EE123
Digital Signal Processing

Lecture 23
Compressed Sensing

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RADIOS

- https://inst.eecs.berkeley.edu/~ee123/sp15/radio.html

Compressive Sampling

Q: What is the rate you need to sample at?

A: At least Nyquist!

Compressive Sampling

Q: What is the rate you need to sample at?

A: Maybe less than Nyquist....

Image Compression

- Images are compressible
  - Standard approach: First collect, then compress

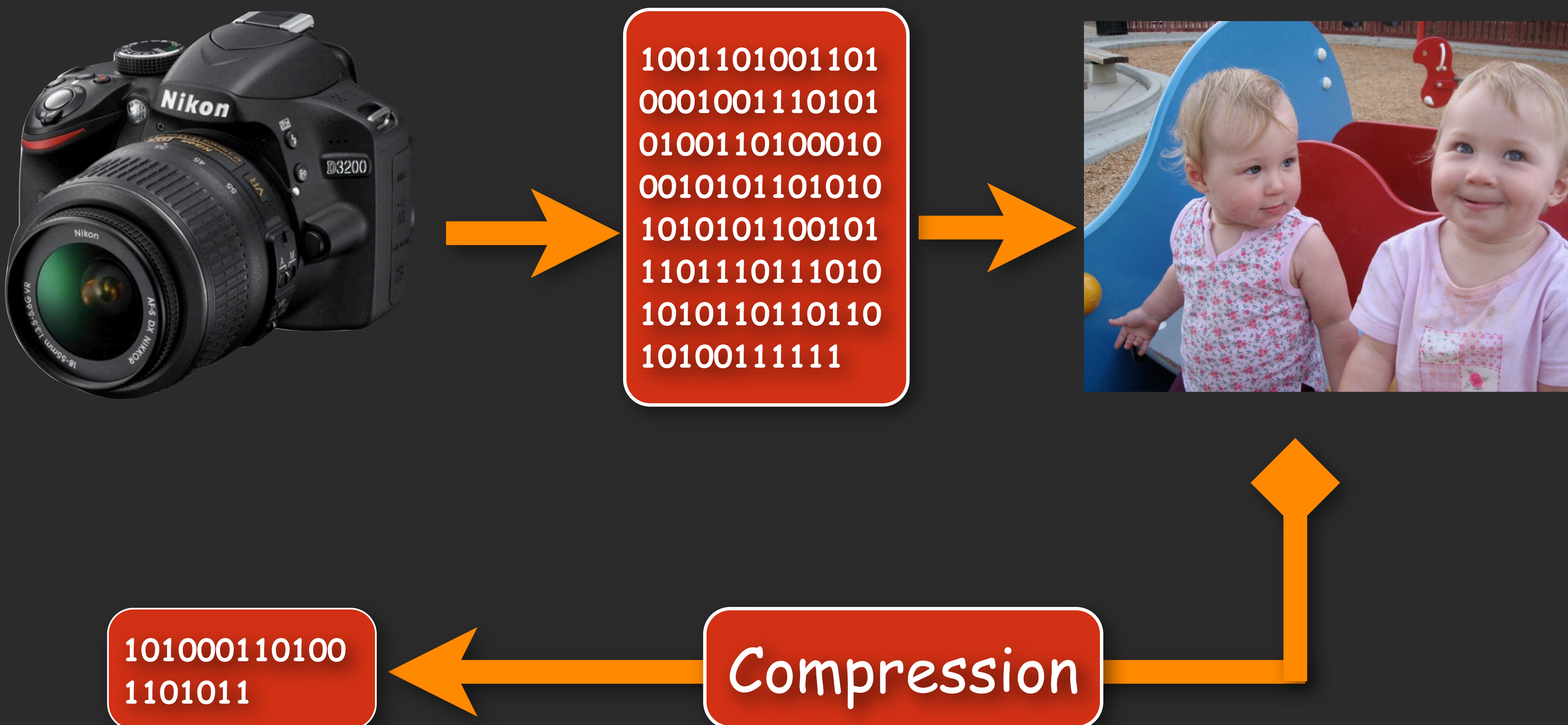
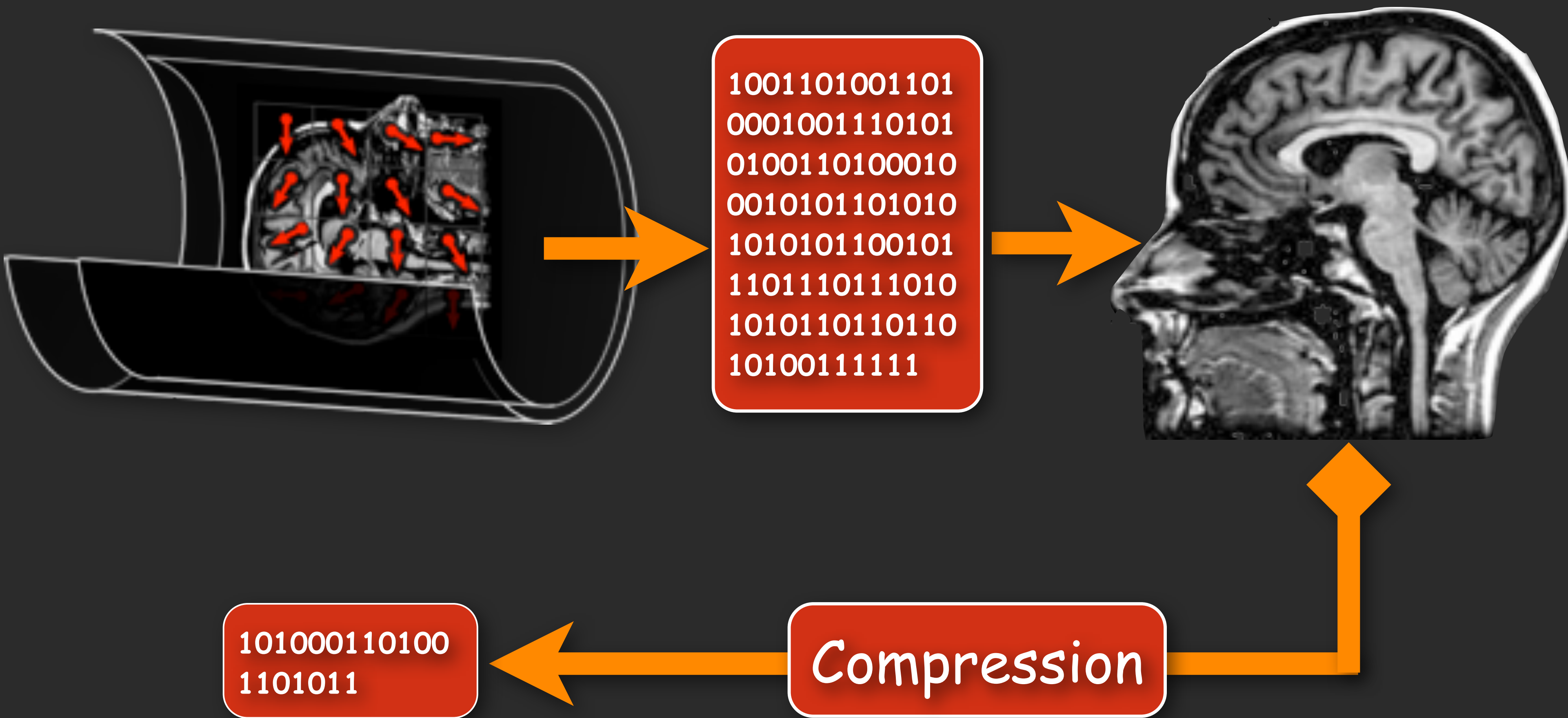


