EE123
Digital Signal Processing

Lecture 20
2D Signals

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Announcements

- Midterm Survey, please fill

Multi-Dimensional Signals

- Our world is more complex than 1D
- Images: $f(x,y)$
- Videos: $f(x,y,t)$
- Dynamic 3D scenes: $f(x,y,z,t)$
  - Medical Imaging
  - 3D Video
  - Computer Graphics

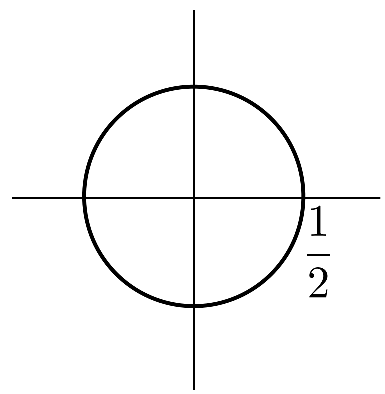
- We will focus on 2D

Continuous-Time 2D functions

- $\delta(x,y)$: Impulse at $x=0$, $y=0$
- $\delta(x)$: Impulse line (vertical or horizontal?)
- $\prod(x,y)$: 2D rect function
- $\cos(2\pi(f_x x + f_y y))$ - Spatial harmonic

- Circularly Symmetic:
  - $\prod(x,y)$: Pillbox

$\sqcap(r)$



Spatial Frequency

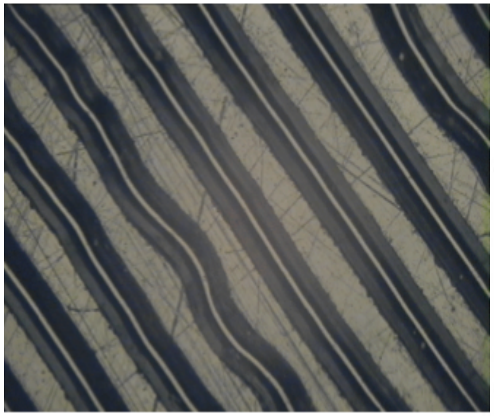
- What is a spatial frequency?
  - Complex Harmonic:

$$e^{j(\Omega_x x+\Omega_y y)} = e^{j2\pi(f_x x+f_y y)}$$

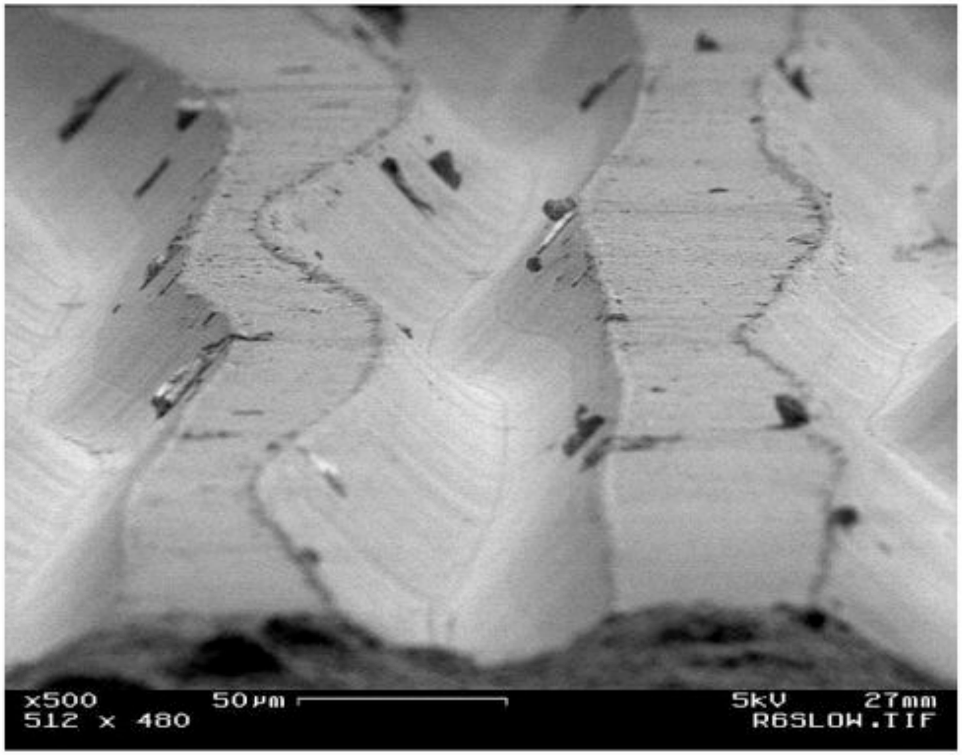
  - Units (for example):
    - x,y - cm
    - f_x, f_y - 1/cm
    - Ω_x, Ω_y - rad/cm

