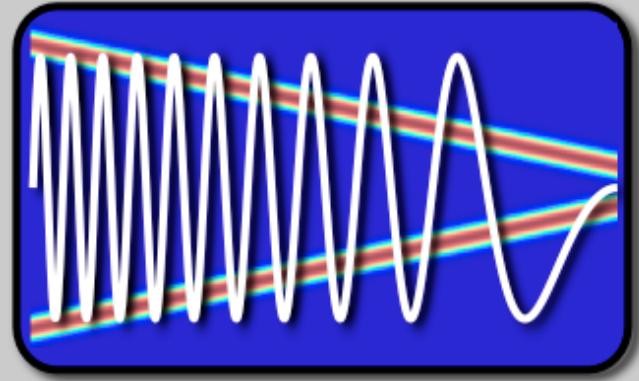


EE123



Digital Signal Processing

Lecture 20
2D Signals

Announcements

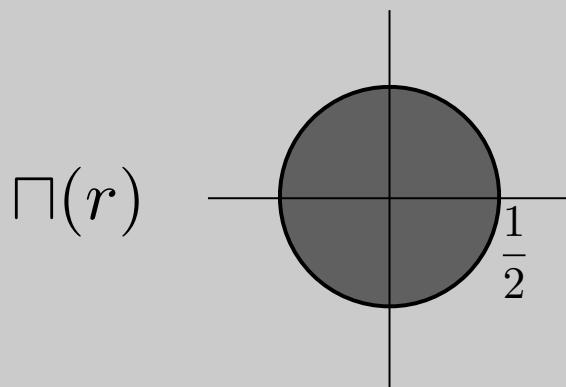
- Midterm Survey, please fill

Multi-Dimensional Signals

- Our world is more complex than 1D
- Images: $f(x,y)$
- Videos: $f(x,y,t)$
- Dynamic 3D scenes: $f(x,y,z,t)$
 - Medical Imaging
 - 3D Video
 - Computer Graphics
- We will focus on 2D

Continuous-Time 2D functions

- $\delta(x,y)$: Impulse at $x=0, y=0$
- $\delta(x)$: Impulse line (vertical or horizontal?)
- $\Pi(x,y)$: 2D rect function
- $\cos(2\pi(f_x x + f_y y))$ - Spatial harmonic
- Circularly Symmetric:
 - $\Pi(r)$: Pillbox



Spatial Frequency

- What is a spatial frequency?

- Complex Harmonic:

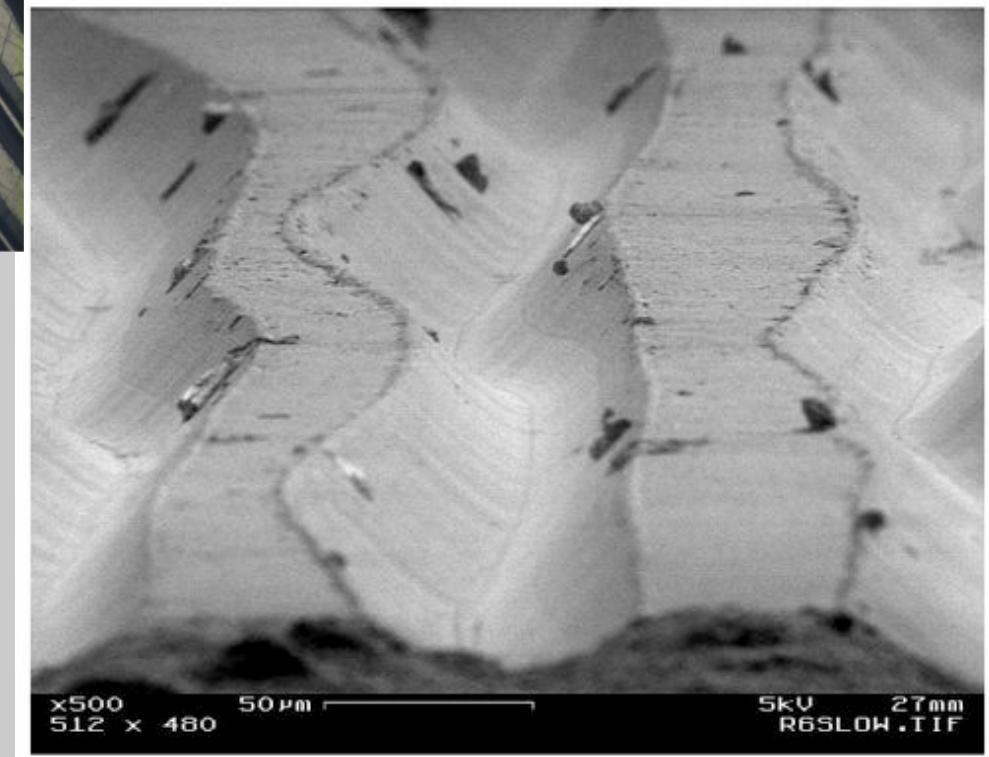
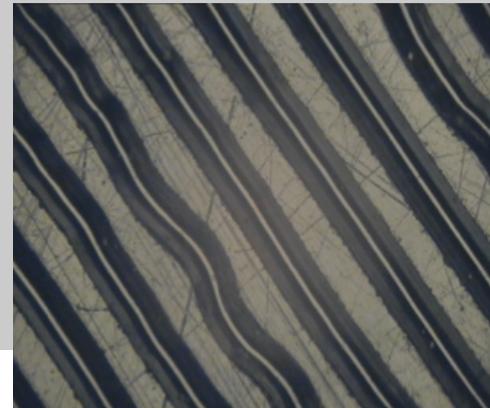
$$e^{j(\Omega_x x + \Omega_y y)} = e^{j2\pi(f_x x + f_y y)}$$

- Units (for example):

- x,y - cm
 - f_x, f_y - 1/cm
 - Ω_x, Ω_y - rad/cm

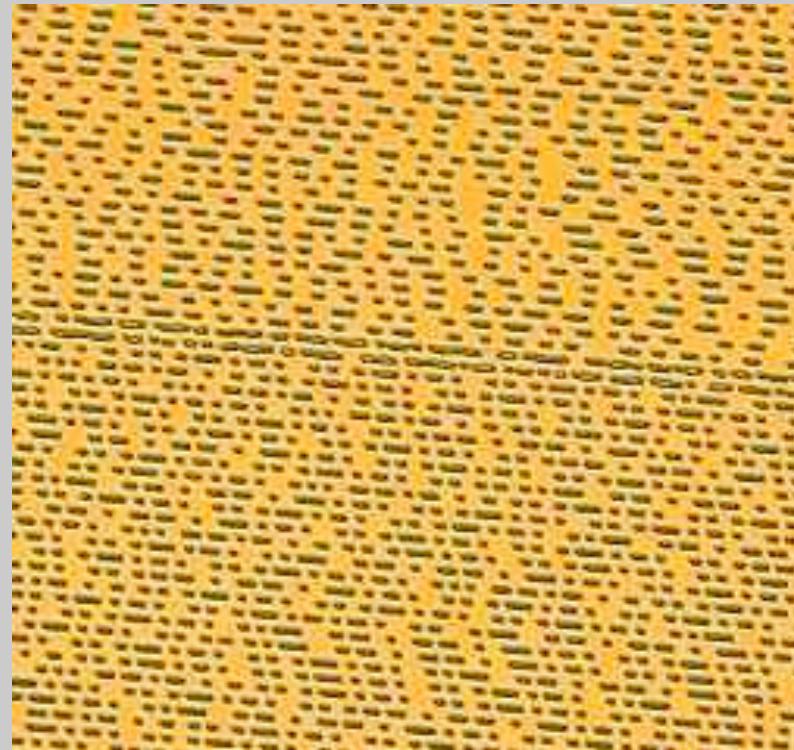
Spatial Frequency

- Vinyl Record
 - Transforms a temporal signal to a spatial signal



Spatial Frequency

- CD ROM
 - encodes digital temporal signals to spatial signals



What is the frequency?