Image Compression

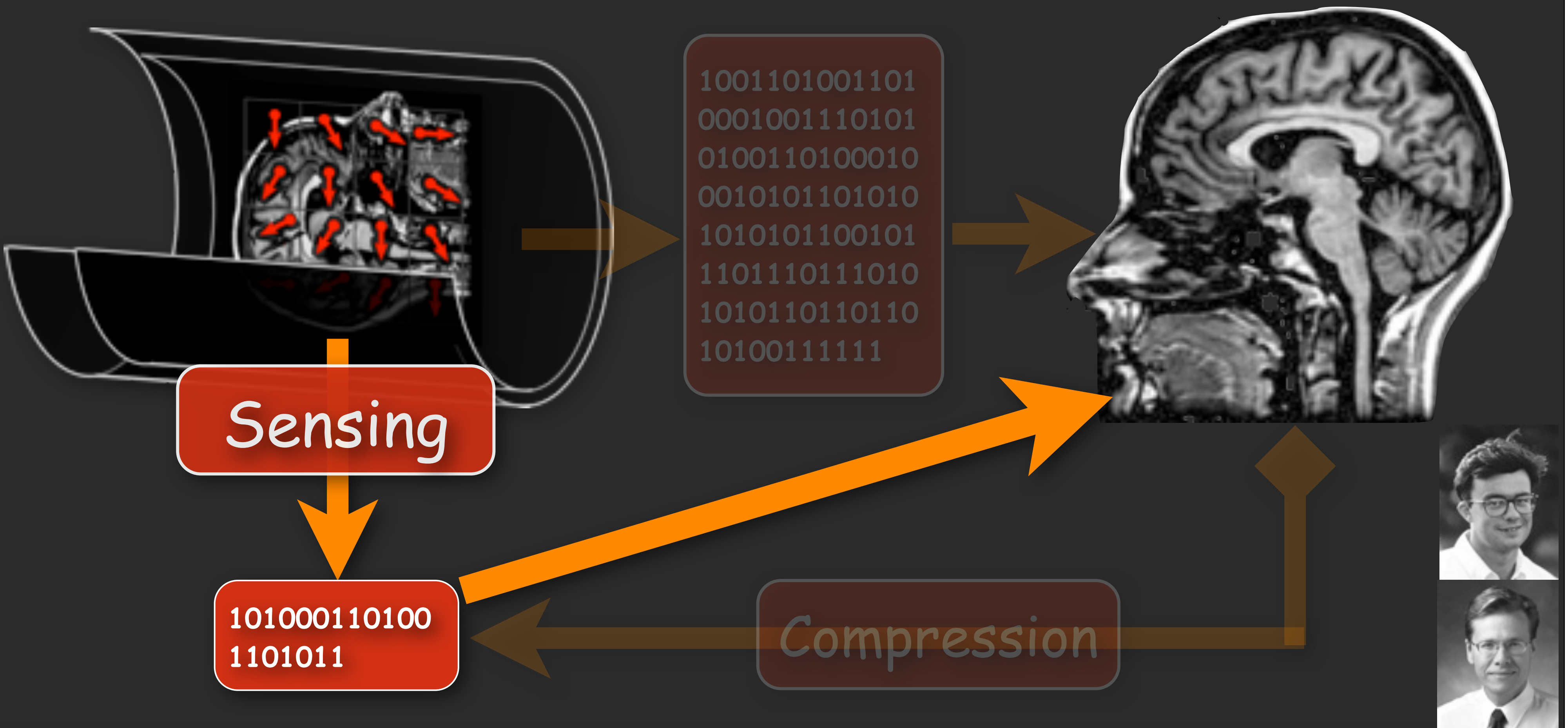
- Medical images are compressible
  - Standard approach: First collect, then compress