Spatial Frequency

- Vinyl Record
  - Transforms a temporal signal to a spatial signal

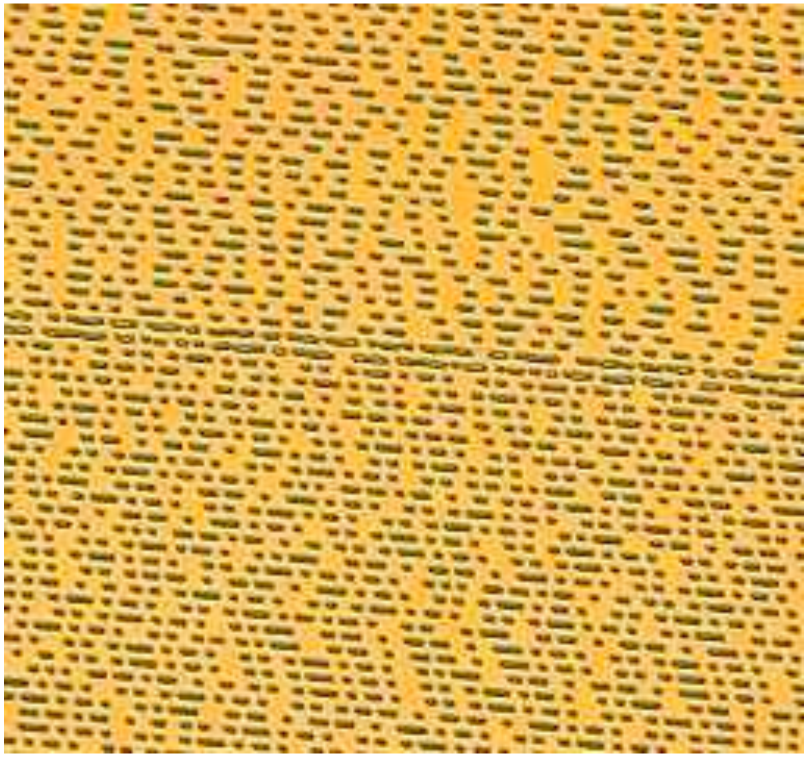


http://offtosognefjord.tumblr.com



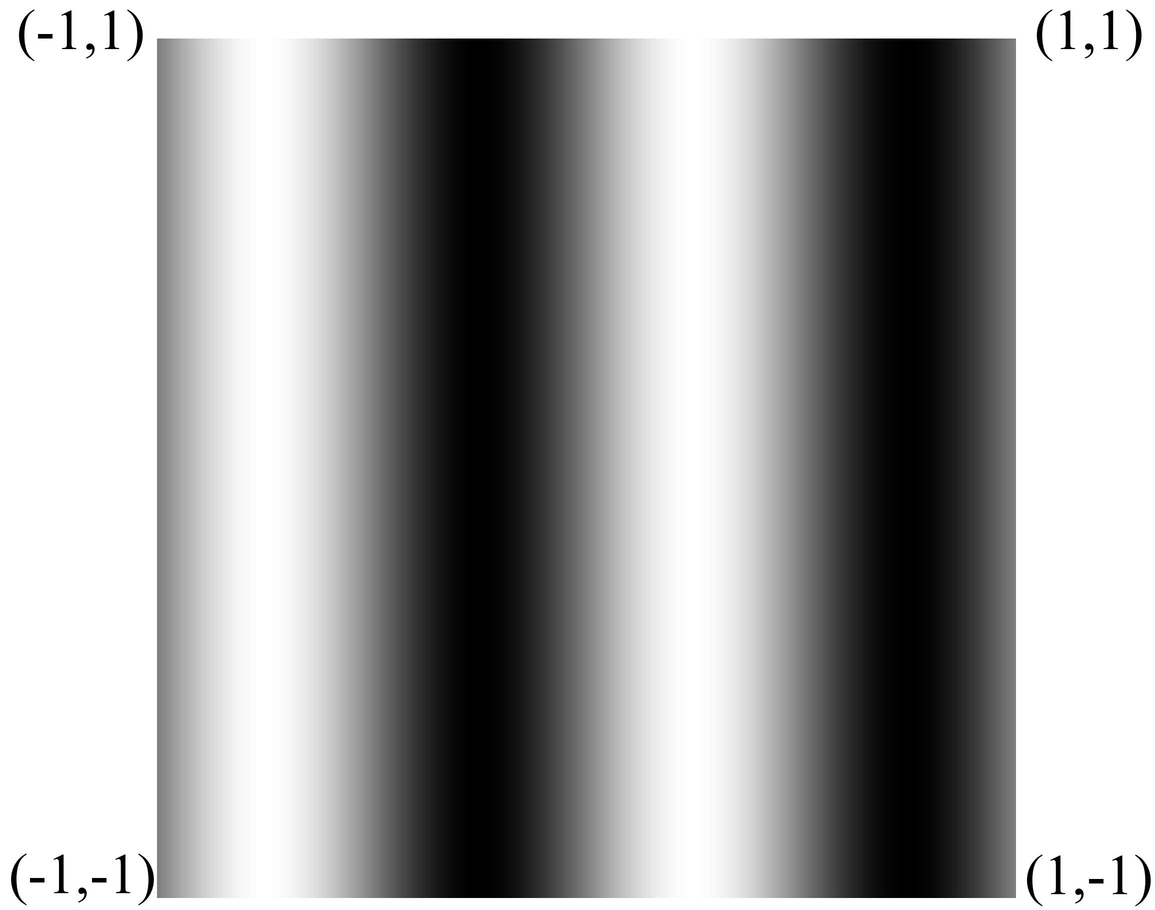
Spatial Frequency

- CD ROM
  - encodes digital temporal signals to spatial signals



What is the frequency?

$$\sin(2\pi(f_x x + f_y y))$$



a) $f_x=2$, $f_y=2$

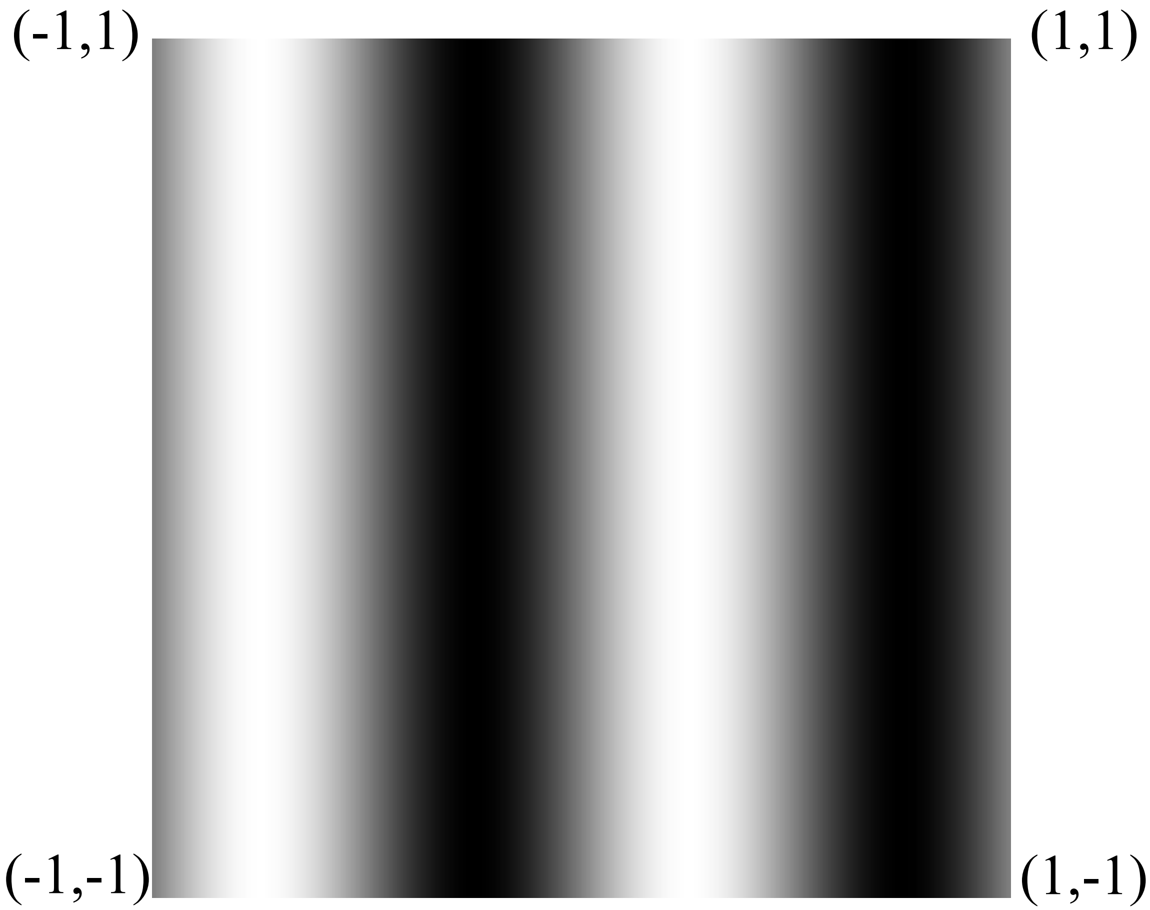
b) $f_x=1$, fy=0

c) $f_x = 4$, $f_y=0$

d) none of the above

What is the frequency?

$$\sin(2\pi(f_x x + f_y y))$$



2 cycles for 2 cm ⇒ f_x=1 cm^{-1}

What is the frequency?

$$\sin(2\pi(f_x x + f_y y)) \quad \text{or} \quad \cos(2\pi(f_x x + f_y y))$$

(-1,1) (1,1)

(-1,-1) (1,-1)

