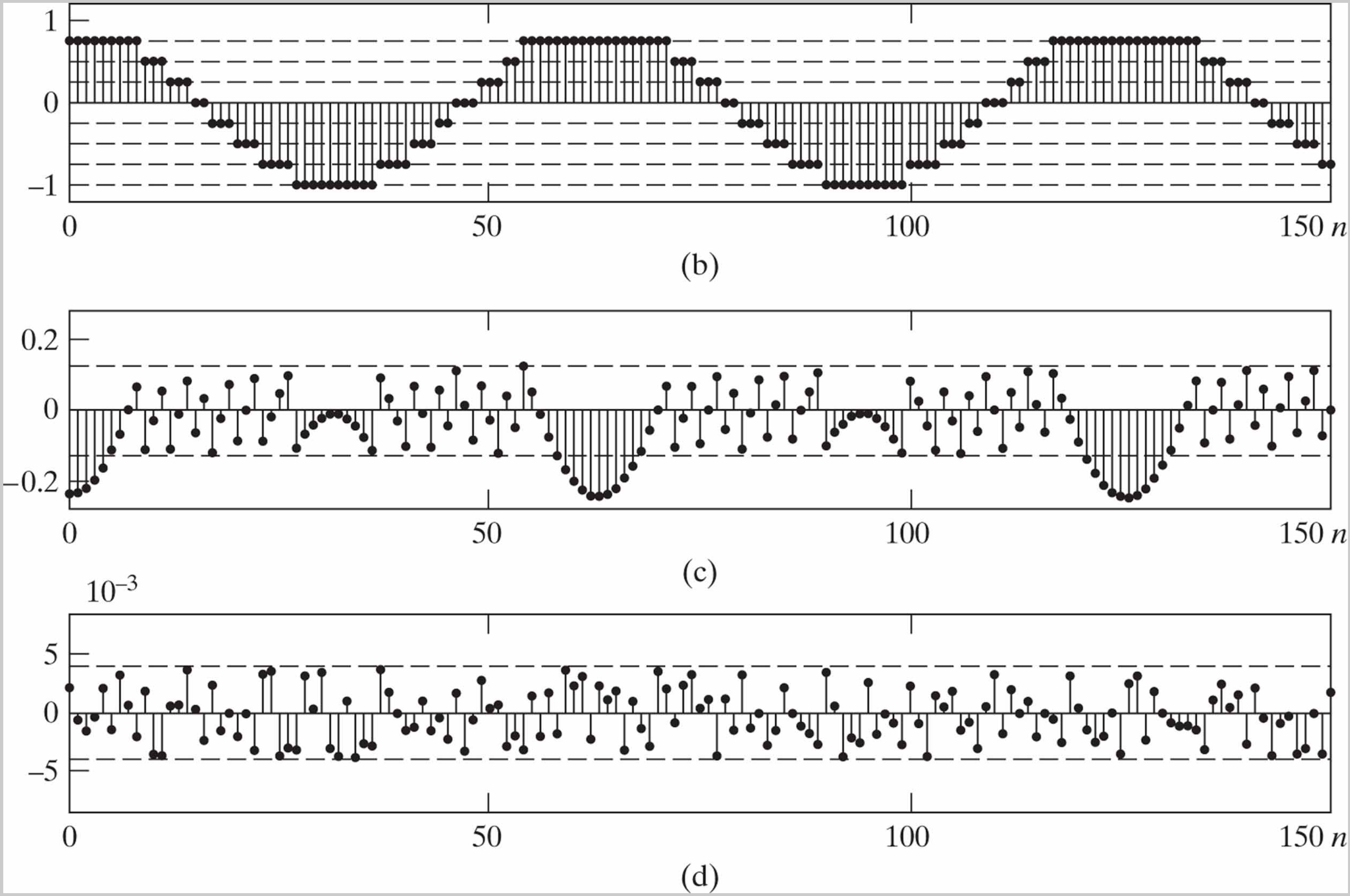
$$(-X_m - \Delta/2) < x[n] \leq (X_m - \Delta/2)$$