$$\sin(2\pi(f_x x + f_y y))$$

(-1,1)

(1,1)

(-1,-1)

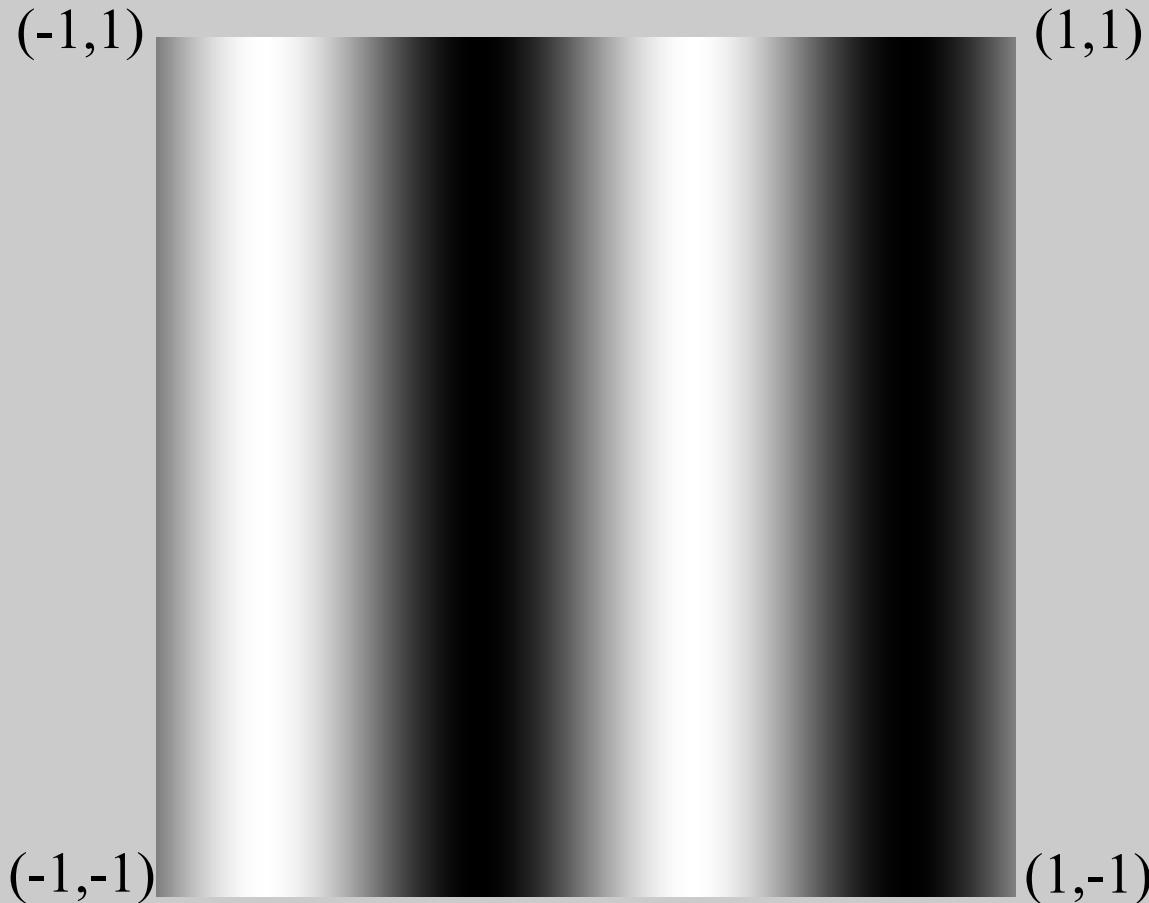
(1,-1)

- a) $f_x=2, f_y=2$
- b) $f_x=1, f_y=0$

- c) $f_x = 4, f_y=0$
- d) none of the above

What is the frequency?

$$\sin(2\pi(f_x x + f_y y))$$



2 cycles for 2 cm $\Rightarrow f_x = 1 \text{ cm}^{-1}$

What is the frequency?

$\sin(2\pi(f_x x + f_y y))$ or $\cos(2\pi(f_x x + f_y y))$

(-1,1)



(1,1)

(-1,-1)

(1,-1)

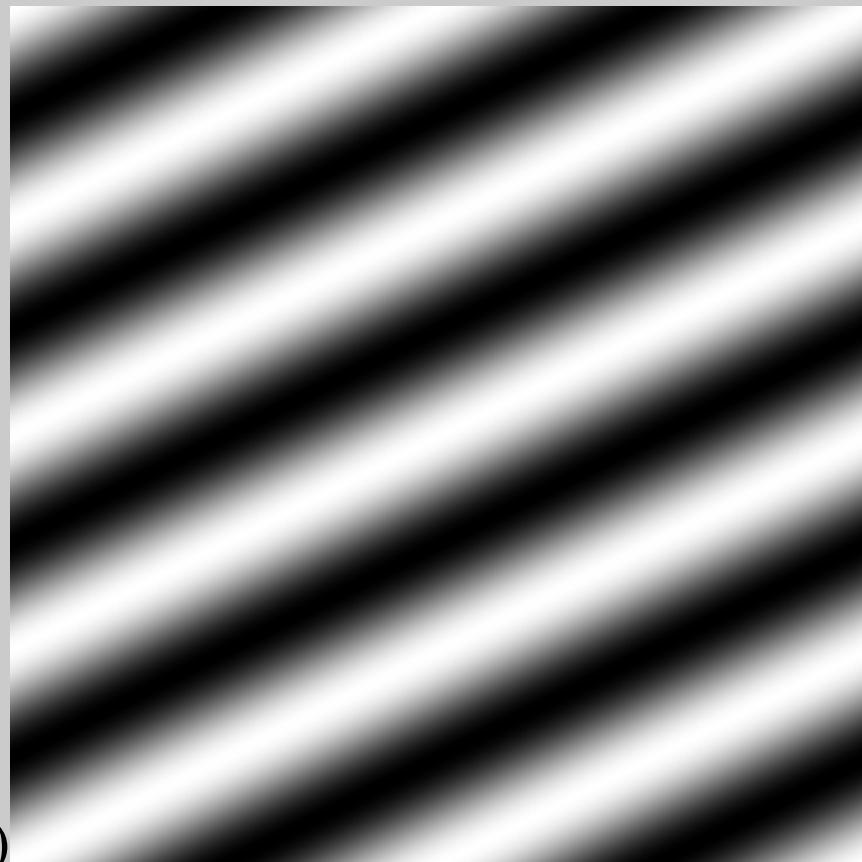
- a) sin, $f_x=0, f_y=2$
- b) cos, $f_x=0, f_y=4$
- c) cos, $f_x = 0, f_y=2$
- d) none of the above