Compressed Sensing

Medical images are compressible

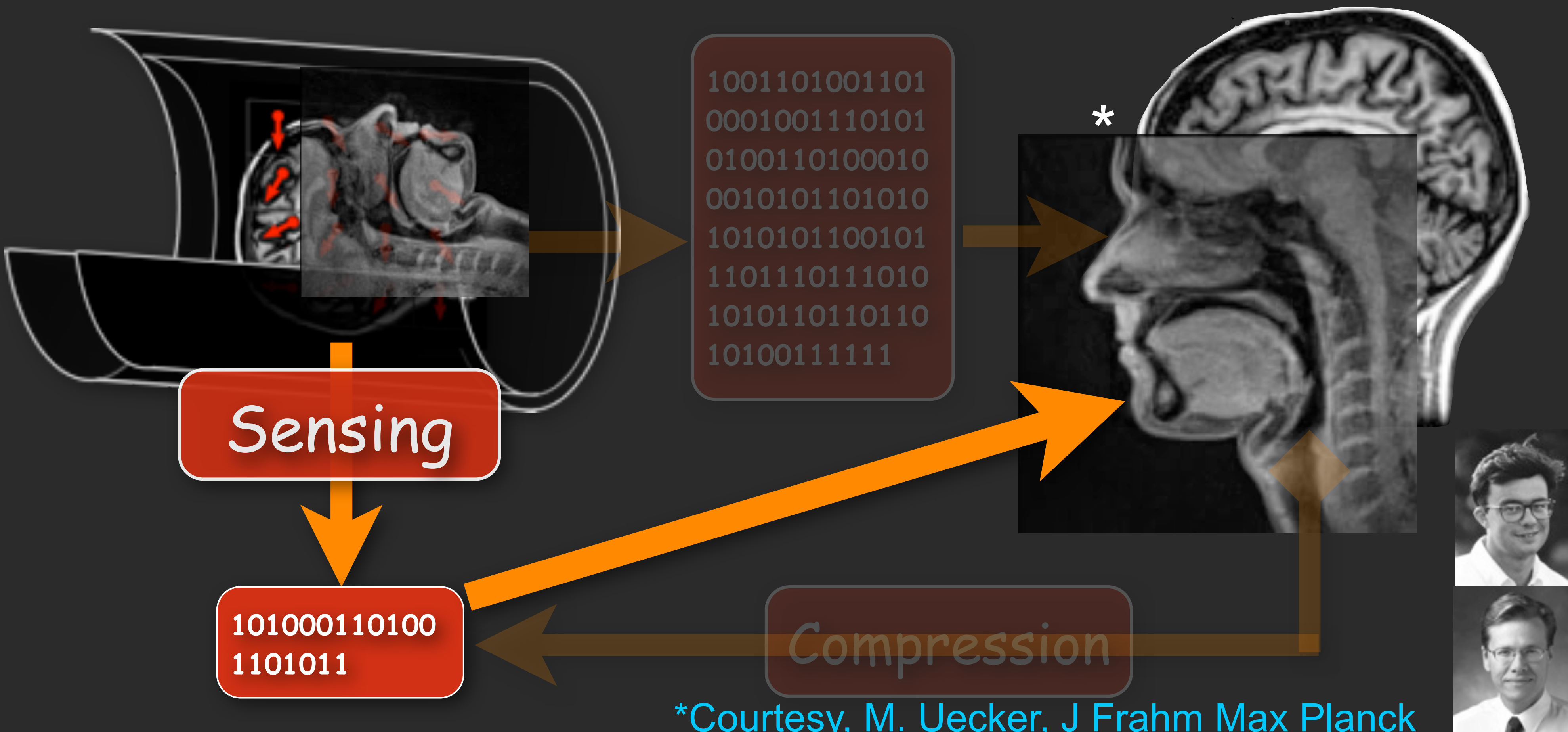
Standard approach: First collect, then compress