Noise Model for Quantization Error

- Assumptions:
  - Model $e[n]$ as a sample sequence of a stationary random process
  - $e[n]$ is not correlated with $x[n]$, *e.g.*, $\mathrm{E}\, e[n]\, x[n] = 0$
  - $e[n]$ not correlated with e[m], e.g., $\mathrm{E}\, e[n]\, x[m] = 0 \;\; | \; \mathrm{m} \neq \mathrm{n}$ (white noise)
  - $e[n] \sim U[\text{-}\Delta/2, \Delta/2]$
- Result:
  - Variance is: $\sigma_e^2 = \dfrac{\Delta^2}{12}$, or $\sigma_e^2 = \dfrac{2^{-2B} X_m^2}{12}$ since $\Delta = 2^{-B} X_m$
  - Assumptions work well for signals that change rapidly, are not clipped and for small Δ

Quantization Noise

Figure 4.57 (*continued*) (b) Quantized samples of the cosine waveform in part (a) with a 3-bit quantizer. (c) Quantization error sequence for 3-bit quantization of the signal in (a). (d) Quantization error sequence for 8-bit quantization of the signal in (a).



SNR of Quantization Noise

- For uniform B+1 bits quantizer: $\sigma_e^2 = \frac{2^{-2B} X_m^2}{12}$

$$\begin{aligned} SNR_Q &= 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right) \\ &= 10\log_{10}\left(\frac{12 \cdot 2^{2B}\sigma_x^2}{X_m^2}\right) \end{aligned}$$

$$\mathrm{SNR}_Q = 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right)$$

Quantizer range

rms of amp

SNR of Quantization Noise

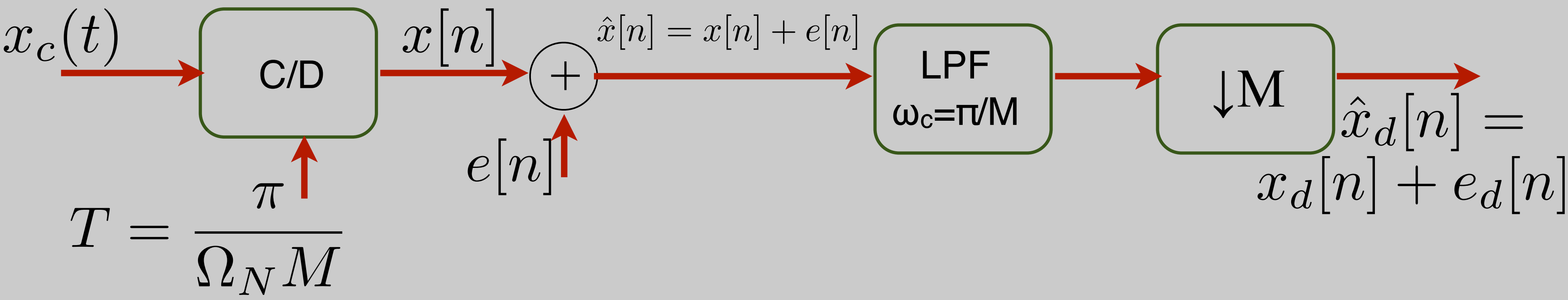
$$\mathrm{SNR}_Q = 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right)$$