What is the frequency?

$$\cos(2\pi(f_x x + f_y y))$$

(-1,1)

(1,1)



(-1,-1)

(1,-1)

This is the answer a)

f_x=1, f_y=2

b) f_x=4, fy=2

c) f_x = 2, f_y=1

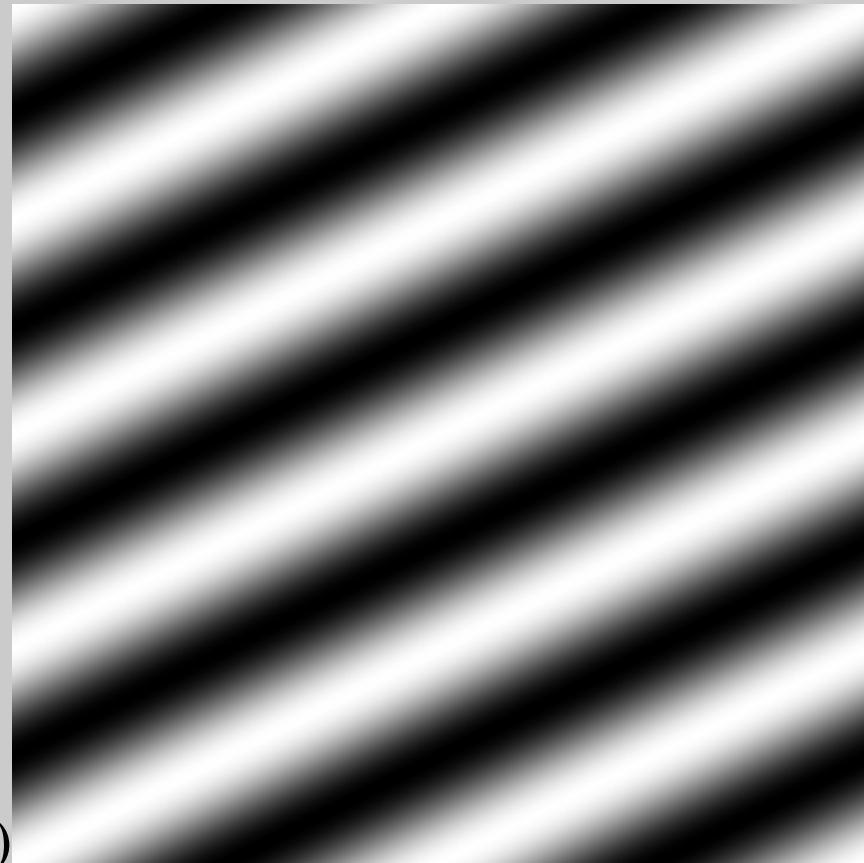
d) f_x=2, fy=4

What is the frequency?

$$\cos(2\pi(f_x x + f_y y))$$

(-1,1)

(1,1)



(-1,-1)

(1,-1)

a) $f_x=1, f_y=2$

b) $f_x=4, f_y=2$

c) $f_x = 2, f_y=1$

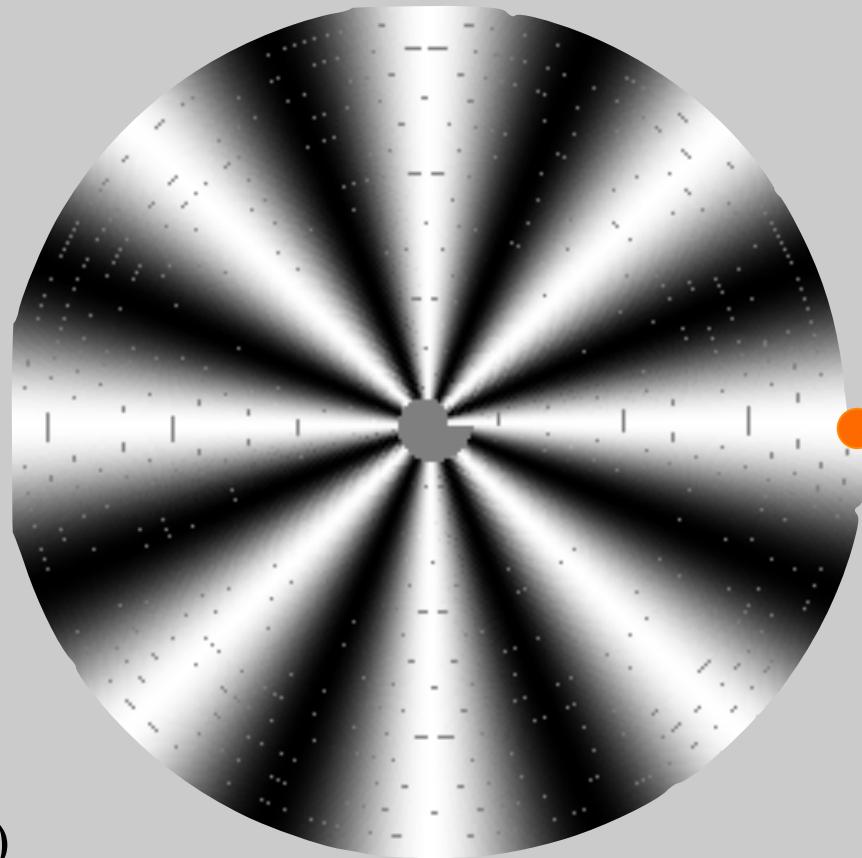
d) $f_x=2, f_y=4$

What is the Temporal Frequency?

Vinyl rotates at 1 Hz

(-1,1)

(1,1)



(-1,-1)