Compressed Sensing

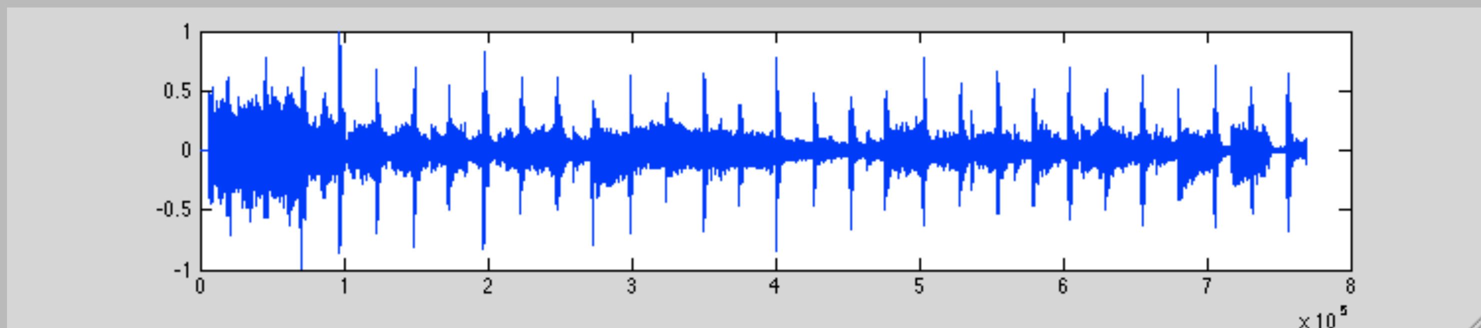
Medical images are compressible

Standard approach: First collect, then compress



*Courtesy, M. Uecker, J Frahm Max Planck

Example I: Audio



Raw audio: 44.1Khz, 16bit, stereo = 1378 Kbit/sec

MP3: 44.1Khz, 16bit, stereo = 128 Kbit/sec

10.76 fold!

Example II: Images

Raw image (RGB): 24 bit/pixel

JPEG : 1280x960, normal = 1.09 bit/pixel

22 fold!

Example III: Videos

Raw Video: (480x360)p x 24b/p x 24fps + 44.1Khz x 16b x 2 = 98,578 Kb/s

MPEG4 : 1300 Kb/s

75 fold!

Compression

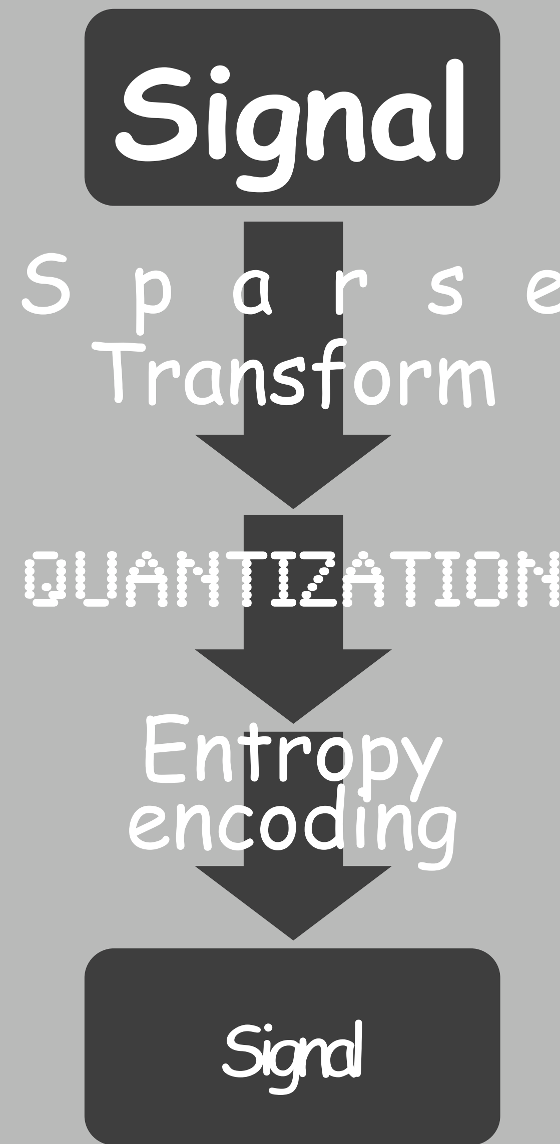
Almost all compression algorithm use transform coding

