EE123 Digital Signal Processing

Lecture 20
2D Signals

Announcements

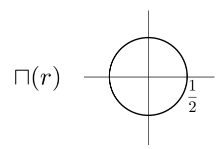
- Midterm Survey, please fill

Multi-Dimensional Signals

- Our world is more complex than 1D
- Images: f(x,y)
- Videos: f(x,y,t)
- Dynamic 3D scenes: f(x,y,z,t)
  - Medical Imaging
  - 3D Video
  - Computer Graphics
- We will focus on 2D

Continuous-Time 2D functions

- $\delta(x,y)$: Impulse at $x=0$, $y=0$
- $\delta(x)$: Impulse line (vertical or horizontal?)
- $\prod(x,y)$: 2D rect function
- $\cos(2\pi(f_x x + f_y y))$ - Spatial harmonic
- Circularly Symmetic:
  - $\prod(x,y)$: Pillbox



Spatial Frequency

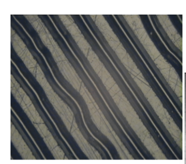
- What is a spatial frequency?
  - Complex Harmonic:

$$e^{j(\Omega_x x+\Omega_y y)} = e^{j2\pi(f_x x+f_y y)}$$

  - Units (for example):
    - x,y - cm
    - f_x, f_y - 1/cm
    - Ω_x, Ω_y - rad/cm

Spatial Frequency

- Vinyl Record
  - Transforms a temporal signal to a spatial signal



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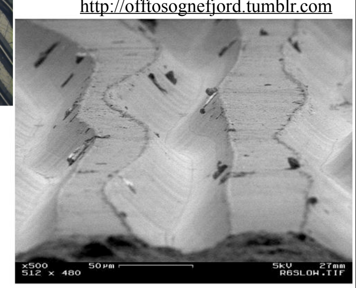
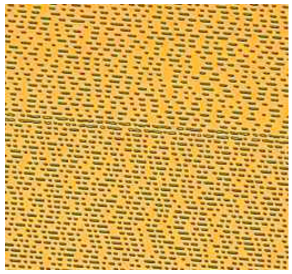


Spatial Frequency

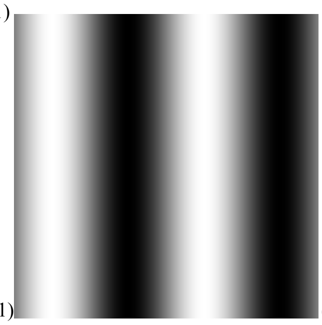
- CD ROM
  - encodes digital temporal signals to spatial signals



What is the frequency?

$\sin(2\pi(f_x x + f_y y))$

(-1,1) (1,1)



(-1,-1) (1,-1)

a) f_x=2, f_y=2

b) f_x=1, fy=0

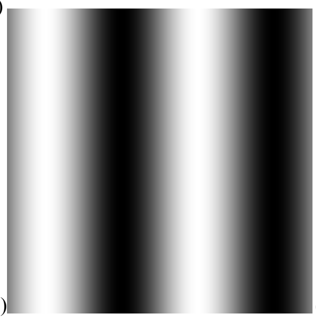
c) f_x = 4, f_y=0

d) none of the above

What is the frequency?

$\sin(2\pi(f_x x + f_y y))$

(-1,1) (1,1)



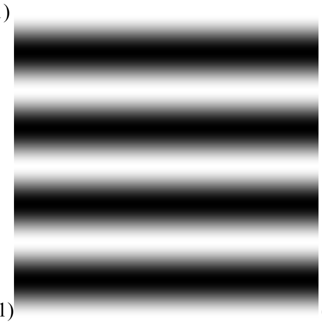
(-1,-1) (1,-1)

2 cycles for 2 cm ⇒ f_x=1 cm^{-1}

What is the frequency?

$\sin(2\pi(f_x x + f_y y))$ or $\cos(2\pi(f_x x + f_y y))$

(-1,1) (1,1)



(-1,-1) (1,-1)

a) sin, f_x=0, f_y=2

b) cos, f_x=0, fy=4

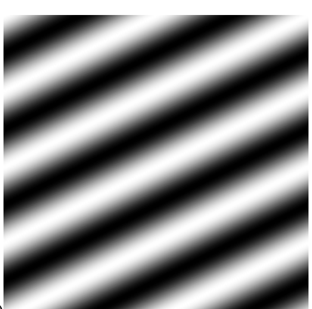
c) cos, f_x = 0, f_y=2

d) none of the above

What is the frequency?

$\cos(2\pi(f_x x + f_y y))$

(-1,1) (1,1)



(-1,-1) (1,-1)

This is the answera) f_x=1, f_y=2

b) f_x=4, fy=2

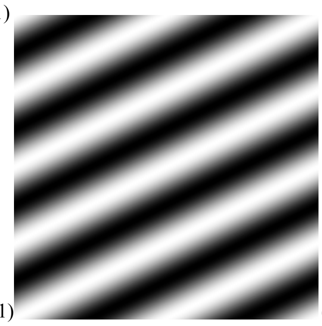
c) f_x = 2, f_y=1

d) f_x=2, fy=4

What is the frequency?