(1,-1)

a) $\cos(2\pi 8t)$

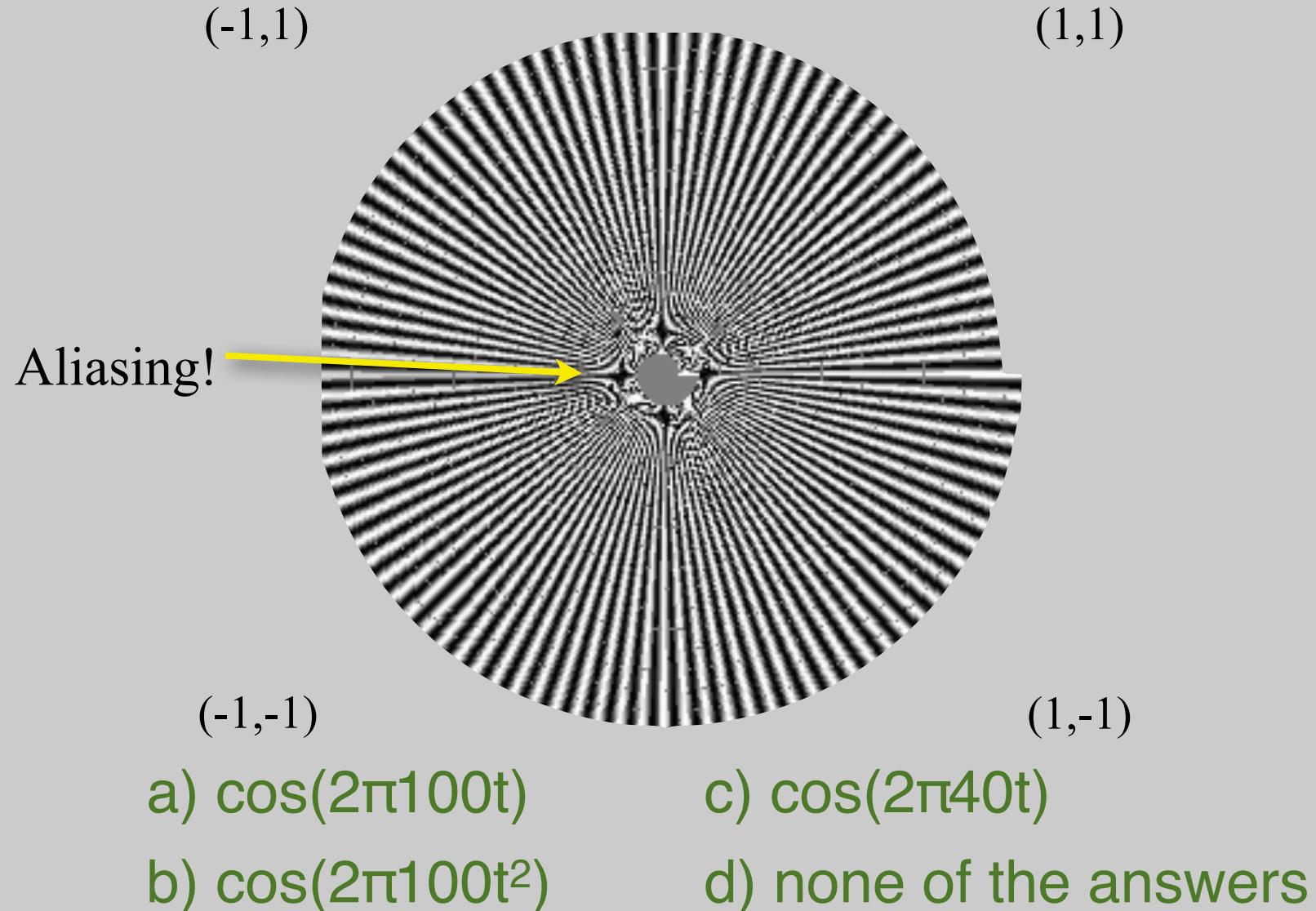
b) $\cos(2\pi 8t^2)$

c) $\cos(2\pi 4t)$

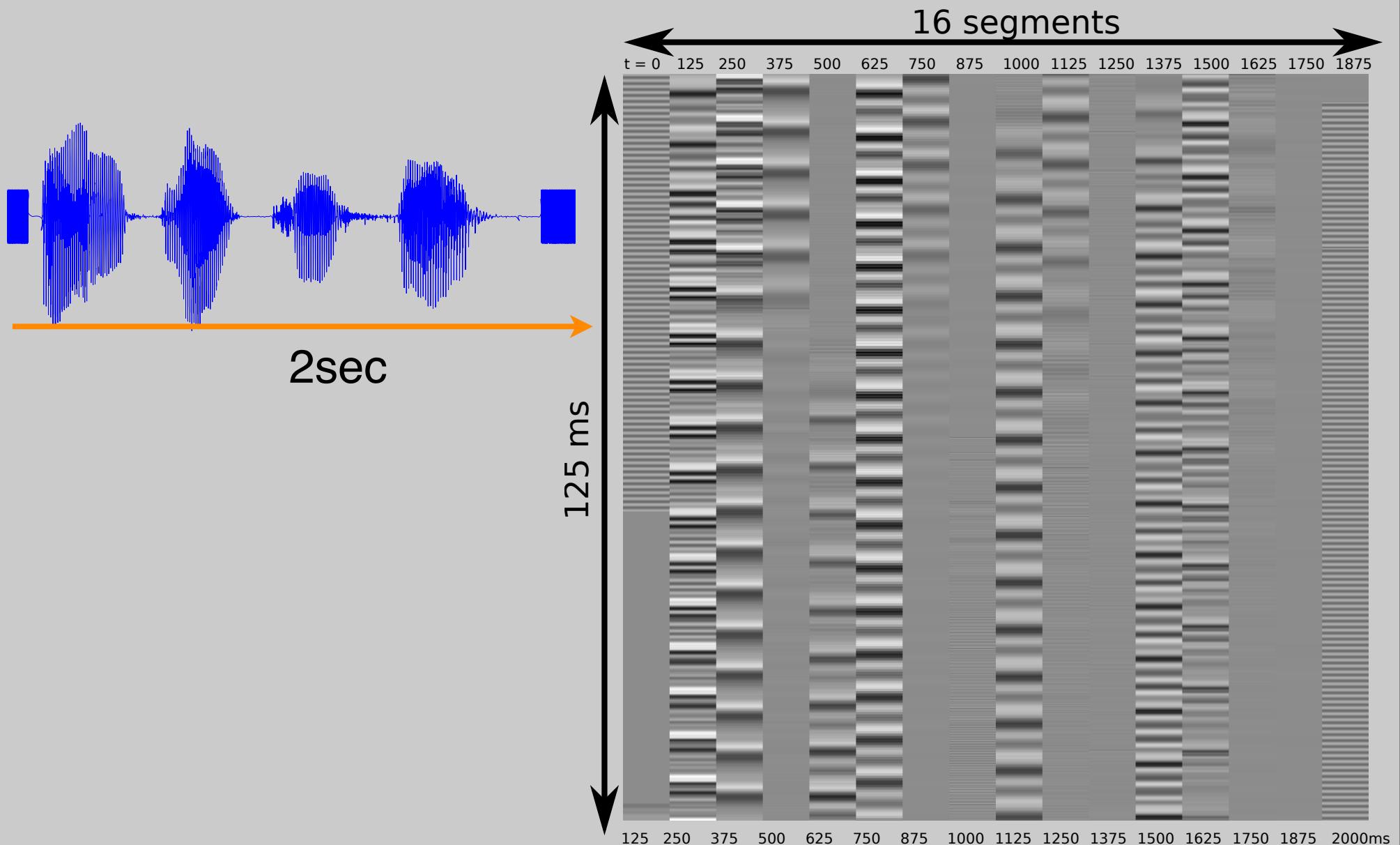
d) $\cos(2\pi 4t^2)$

What is the Temporal Frequency?