mp3: DCT

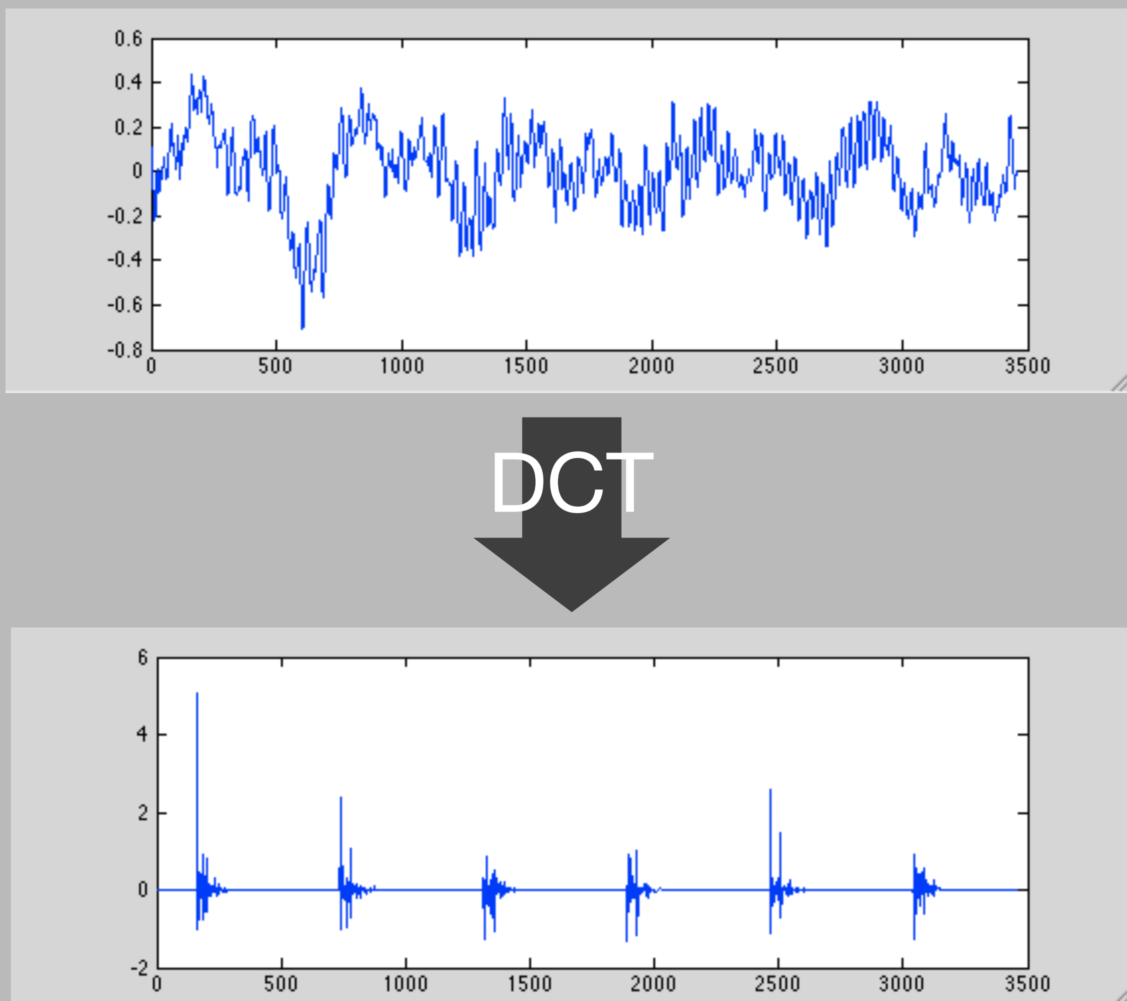
JPEG: DCT

JPEG2000: Wavelet

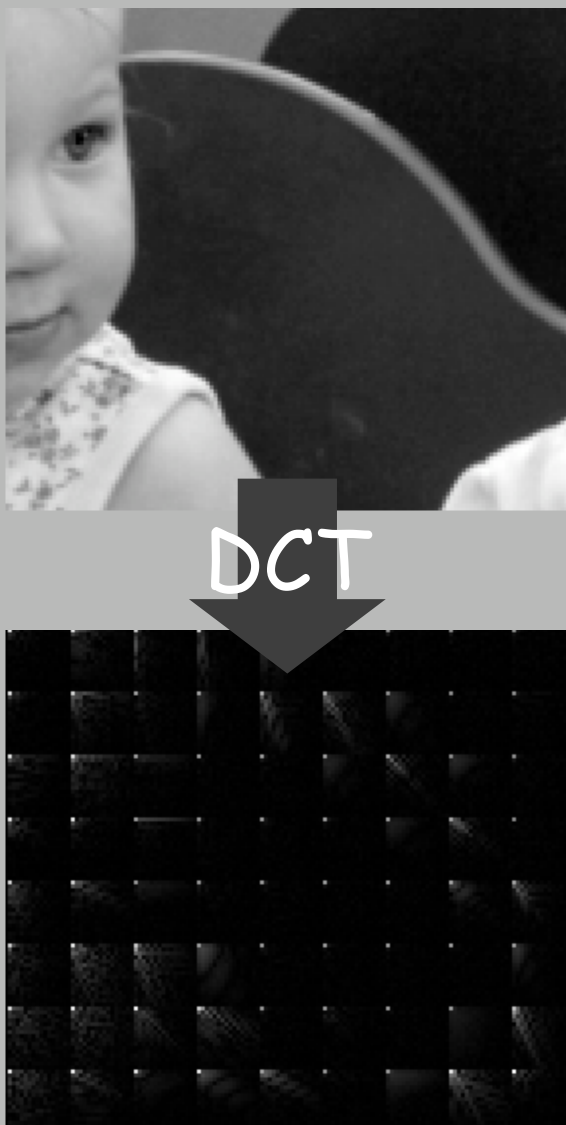
MPEG: DCT & time-difference



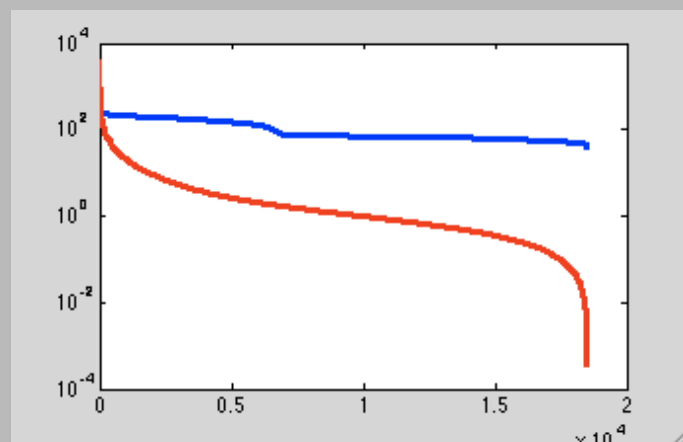