$\cos(2\pi(f_x x + f_y y))$

(-1,1) (1,1)



(-1,-1) (1,-1)

a) f_x=1, f_y=2

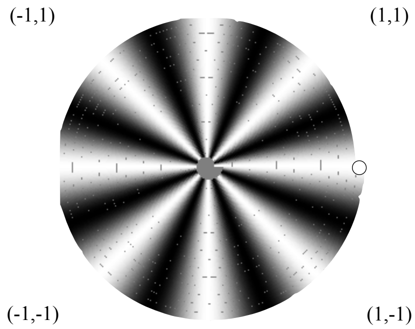
b) f_x=4, fy=2

c) f_x = 2, f_y=1

d) f_x=2, fy=4

What is the Temporal Frequency?

Vinyl rotates at 1 Hz



a) cos(2π8t)

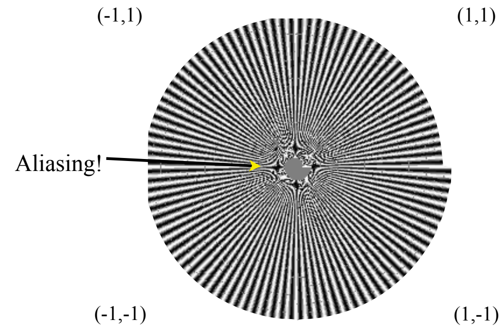
b) cos(2π8t²)

c) cos(2π4t)

d) cos(2π4t²)

What is the Temporal Frequency?

Vinyl rotates at 1 Hz



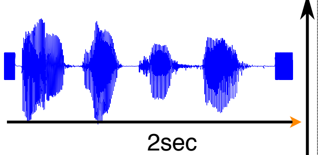
a) cos(2π100t)

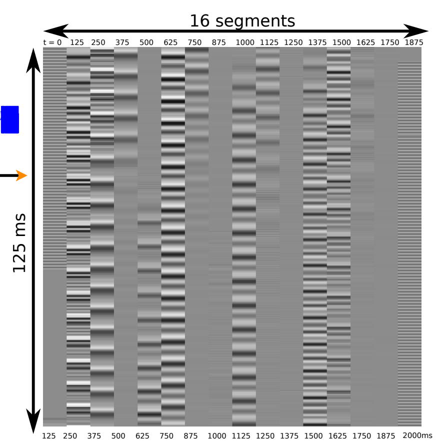
b) cos(2π100t²)

c) cos(2π40t)

d) none of the answers

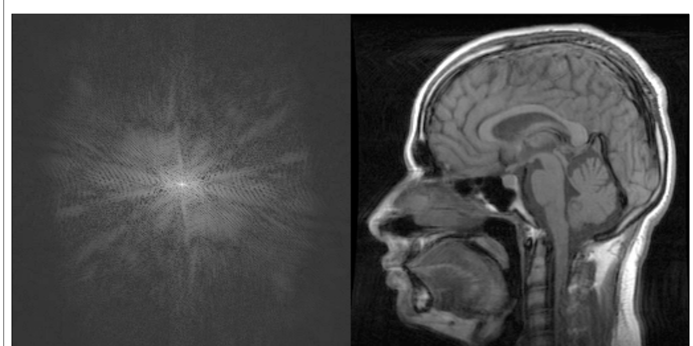
Challenge: What is the password?