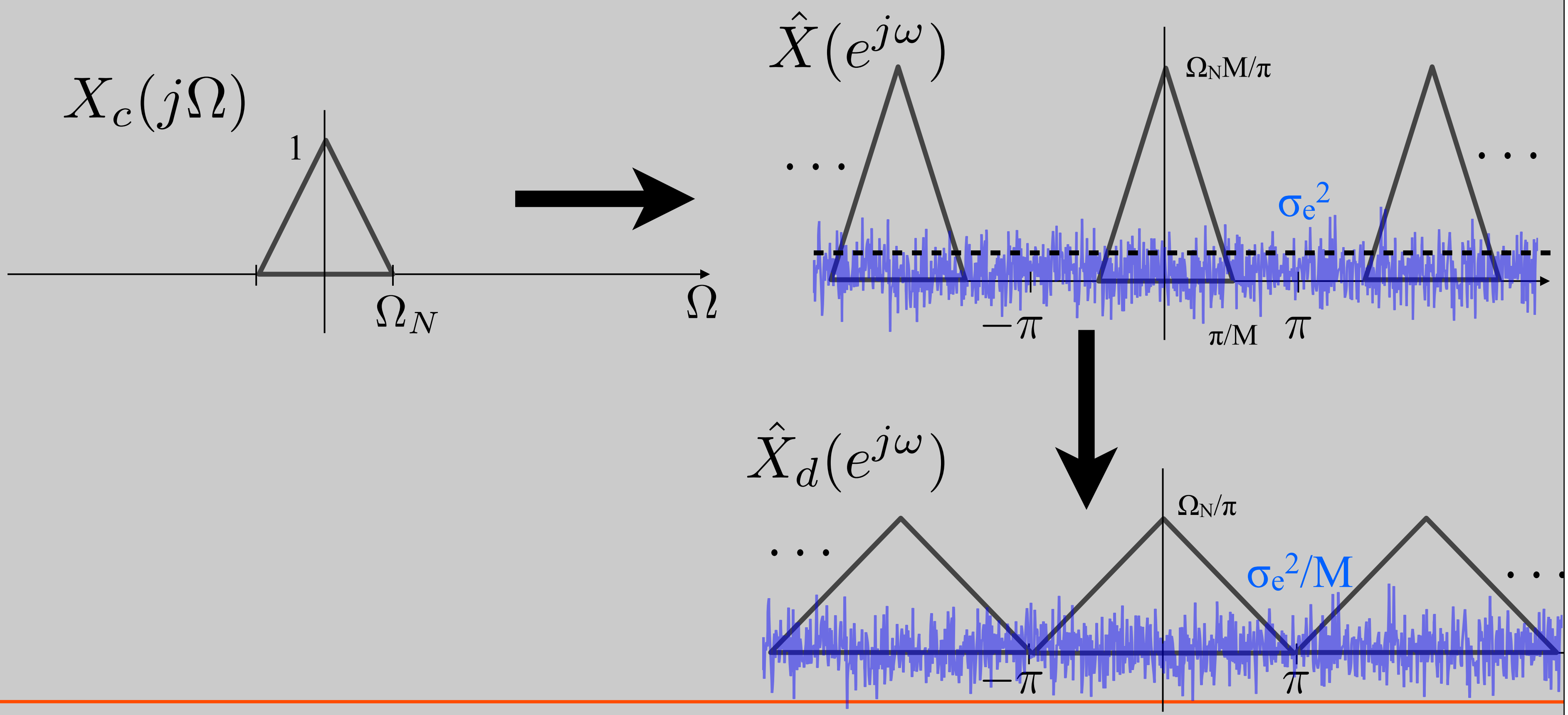
Quantizer range

rms of amp

- Improvement of 6dB with every bit
- The range of the quantization must be adapted to the rms amplitude of the signal
  - Tradeoff between clipping and noise!
  - Often use pre-amp
  - Sometimes use analog auto gain controller (AGC)
  - If $\sigma_x = X_m/4$ then $\mathrm{SNR}_Q \approx 6B - 1.25\mathrm{dB}$
    so SNR of 90-96 dB requires 16-bits (audio)

Quantization noise in Oversampled ADC





Quantization noise in Oversampled ADC

- Energy of $x_d[n]$ equals energy of x[n]
  - No filtering of signal!
- Noise std is reduced by factor of M

$$\mathrm{SNR}_Q = 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right) + 10\log_{10} M$$

- For doubling of M we get 3dB improvement, which is the same as 1/2 a bit of accuracy
  - With oversampling of 16 with 8bit ADC we get the same quantization noise as 10bit ADC!

Practical ADC (Ch. 4.8.4)

D.T

$$x[n] = x(t)|_{t=nT} \longrightarrow \boxed{\text{sinc pulse generator}} \longrightarrow x_r(t) = \sum_{n=-\infty}^{\infty} x[n]\mathrm{sinc}\left(\frac{t-nT}{T}\right)$$

C.T

- Scaled train of sinc pulses
- Difficult to generate sinc ⇒ Too long!