a) sin, f_x=0, f_y=2

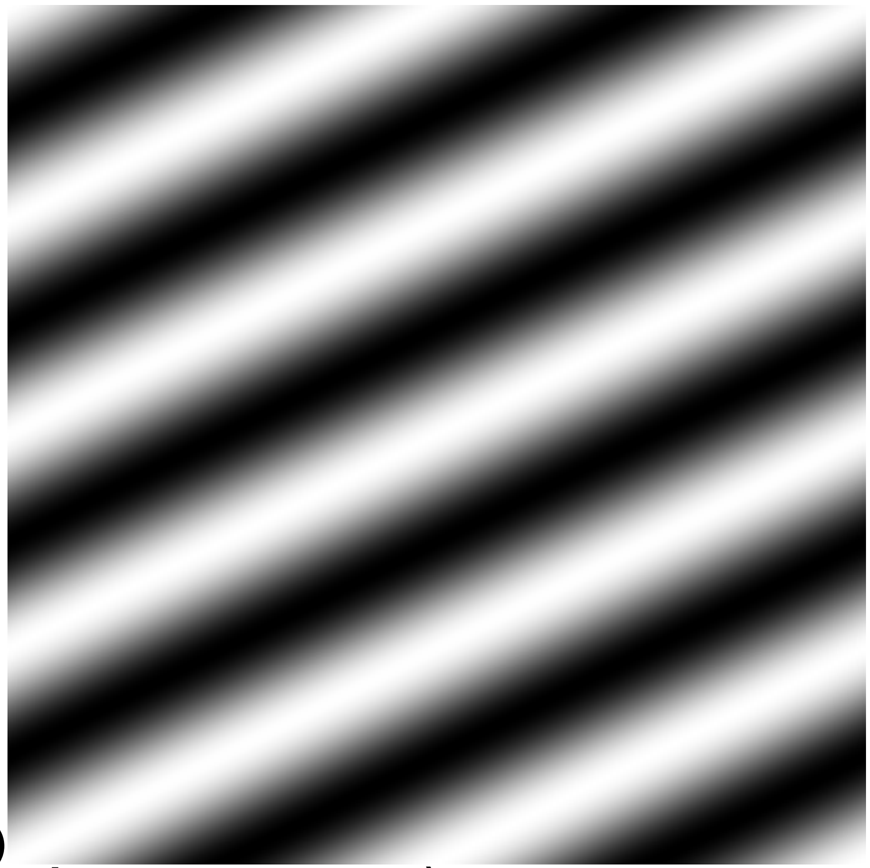
b) cos, f_x=0, fy=4

c) cos, f_x = 0, f_y=2

d) none of the above

What is the frequency?

$$\cos(2\pi(f_x x + f_y y))$$



This is the answera) $f_x=1$, $f_y=2$

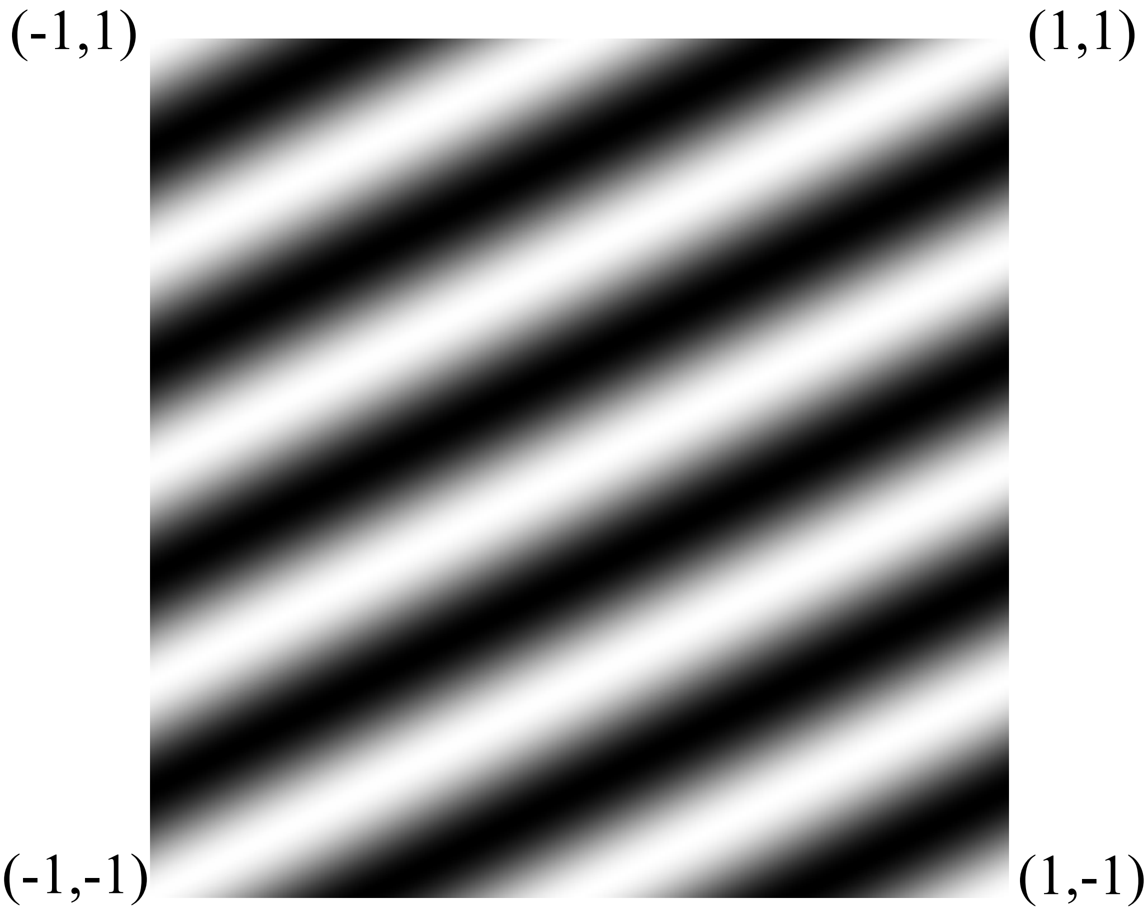
b) $f_x=4$, fy=2

c) $f_x = 2$, $f_y=1$

d) $f_x=2$, fy=4

What is the frequency?

$$\cos(2\pi(f_x x + f_y y))$$



a) f_x=1, f_y=2

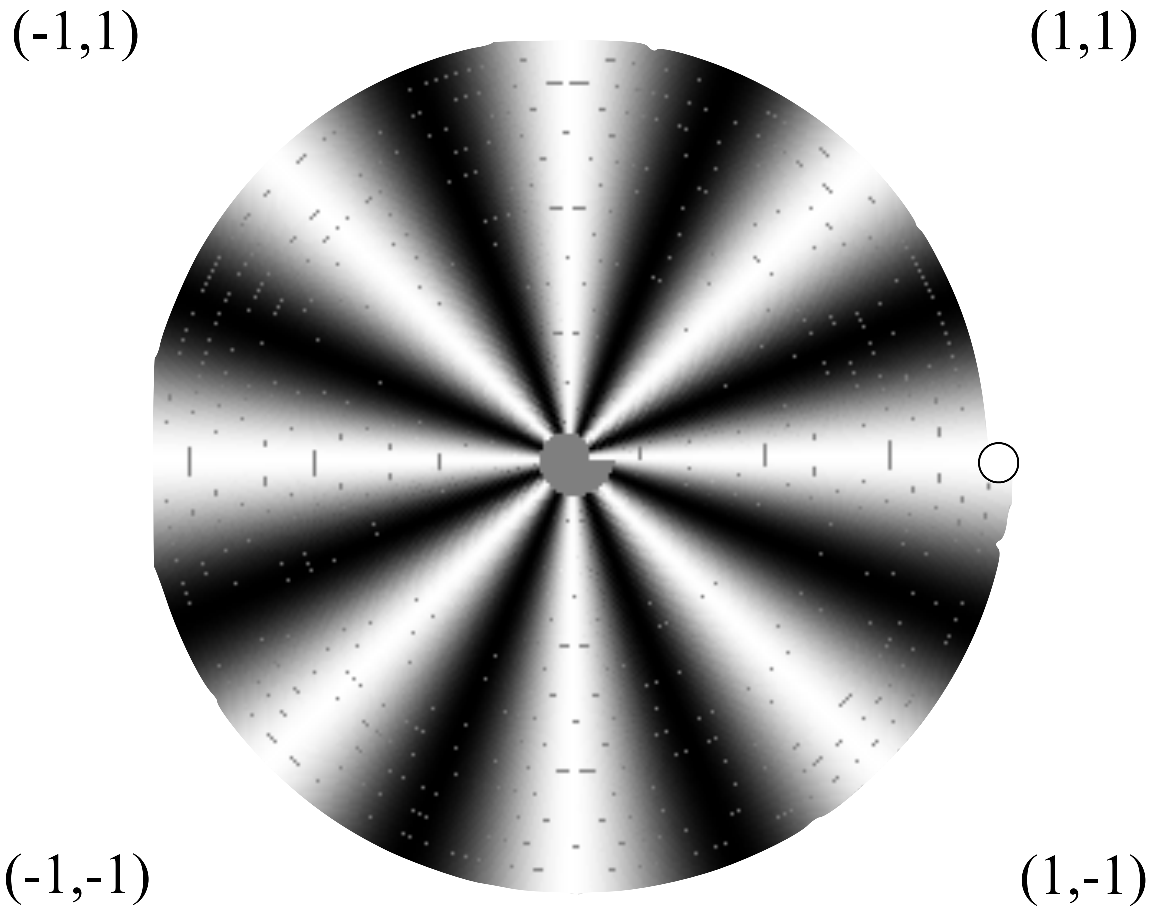
c) f_x = 2, f_y=1

b) f_x=4, fy=2

d) f_x=2, fy=4

What is the Temporal Frequency?