Sparse Transform



Sparse Transform

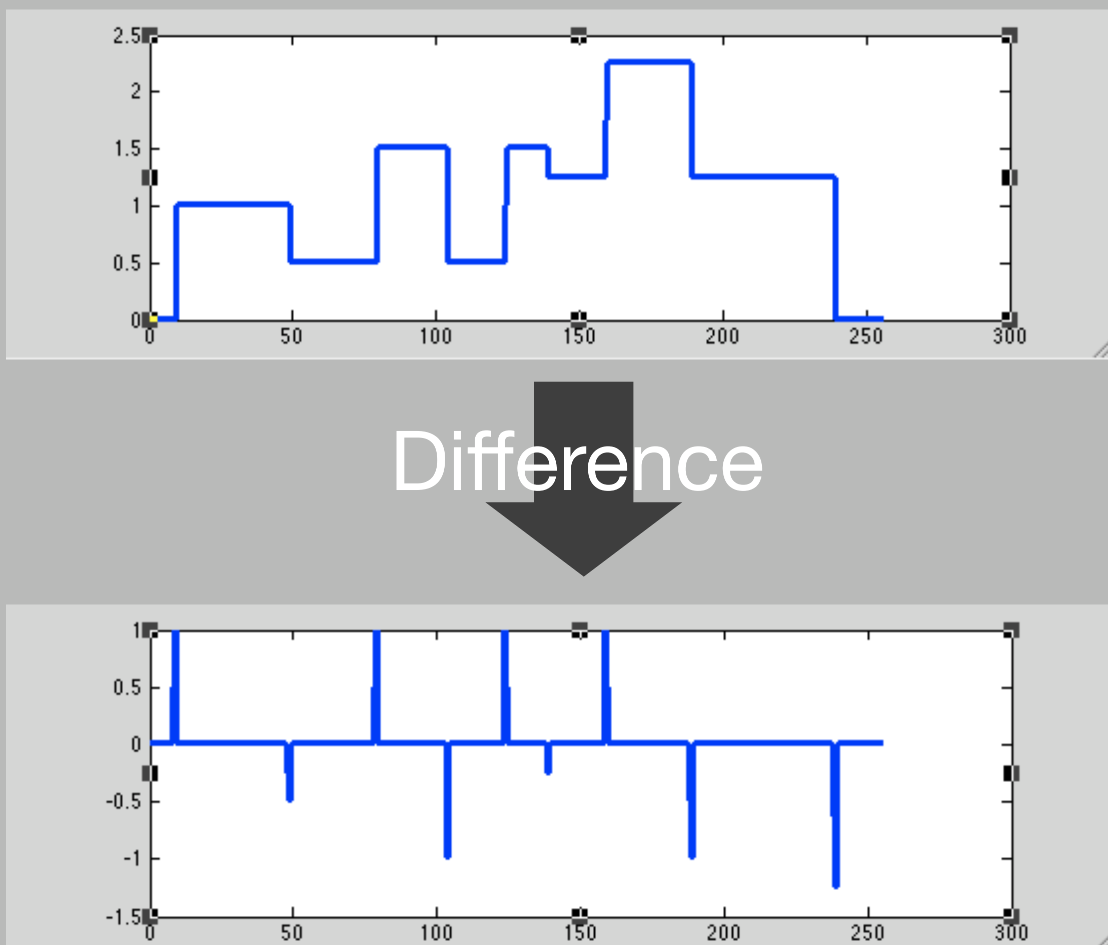


sorted coefficients



Sparse Transform

What sparsifying transform would you use here?