Practical ADC

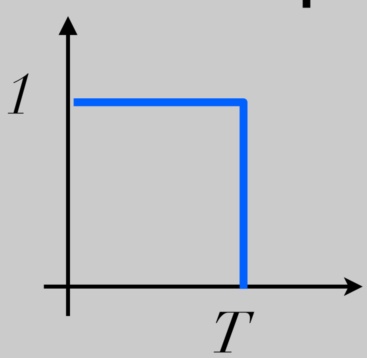
C.T analog processing

D.T

$x[n] = x(t)|_{t=nT} \rightarrow$ Interp. Filter $h_0(t) \Leftrightarrow H_0(j\Omega)$ $\rightarrow$ Recon. Filter $h_r(t) \Leftrightarrow H_r(j\Omega)$ $\rightarrow x_r(t)$

$= \sum x[n] h_0(t - nT)$

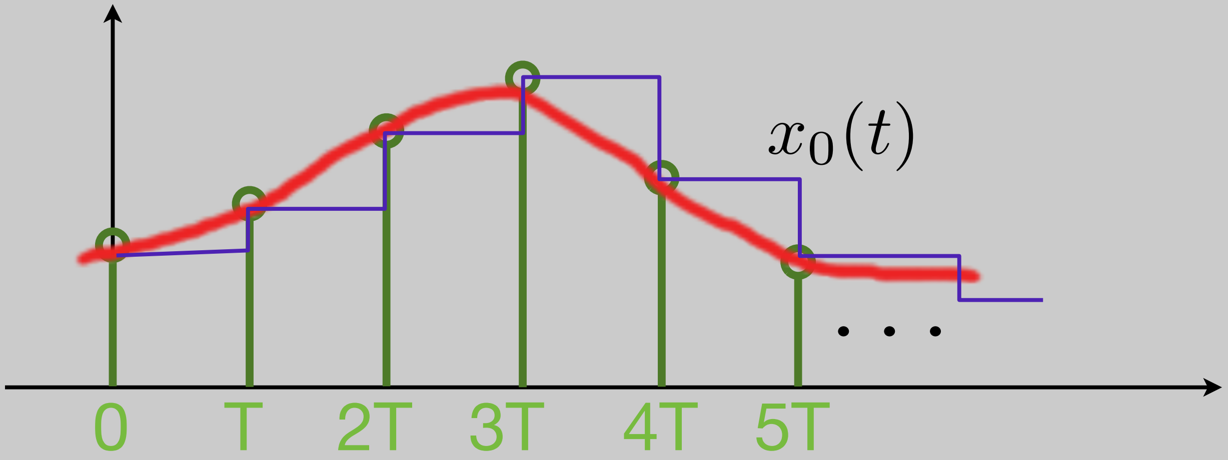
- $h_0(t)$ is finite length pulse $\Rightarrow$ easy to implement
- For example: zero-order hold



$$H_0(j\Omega) = Te^{-j\Omega\frac{T}{2}} \mathrm{sinc}(\frac{\Omega}{\Omega_s})$$