Vinyl rotates at 1 Hz

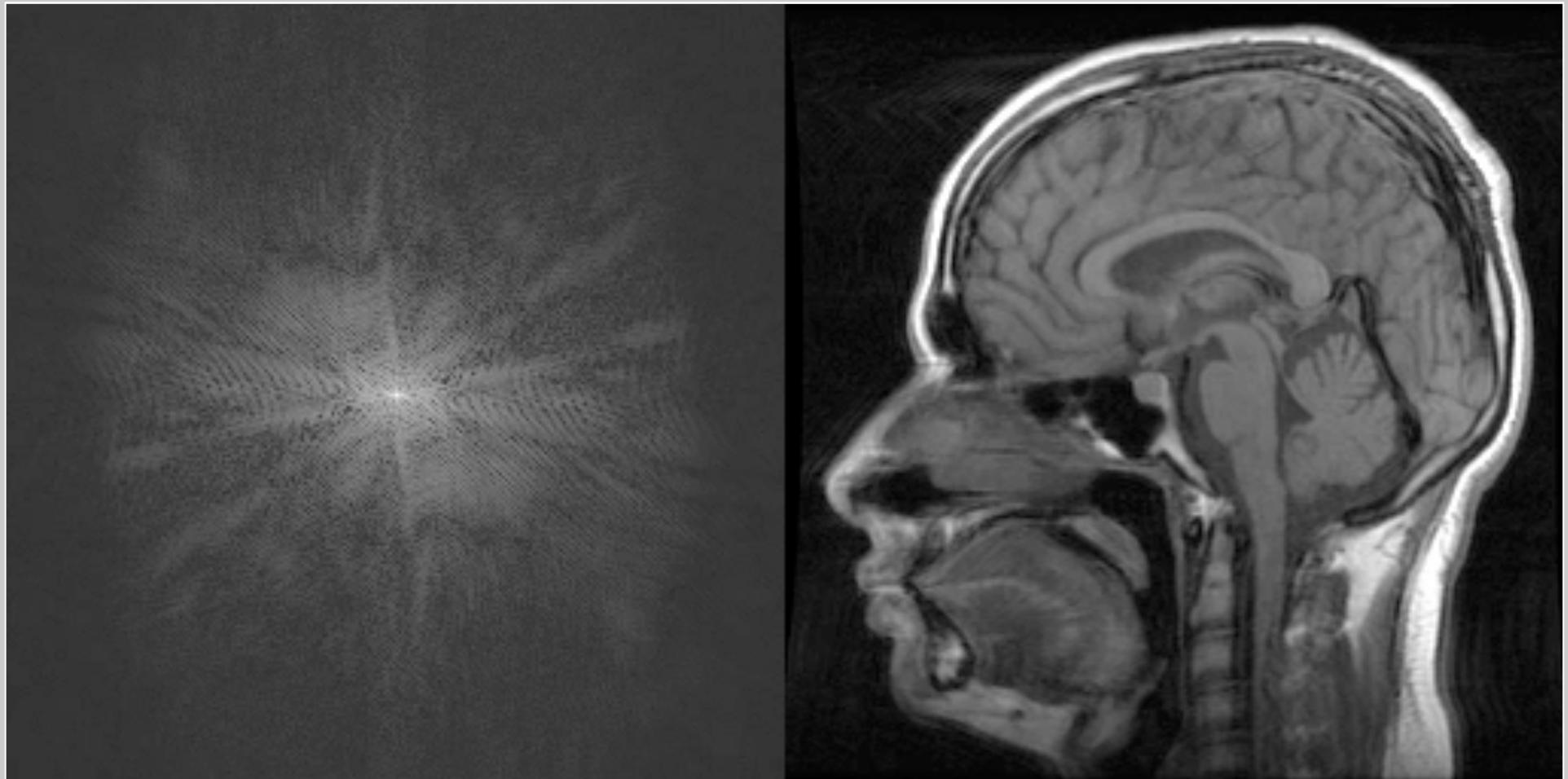


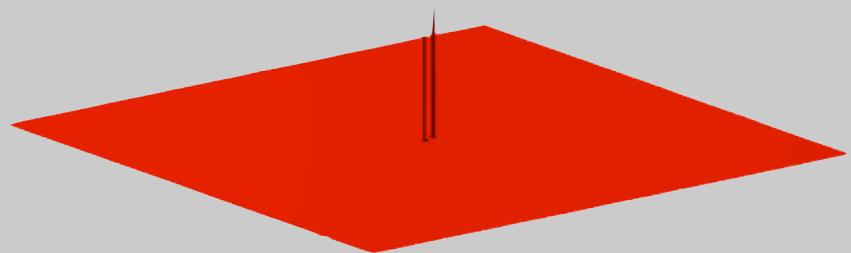
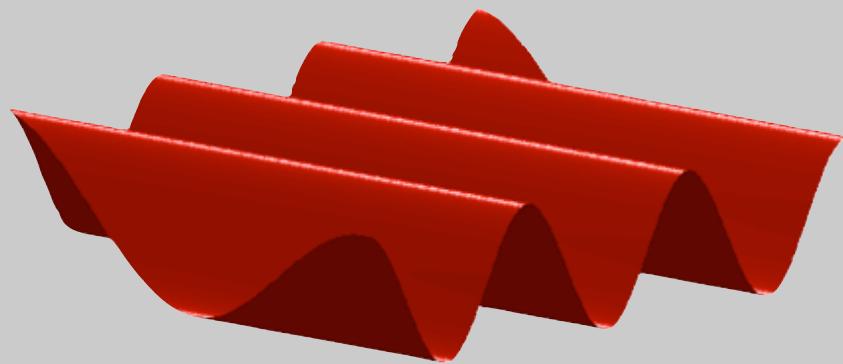
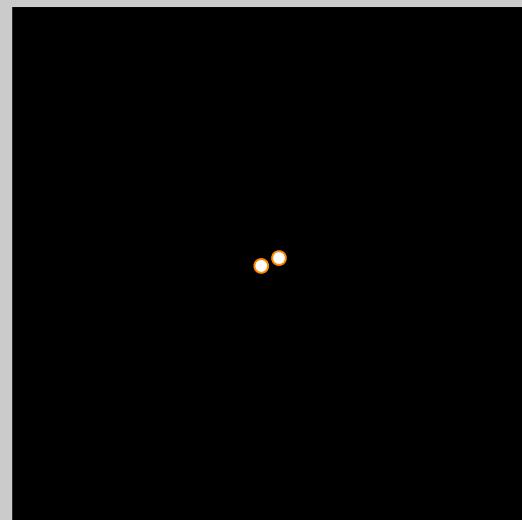
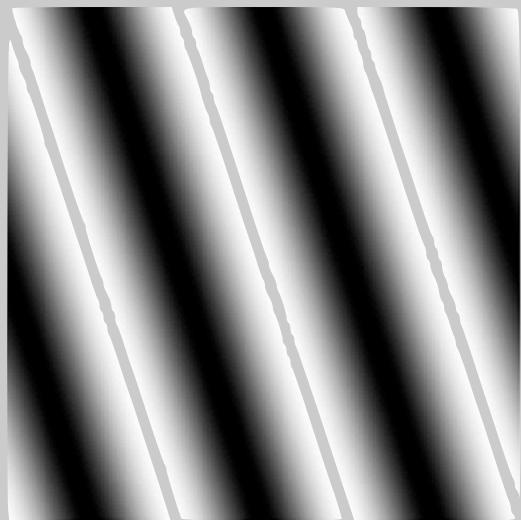
Challenge: What is the password?