Vinyl rotates at 1 Hz



a) $\cos(2\pi 8t)$

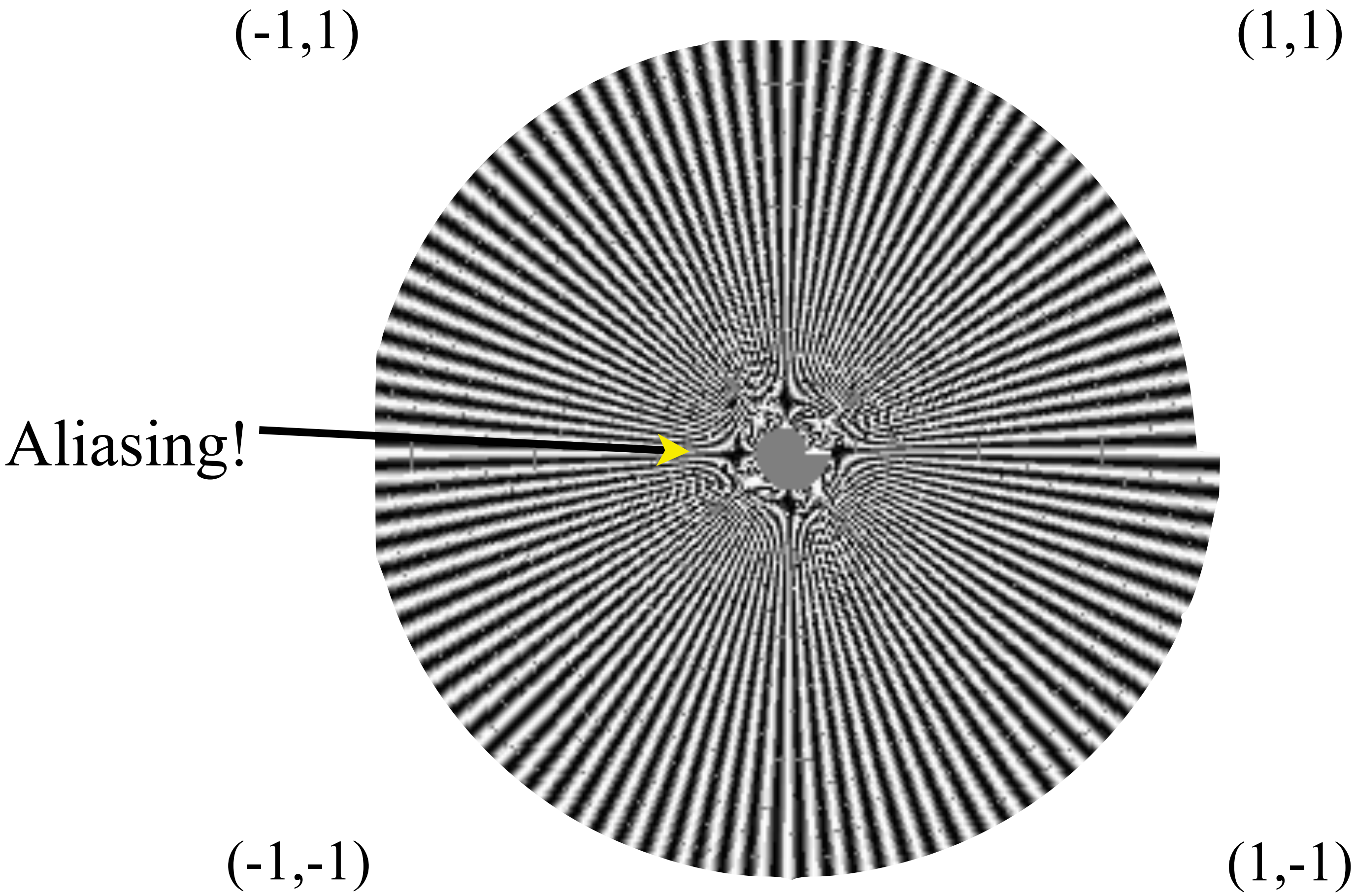
b) $\cos(2\pi 8t^2)$

c) $\cos(2\pi 4t)$

d) $\cos(2\pi 4t^2)$

What is the Temporal Frequency?