Practical ADC

Zero-Order-Hold interpolation



$$x_0(t) = \sum_{n=-\infty}^{\infty} x[n]h_0(t - nT) = x_0(t) * x_s(t)$$

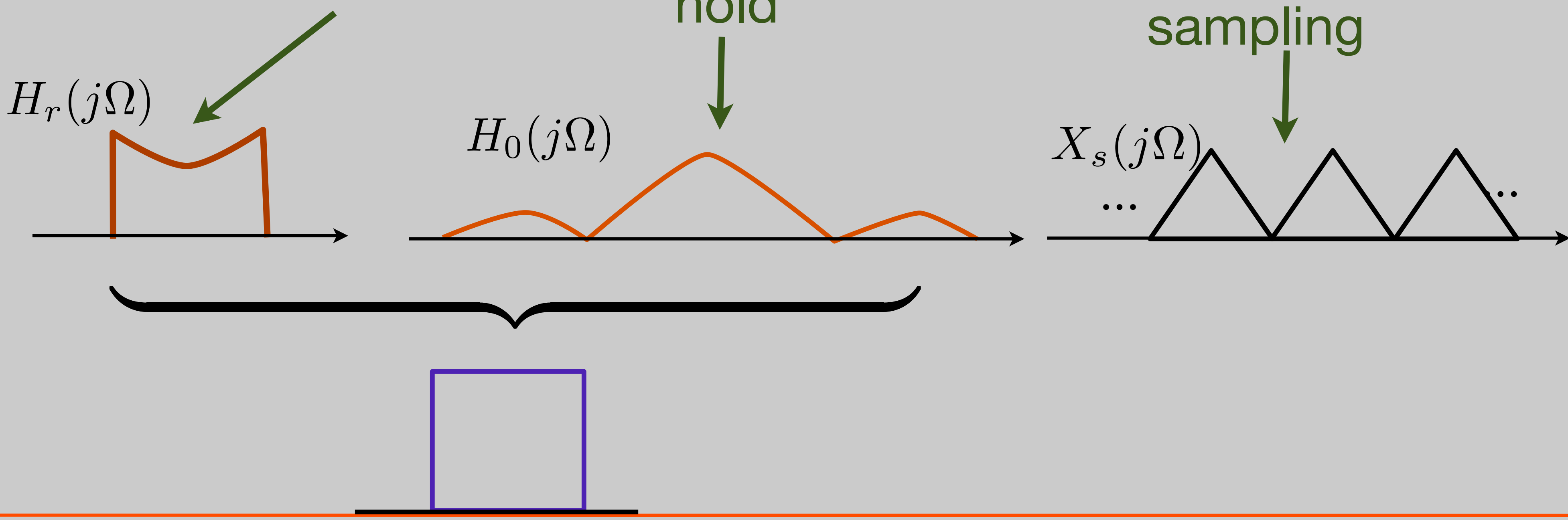
Taking a FT:

$$\begin{aligned} X_(j\Omega) &= H_0(j\Omega)X_s(j\Omega) \\ &= H_0(j\Omega)\frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\Omega - k\Omega_s)) \end{aligned}$$

Practical ADC

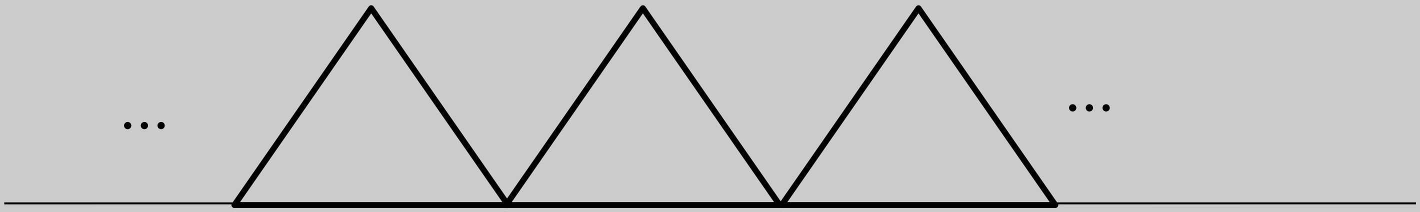
Output of the reconstruction filter:

$$
\begin{aligned}
X_r(j\Omega) &= H_r(j\Omega)\cdot H_0(j\Omega)\cdot X_s(j\Omega)\\
&= \underbrace{H_r(j\Omega)}_{\text{recon filter}}\cdot \underbrace{Te^{-j\Omega\frac{T}{2}}\operatorname{sinc}(\frac{\Omega}{\Omega_s})}_{\text{from zero-order hold}}\cdot \underbrace{\frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\Omega-k\Omega_s))}_{\text{Shifted copies from sampling}}
\end{aligned}
$$