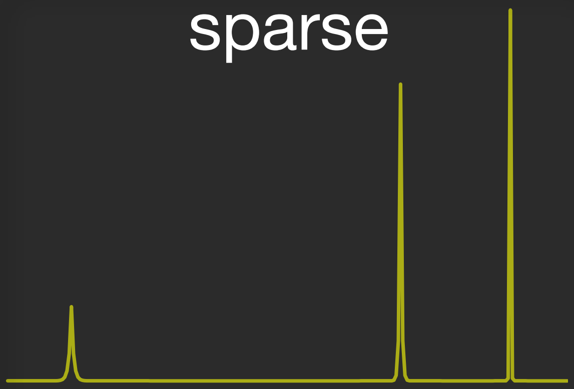
S p a r s i t y

&

Compressibility

Sparsity and Noise

sparse



not sparse

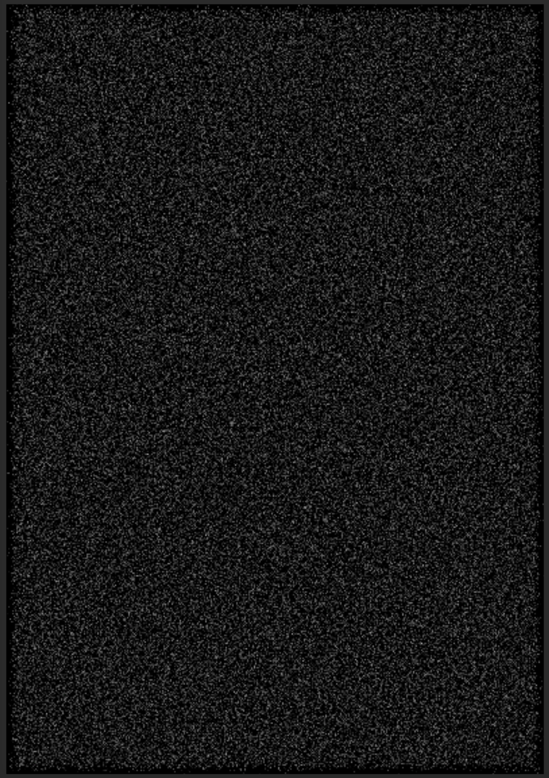