Vinyl rotates at 1 Hz



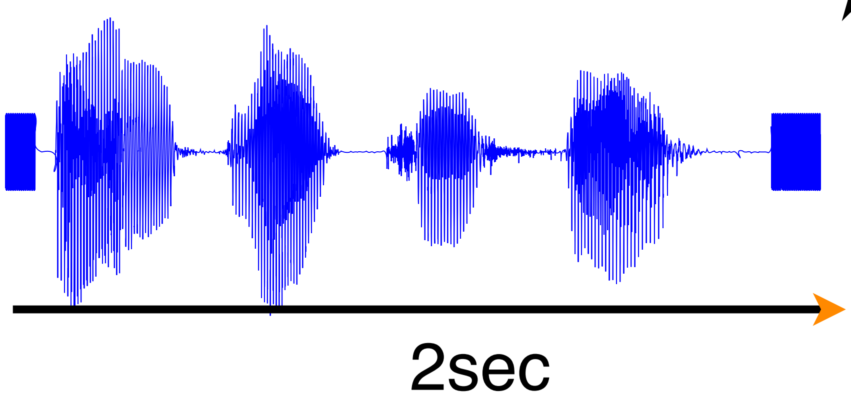
a) $\cos(2\pi 100t)$

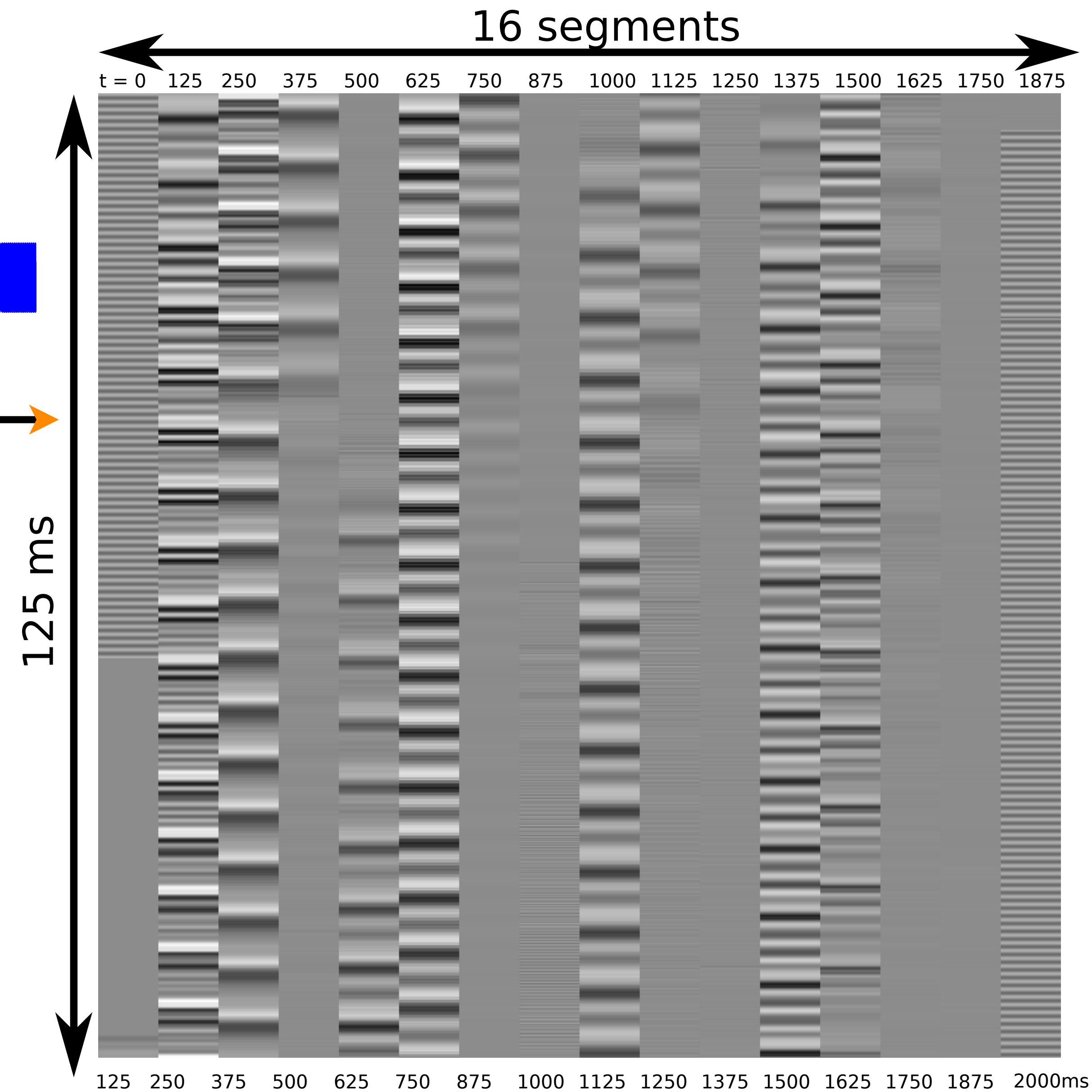
b) $\cos(2\pi 100t^2)$

c) $\cos(2\pi 40t)$

d) none of the answers

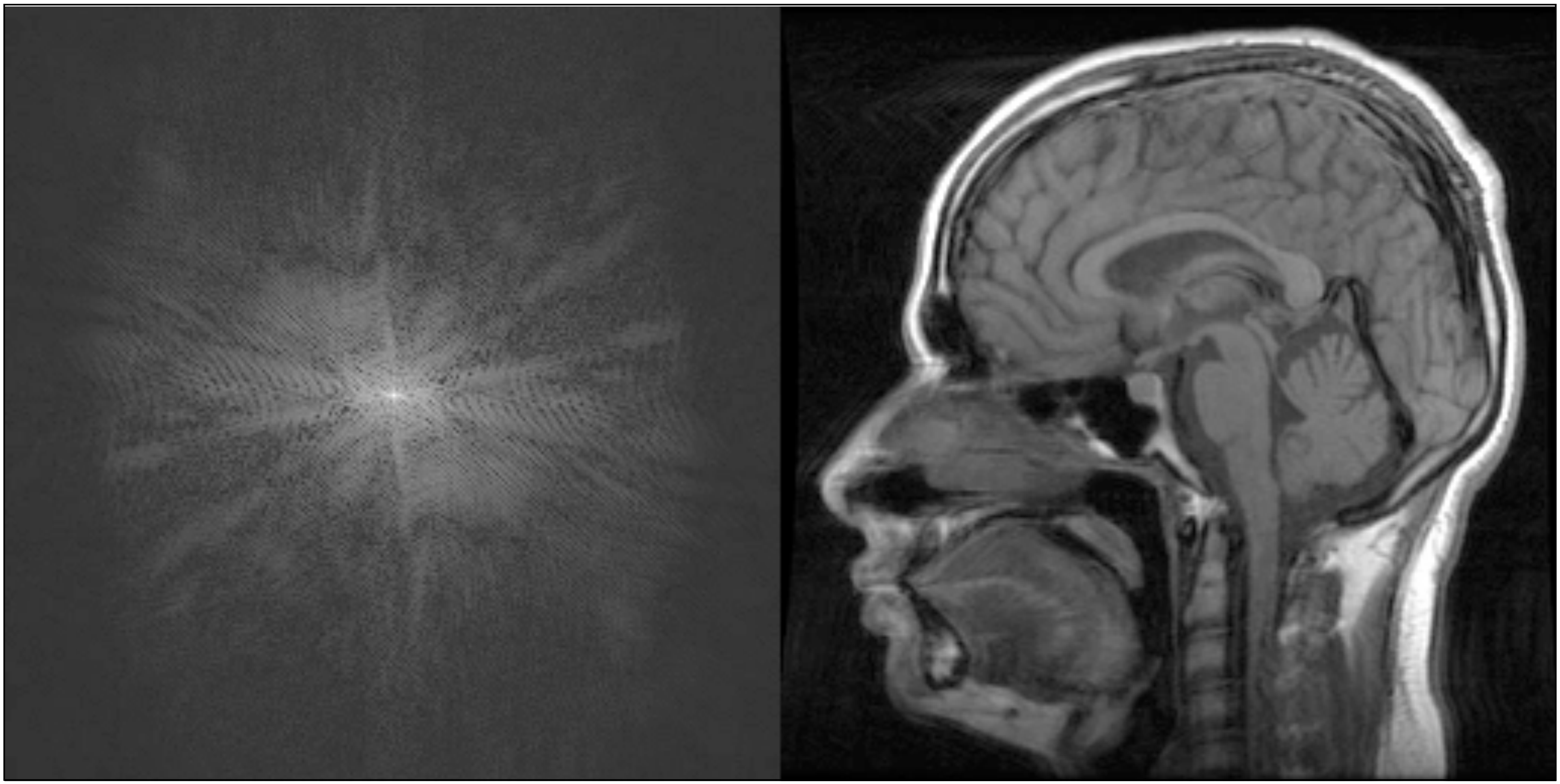
Challenge: What is the password?