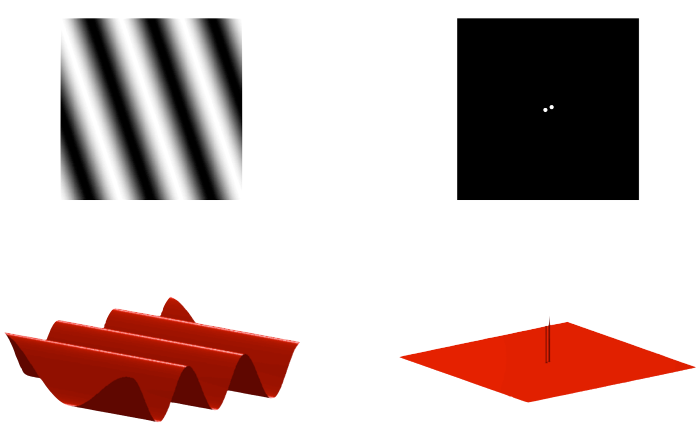


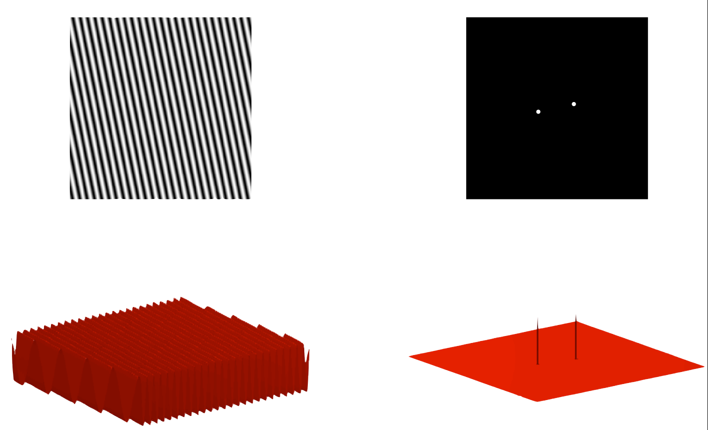


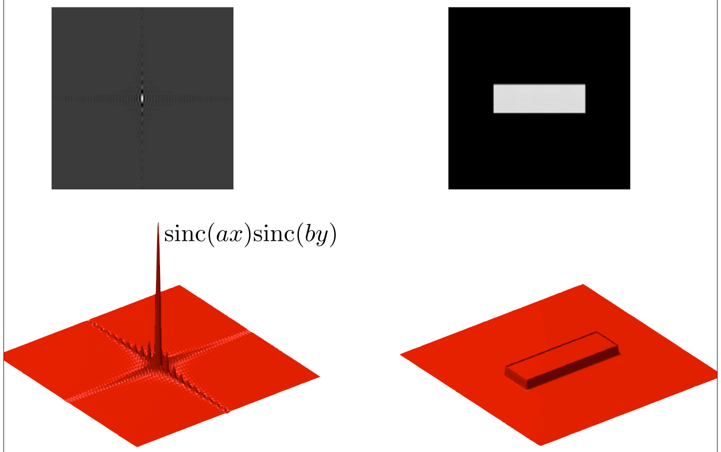
2D Fourier Transform

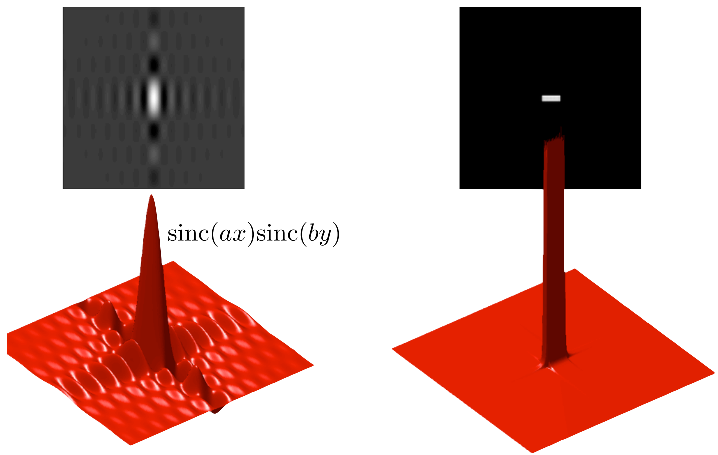
$$F(f_x, f_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(f_x x + f_y y)}dxdy$$











2D DTFT

$$F(\omega_x,\omega_y) = \sum_{n_x=-\infty}^{\infty}\sum_{n_y=-\infty}^{\infty} f[n_x,n_y]e^{-j(\omega_x n_x+\omega_y n_y)}$$

$$-\pi \le \omega_x, \omega_y \le \pi$$

$$F(\kappa_x,\kappa_y) = \sum_{n_x=-\infty}^{\infty}\sum_{n_y=-\infty}^{\infty} f[n_x,n_y]e^{-j2\pi(\kappa_x n_x+\kappa_y n_y)}$$

$$-0.5 \le \kappa_x, \kappa_y \le 0.5$$

- I prefer 2nd
- “Massaging” the DTFT leads to separable transforms in each axis

2D - DFT

- Similarly to 1D:
  - Forward:

$$F[k_x,k_y] = \sum_{n_x=0}^{N-1}\sum_{n_y=0}^{M-1} f[n_x,n_y]e^{-j2\pi(n_x k_x/N+n_y k_y/M)}$$

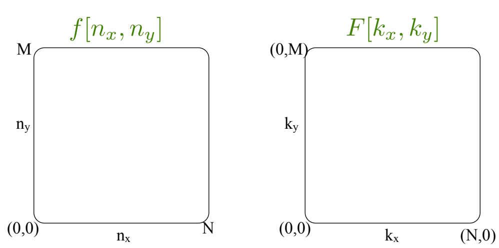
$$\kappa_x = k_x/N, \kappa_y = k_y/M$$

  - Inverse:

$$f[n_x,n_y] = \frac{1}{NM}\sum_{k_x=0}^{N-1}\sum_{k_y=0}^{M-1} F[k_x,k_y]e^{+j2\pi(n_x k_x/N+n_y k_y/M)}$$

2D - DFT

$$F[k_x,k_y] = \sum_{n_x=0}^{N-1}\sum_{n_y=0}^{M-1} f[n_x,n_y]e^{-j2\pi(n_x k_x/N+n_y k_y/M)}$$



Need to fftshift in 2D to get it to look like DTFT.

Properties of 2D DFT

- Circular Convolution

$$f[n_x,n_y] ** h[n_x,n_y] = F[k_x,k_y]H[k_x,k_y]$$

- Circular shift

$$f[(n_x-m_x)_N,(n_y-m_y)_M] = e^{-j2\pi(k_x m_x/N+k_y m_y/M)}F[k_x,k_y]$$