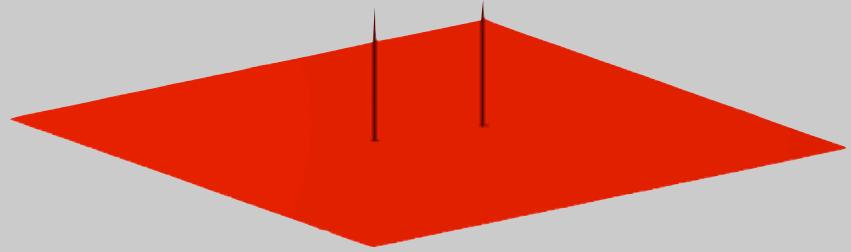
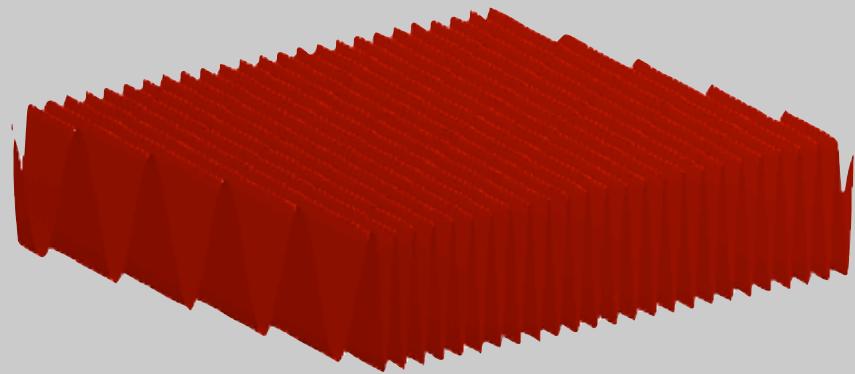
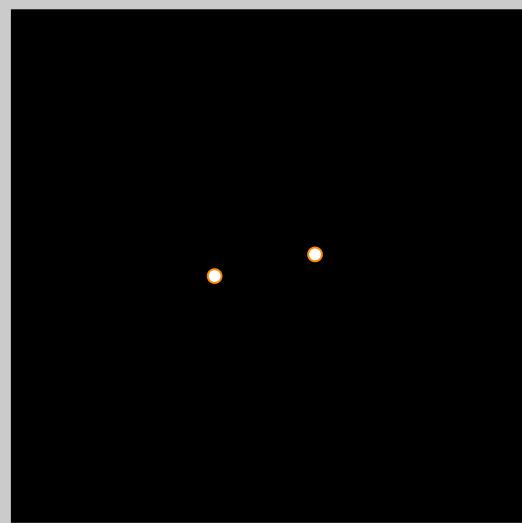
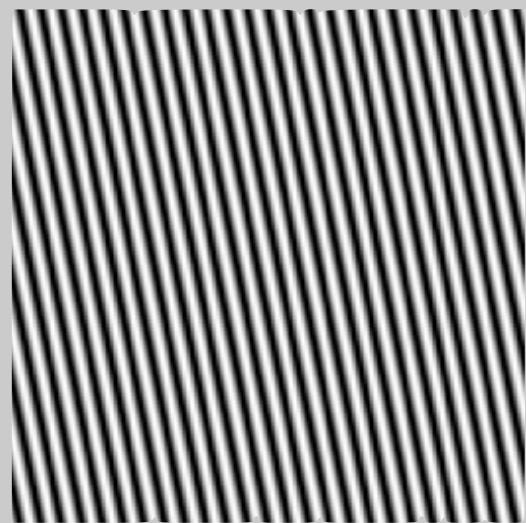
2D Fourier Transform

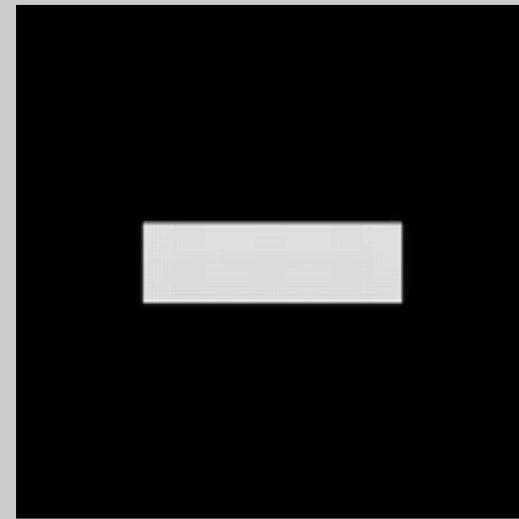
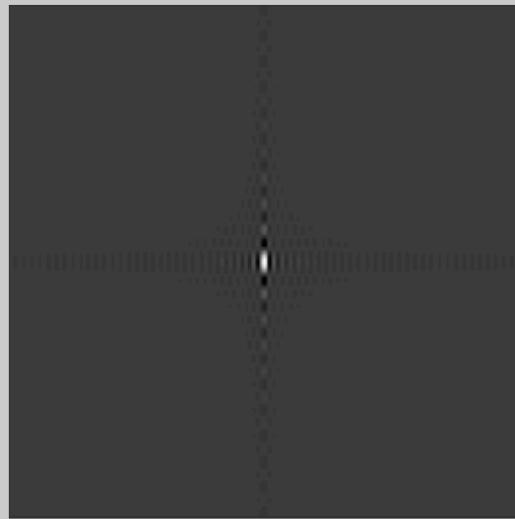
$$F(f_x, f_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(f_x x + f_y y)} dx dy$$