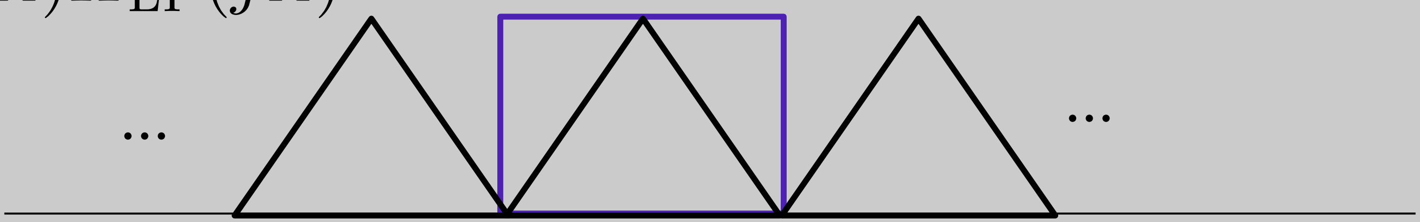
Practical ADC

$X_s(j\Omega)$



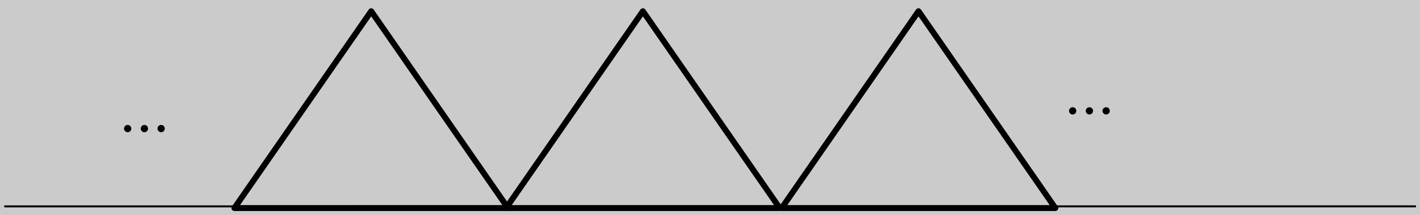
Ideally:

$X_s(j\Omega)H_{\mathrm{LP}}(j\Omega)$



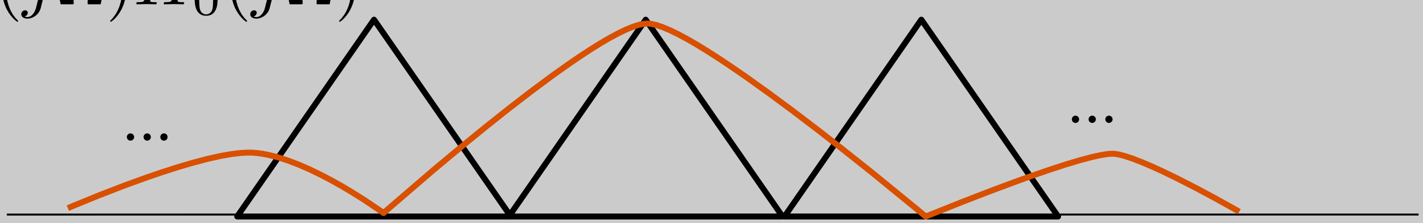
Practical ADC

$X_s(j\Omega)$



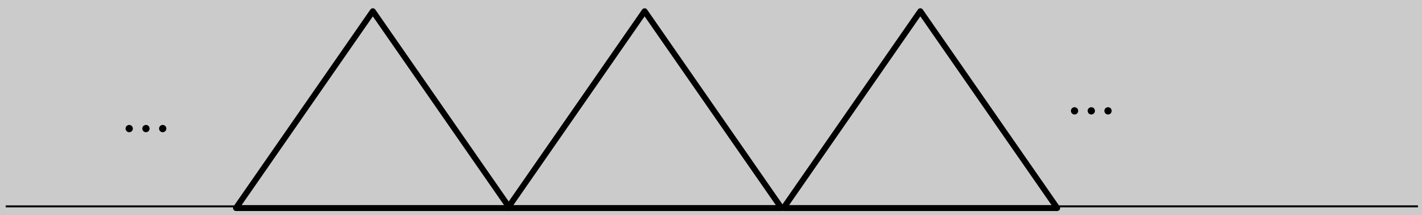
Practically:

$X_s(j\Omega)H_0(j\Omega)$



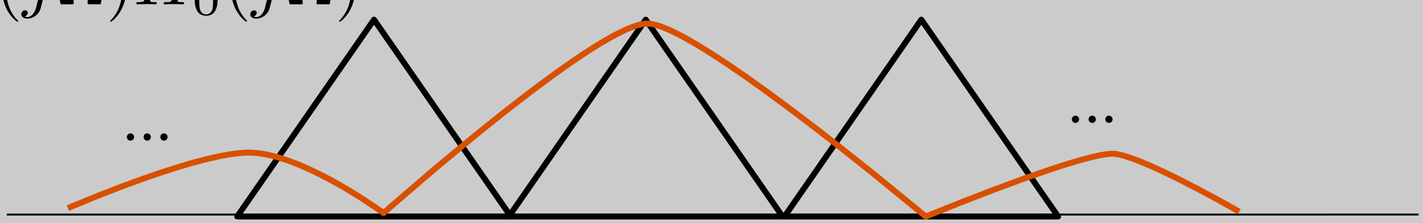
Practical ADC

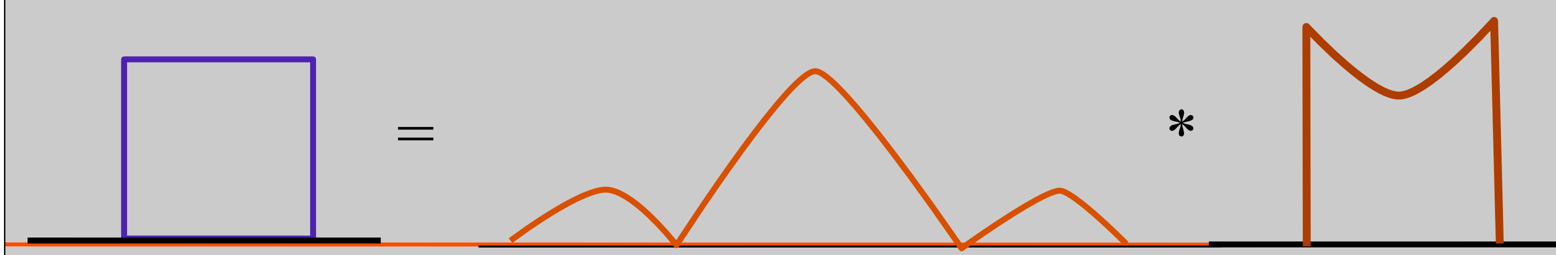
$X_s(j\Omega)$



Practically:

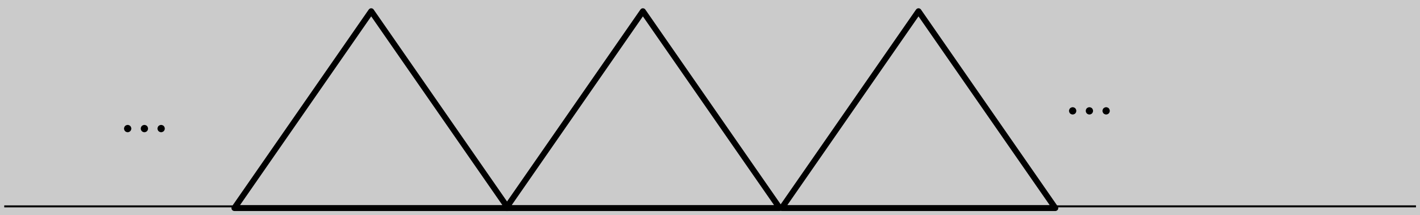
$X_s(j\Omega)H_0(j\Omega)$





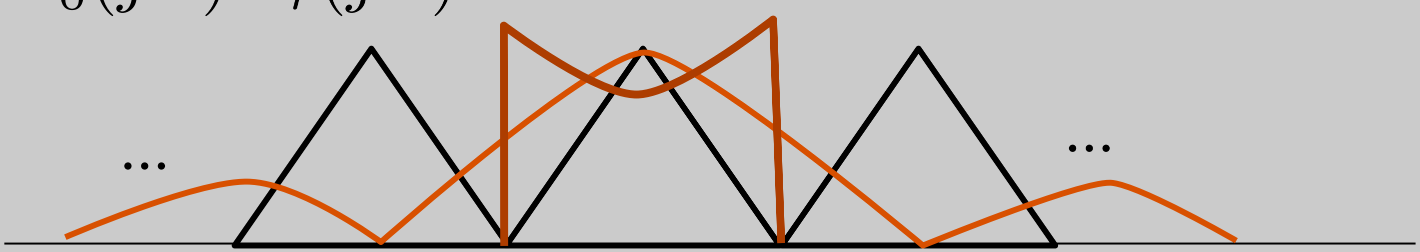
Practical ADC

$X_s(j\Omega)$

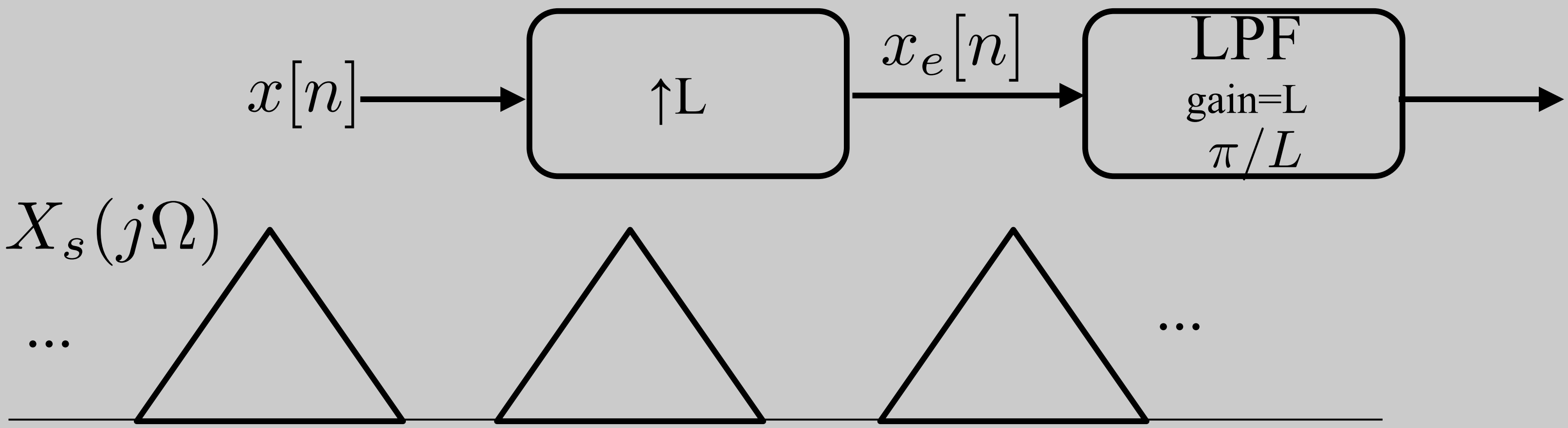


Practically:

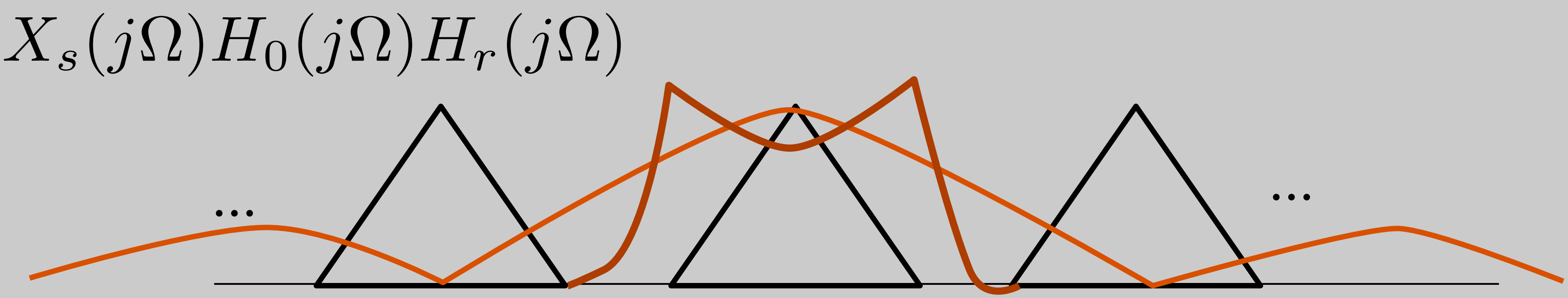
$X_s(j\Omega)H_0(j\Omega)H_r(j\Omega)$



Easier Implementation with Digital upsampling



Practically:



Easier Implementation with Digital upsampling

easier implementing
with analog components

Need analog components
made of Nonobtainium