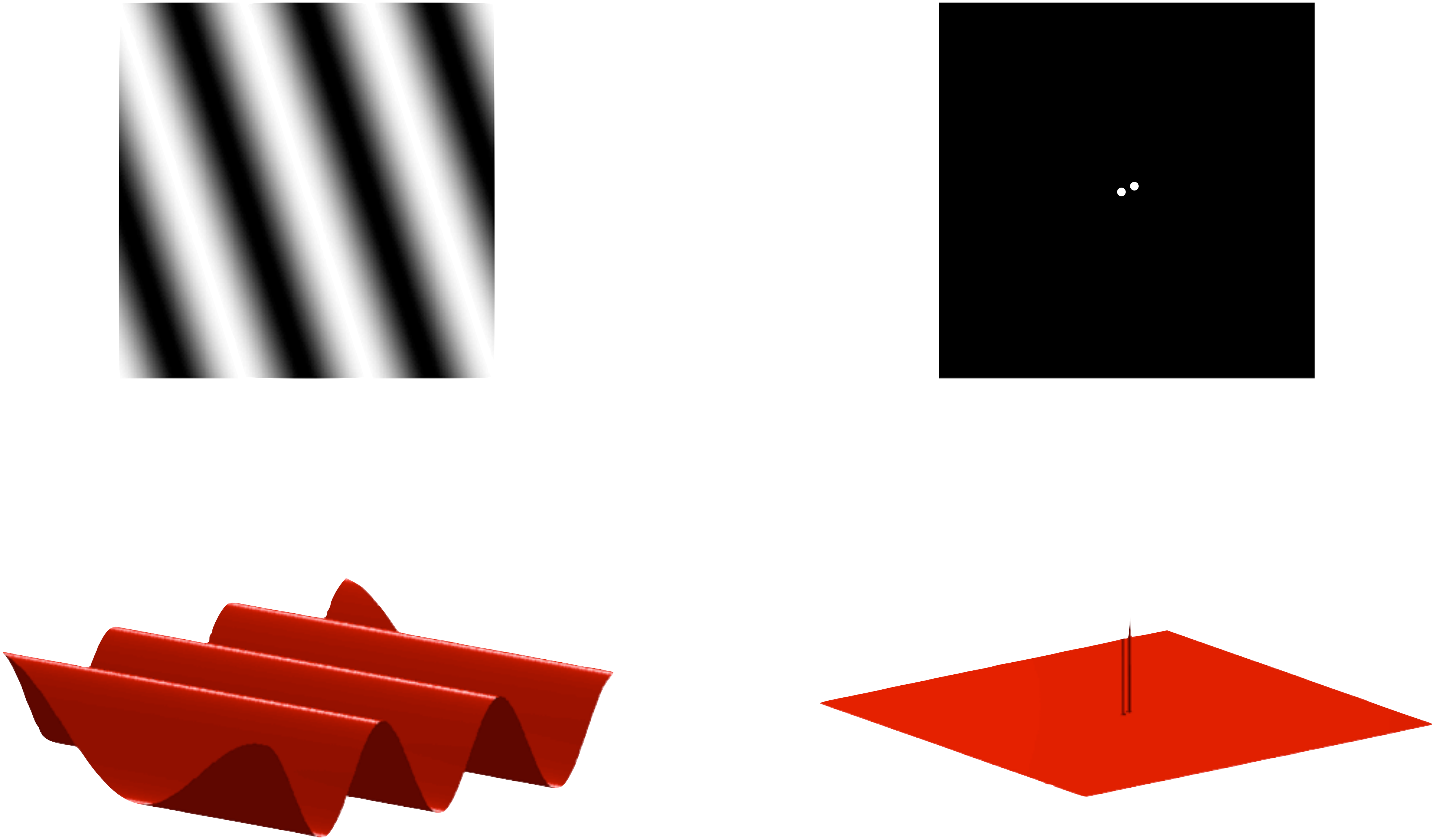


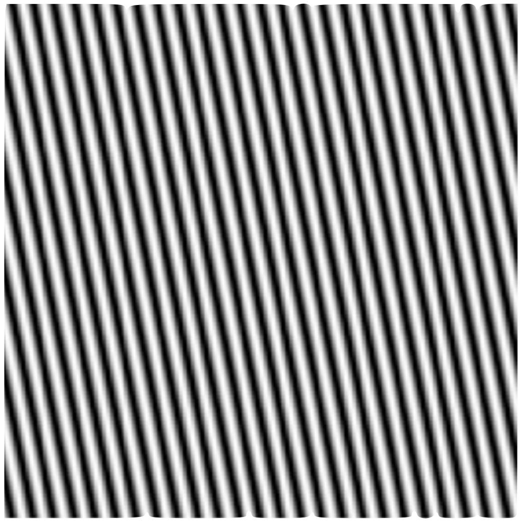


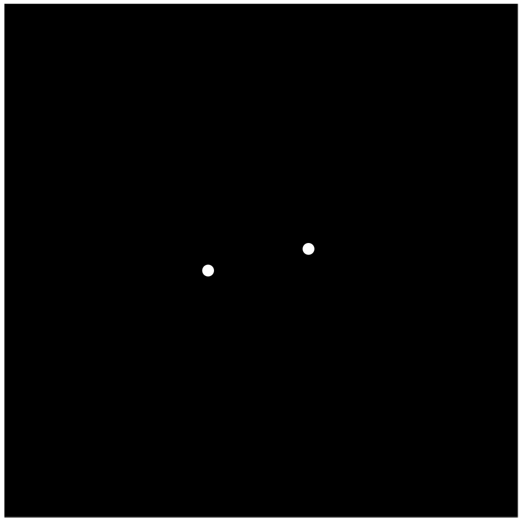
2D Fourier Transform

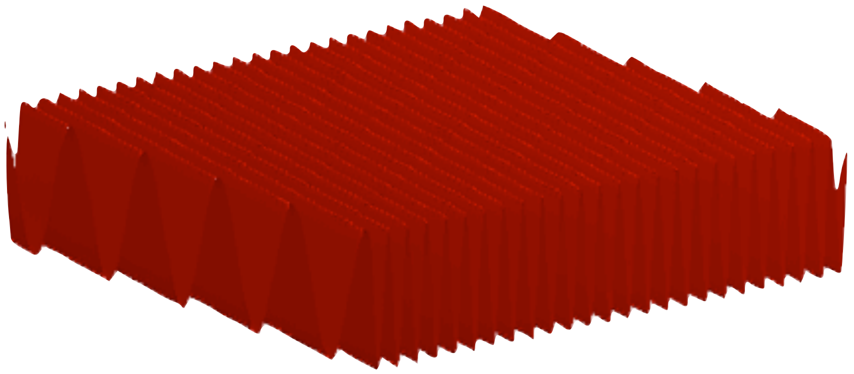
$$F(f_x, f_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(f_x x + f_y y)}dxdy$$