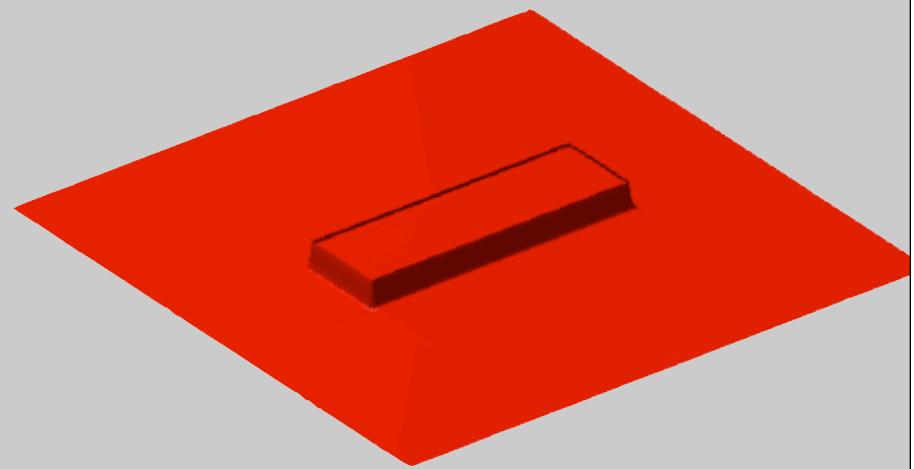
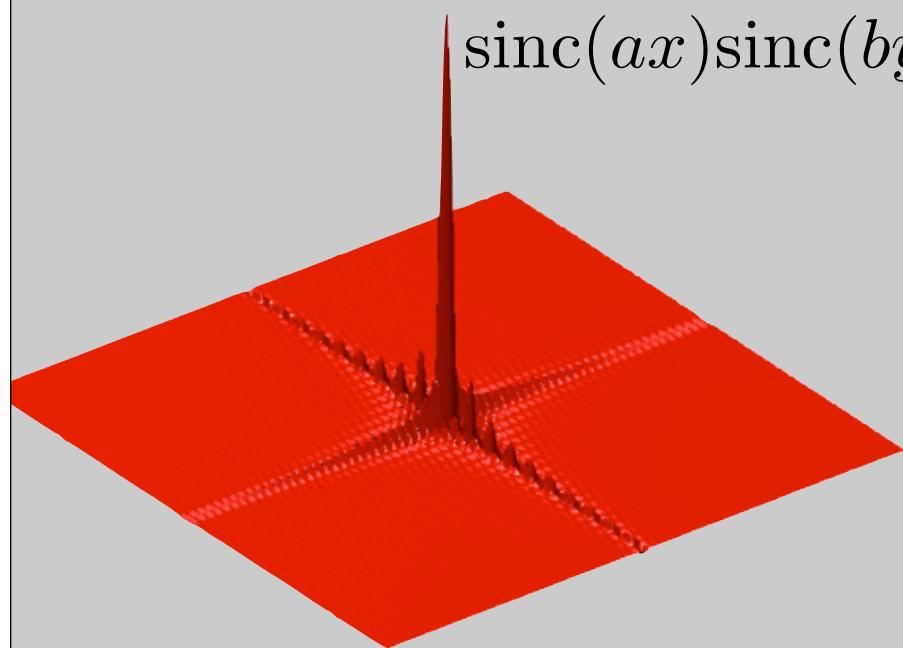


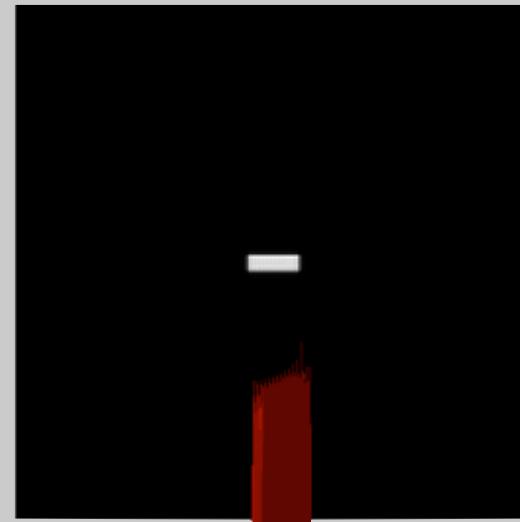
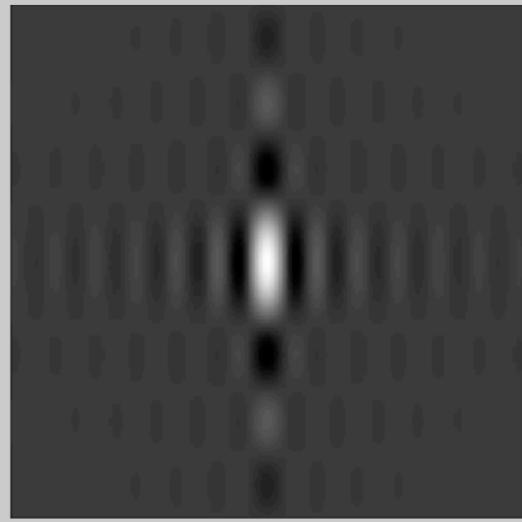
M. Lustig, EECS UC Berkeley



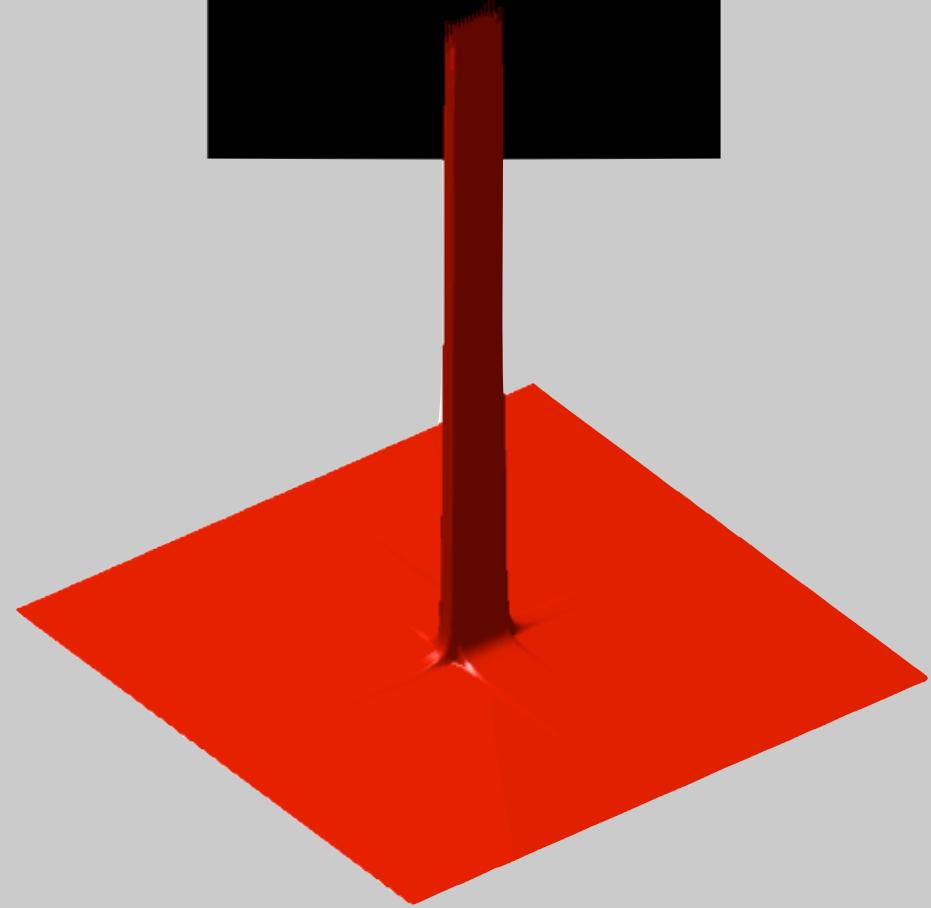
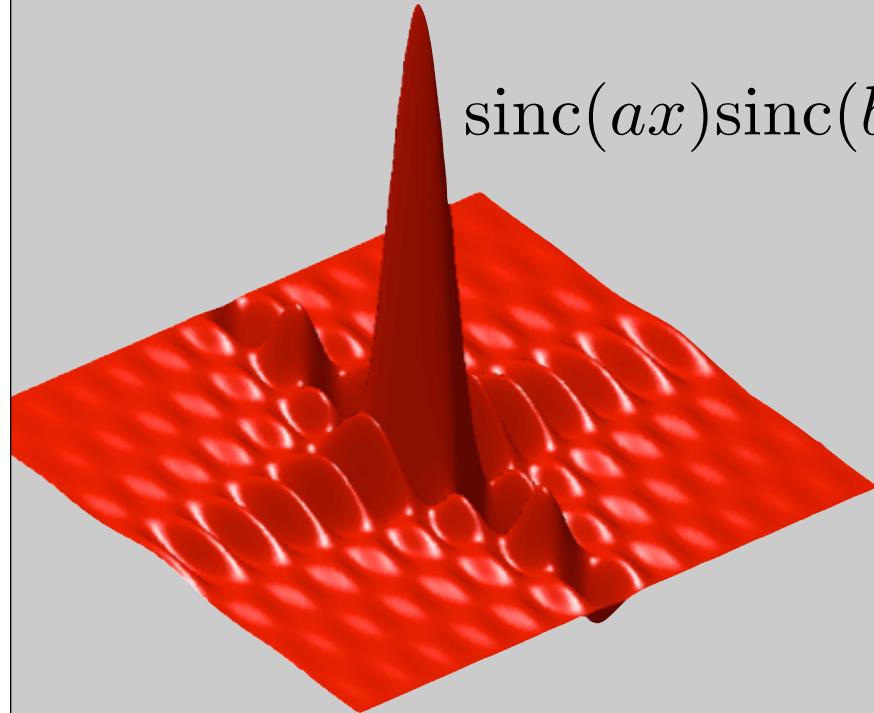