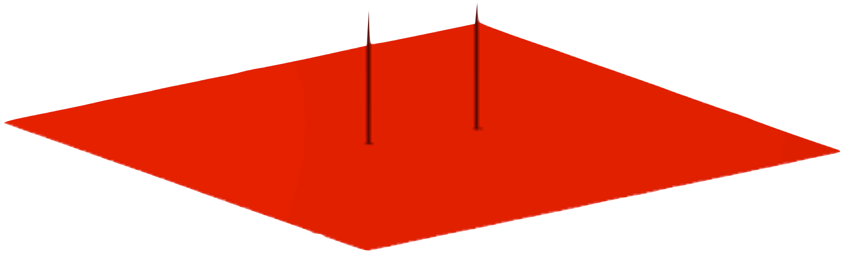


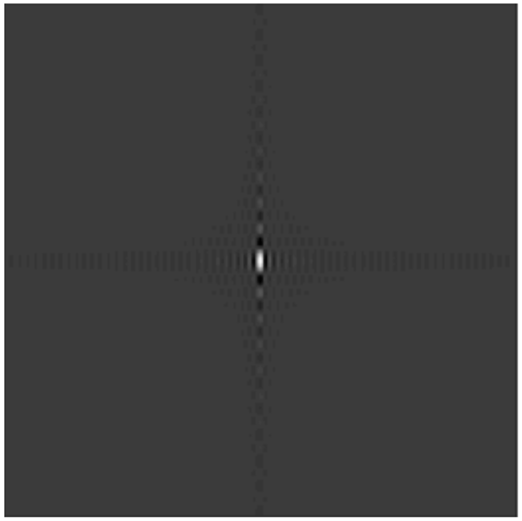


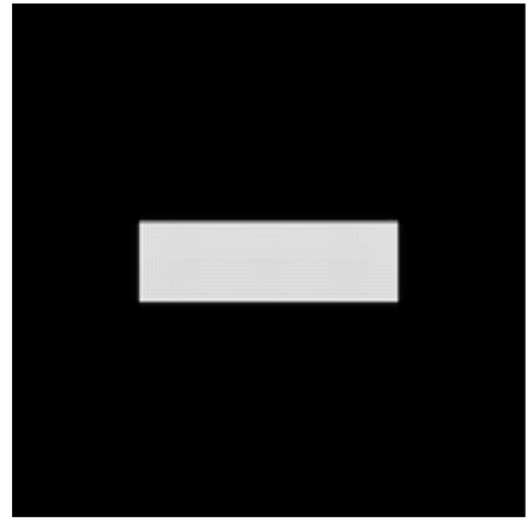




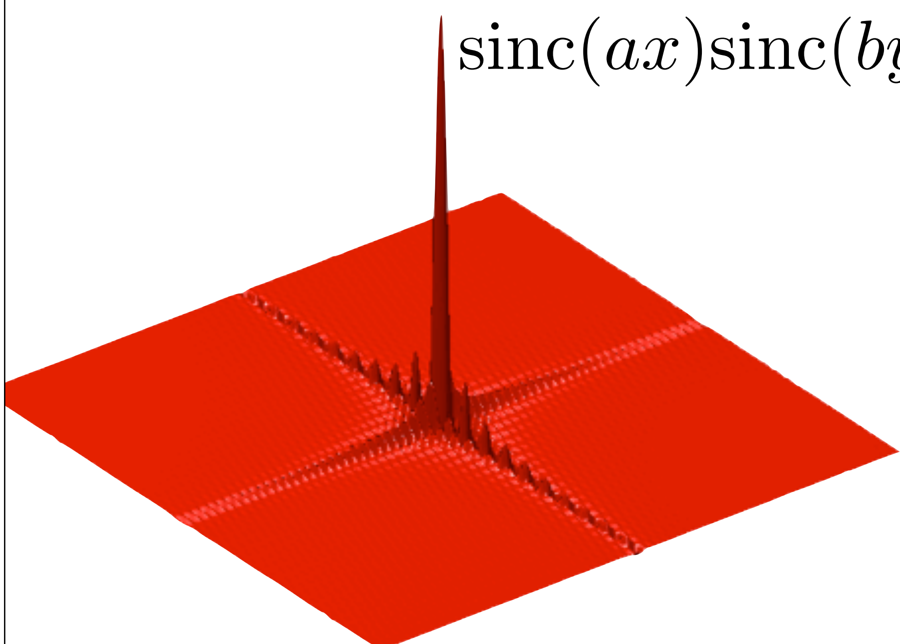


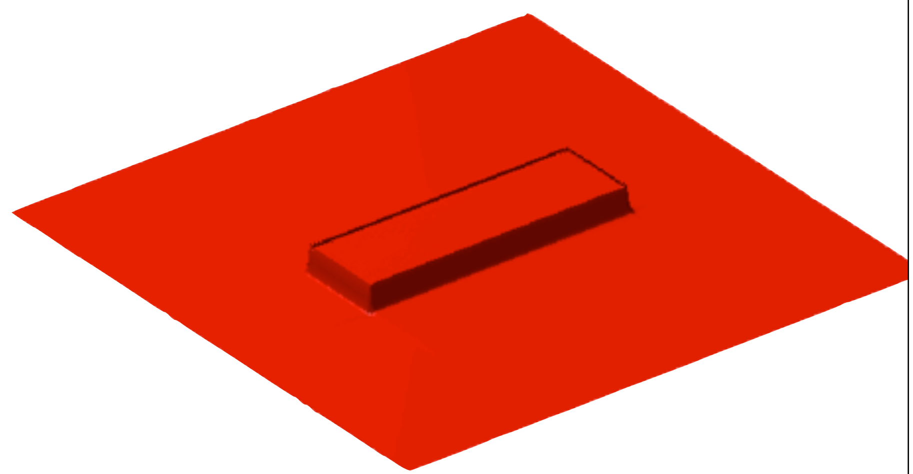


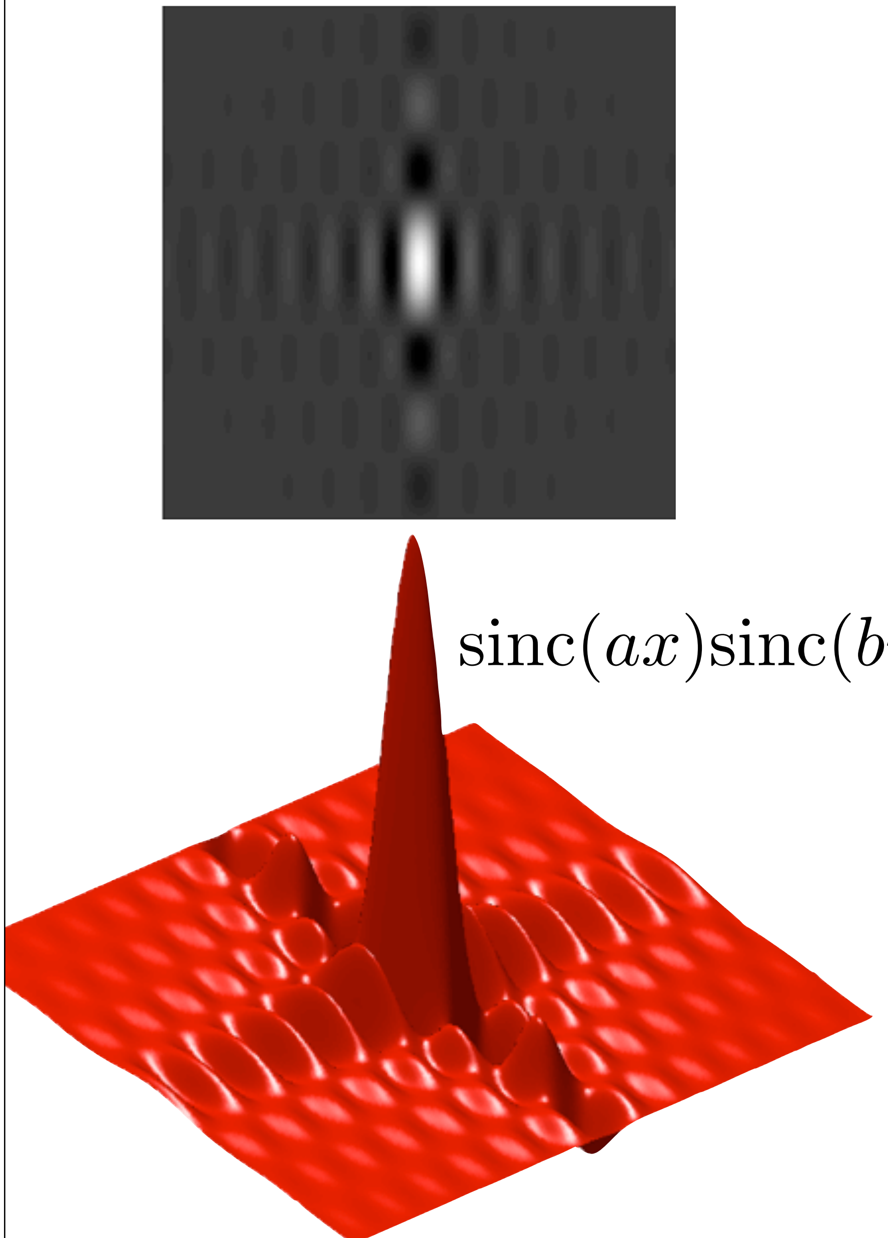




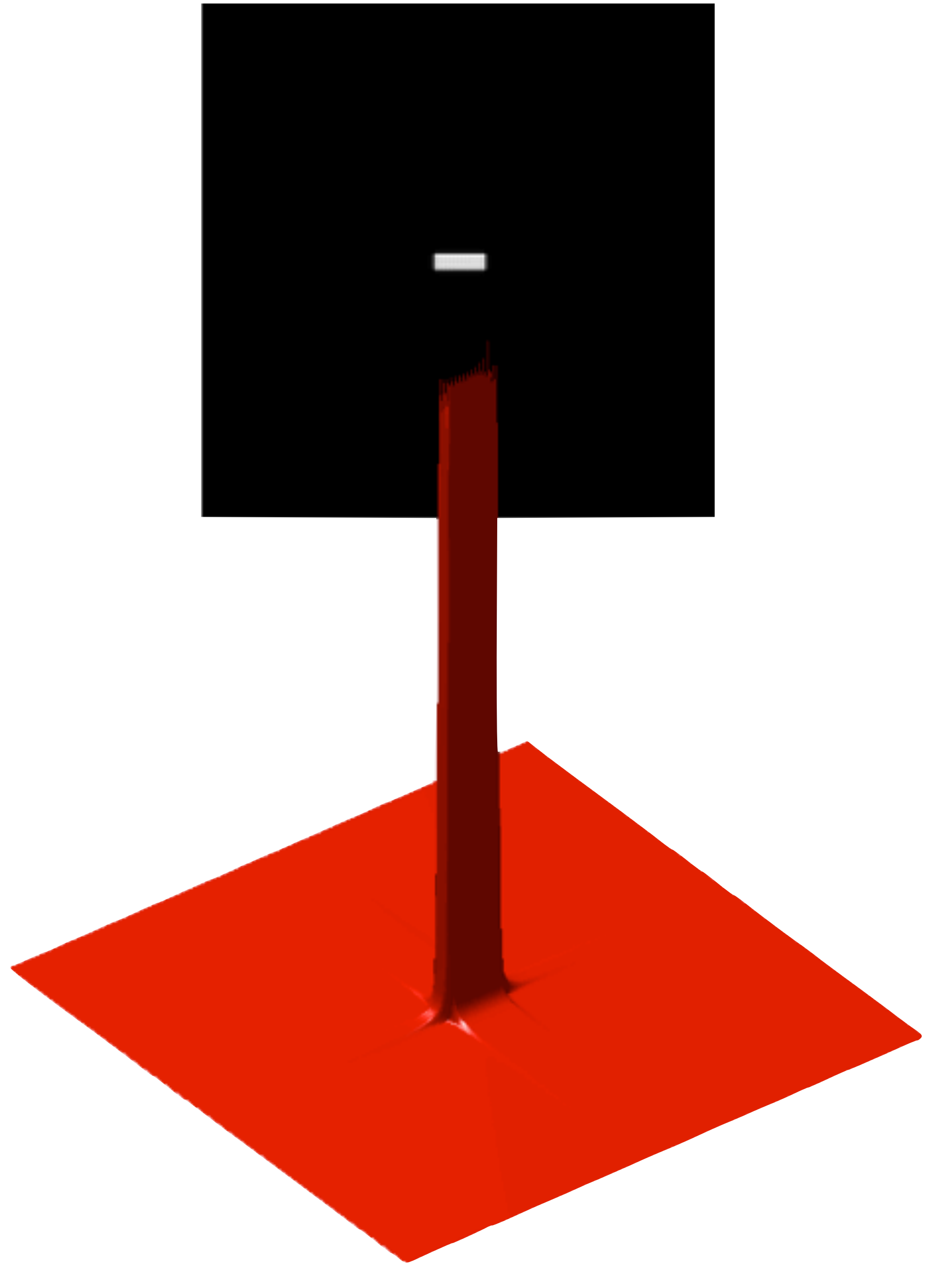
$\mathrm{sinc}(ax)\mathrm{sinc}(by)$







$\mathrm{sinc}(ax)\mathrm{sinc}(by)$



2D DTFT

$$F(\omega_x, \omega_y) = \sum_{n_x=-\infty}^{\infty} \sum_{n_y=-\infty}^{\infty} f[n_x, n_y] e^{-j(\omega_x n_x + \omega_y n_y)}$$

$$-\pi \leq \omega_x, \omega_y \leq \pi$$

$$F(\kappa_x, \kappa_y) = \sum_{n_x=-\infty}^{\infty} \sum_{n_y=-\infty}^{\infty} f[n_x, n_y] e^{-j2\pi(\kappa_x n_x + \kappa_y n_y)}$$

$$-0.5 \leq \kappa_x, \kappa_y \leq 0.5$$

- I prefer 2nd