$$\text{sinc}(ax)\text{sinc}(by)$$





$$\text{sinc}(ax)\text{sinc}(by)$$



2D DTFT

$$F(\omega_x, \omega_y) = \sum_{n_x=-\infty}^{\infty} \sum_{n_y=-\infty}^{\infty} f[n_x, n_y] e^{-j(\omega_x n_x + \omega_y n_y)}$$
$$-\pi \leq \omega_x, \omega_y \leq \pi$$

$$F(\kappa_x, \kappa_y) = \sum_{n_x=-\infty}^{\infty} \sum_{n_y=-\infty}^{\infty} f[n_x, n_y] e^{-j2\pi(\kappa_x n_x + \kappa_y n_y)}$$
$$-0.5 \leq \kappa_x, \kappa_y \leq 0.5$$

- I prefer 2nd
- “Massaging” the DTFT leads to separable transforms in each axis

2D - DFT

- Similarly to 1D:

- Forward:

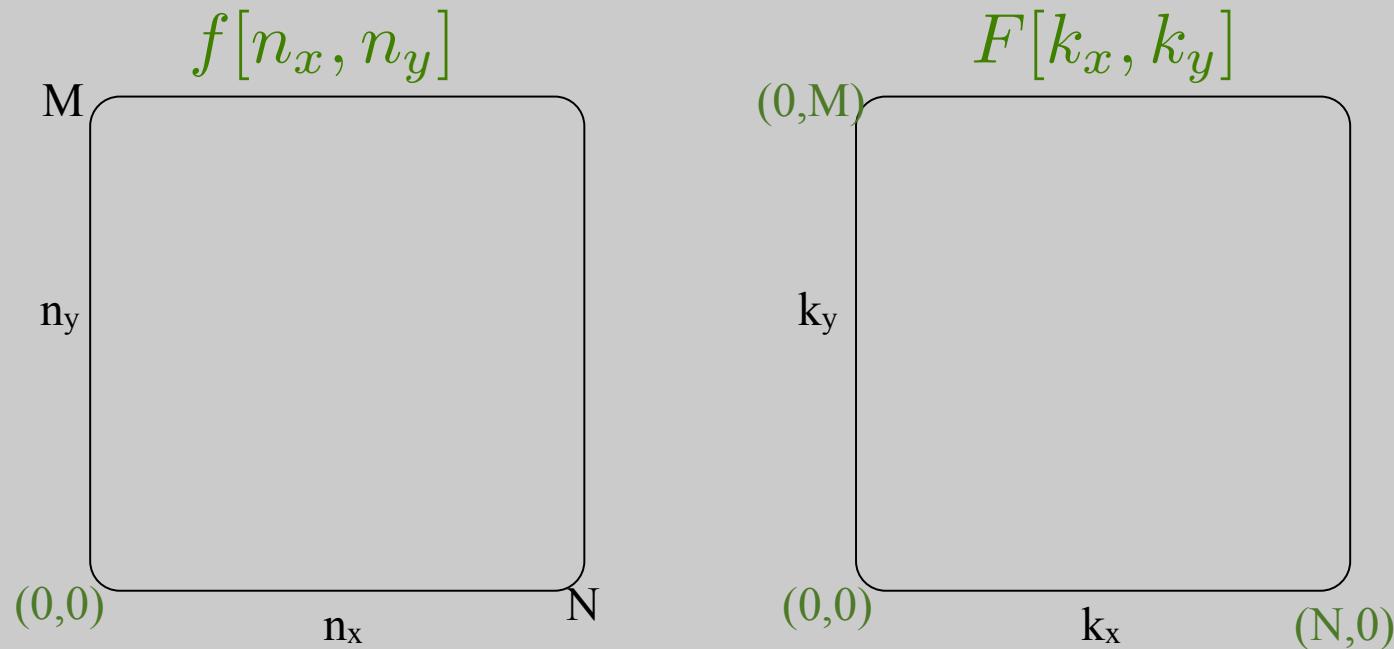
$$F[k_x, k_y] = \sum_{n_x=0}^{N-1} \sum_{n_y=0}^{M-1} f[n_x, n_y] e^{-j2\pi(n_x k_x / N + n_y k_y / M)}$$
$$\kappa_x = k_x / N, \kappa_y = k_y / M$$

- Inverse:

$$f[n_x, n_y] = \frac{1}{NM} \sum_{k_x=0}^{N-1} \sum_{k_y=0}^{M-1} F[k_x, k_y] e^{+j2\pi(n_x k_x / N + n_y k_y / M)}$$

2D - DFT

$$F[k_x, k_y] = \sum_{n_x=0}^{N-1} \sum_{n_y=0}^{M-1} f[n_x, n_y] e^{-j2\pi(n_x k_x / N + n_y k_y / M)}$$



Need to `fftshift` in 2D to get it to look like DTFT.

Properties of 2D DFT

- Circular Convolution

$$f[n_x, n_y] * * h[n_x, n_y] = F[k_x, k_y] H[k_x, k_y]$$

- Circular shift

$$f[(n_x - m_x)_N, (n_y - m_y)_M] = e^{-j2\pi(k_x m_x / N + k_y m_y / M)} F[k_x, k_y]$$