- "Massaging" the DTFT leads to separable transforms in each axis

2D - DFT

- Similarly to 1D:
  - Forward:

$$F[k_x, k_y] = \sum_{n_x=0}^{N-1} \sum_{n_y=0}^{M-1} f[n_x, n_y] e^{-j2\pi(n_x k_x/N + n_y k_y/M)}$$

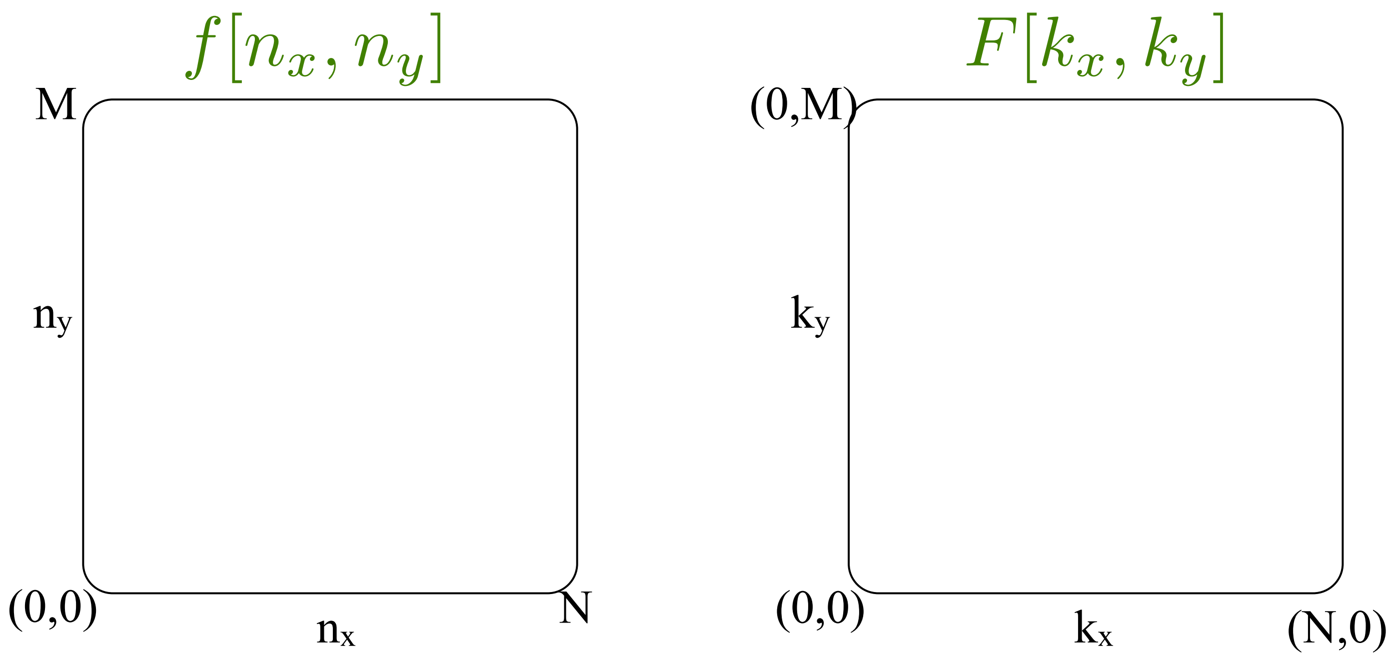
$$\kappa_x = k_x/N, \kappa_y = k_y/M$$

  - Inverse:

$$f[n_x, n_y] = \frac{1}{NM} \sum_{k_x=0}^{N-1} \sum_{k_y=0}^{M-1} F[k_x, k_y] e^{+j2\pi(n_x k_x/N + n_y k_y/M)}$$

2D - DFT

$$F[k_x, k_y] = \sum_{n_x=0}^{N-1} \sum_{n_y=0}^{M-1} f[n_x, n_y] e^{-j2\pi(n_x k_x/N + n_y k_y/M)}$$



Need to fftshift in 2D to get it to look like DTFT.

Properties of 2D DFT

- Circular Convolution

$$f[n_x, n_y] * *h[n_x, n_y] = F[k_x, k_y]H[k_x, k_y]$$

- Circular shift

$$f[(n_x - m_x)_N, (n_y - m_y)_M] = e^{-j2\pi(k_x m_x/N + k_y m_y/M)}F[k_x, k_y]$$