*image courtesy of J. Trzasko

Sparsity and Noise

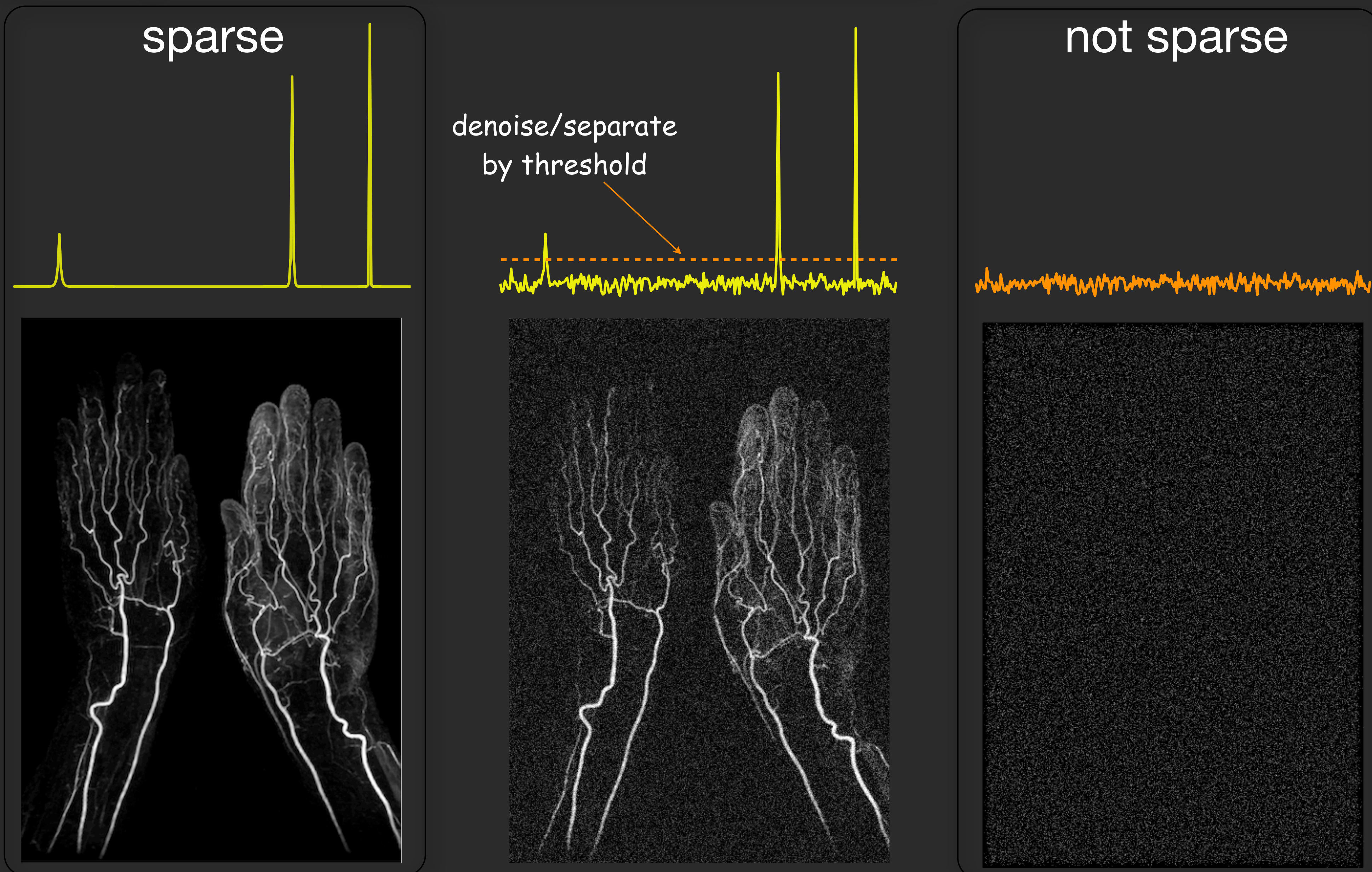
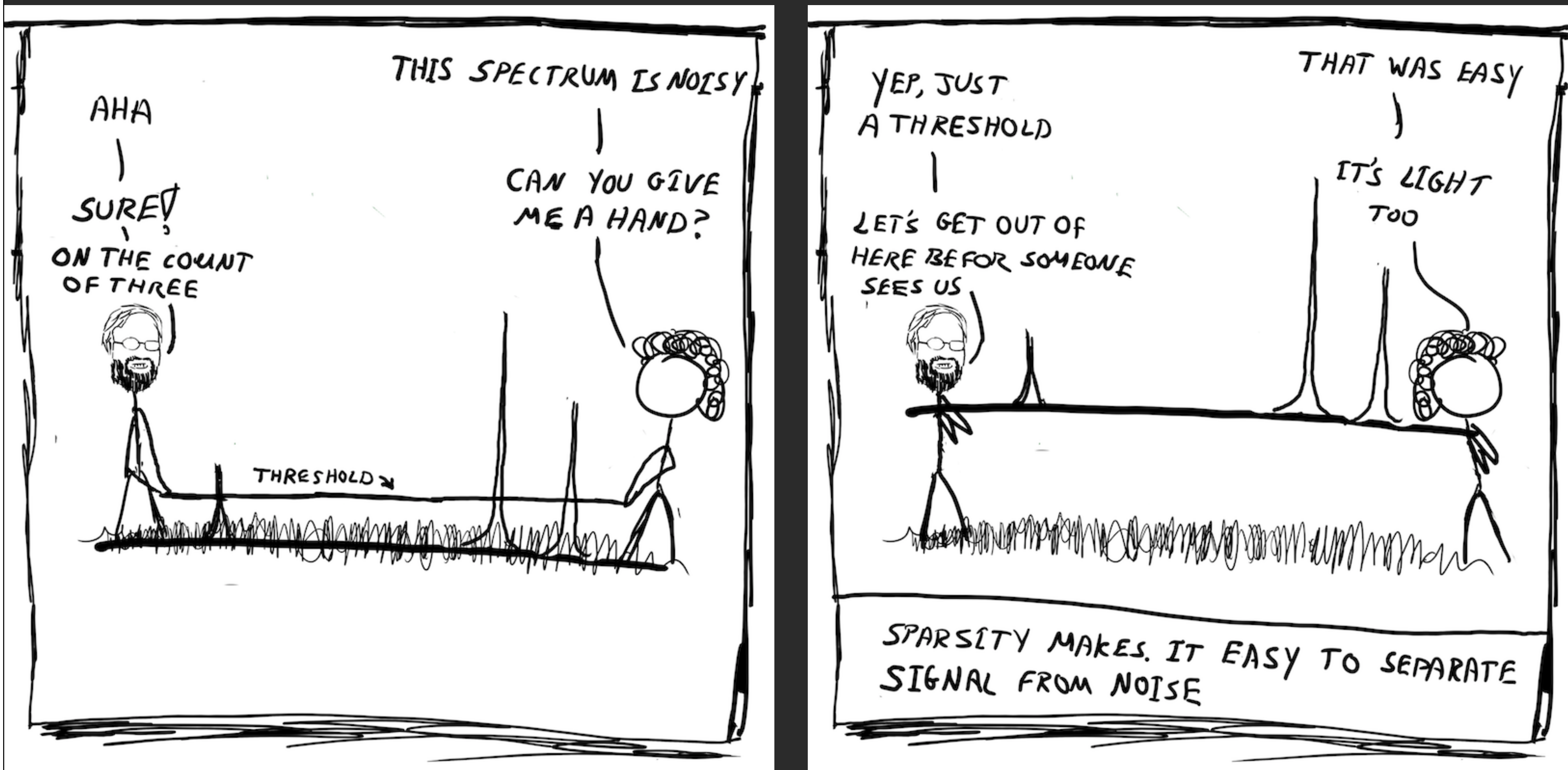


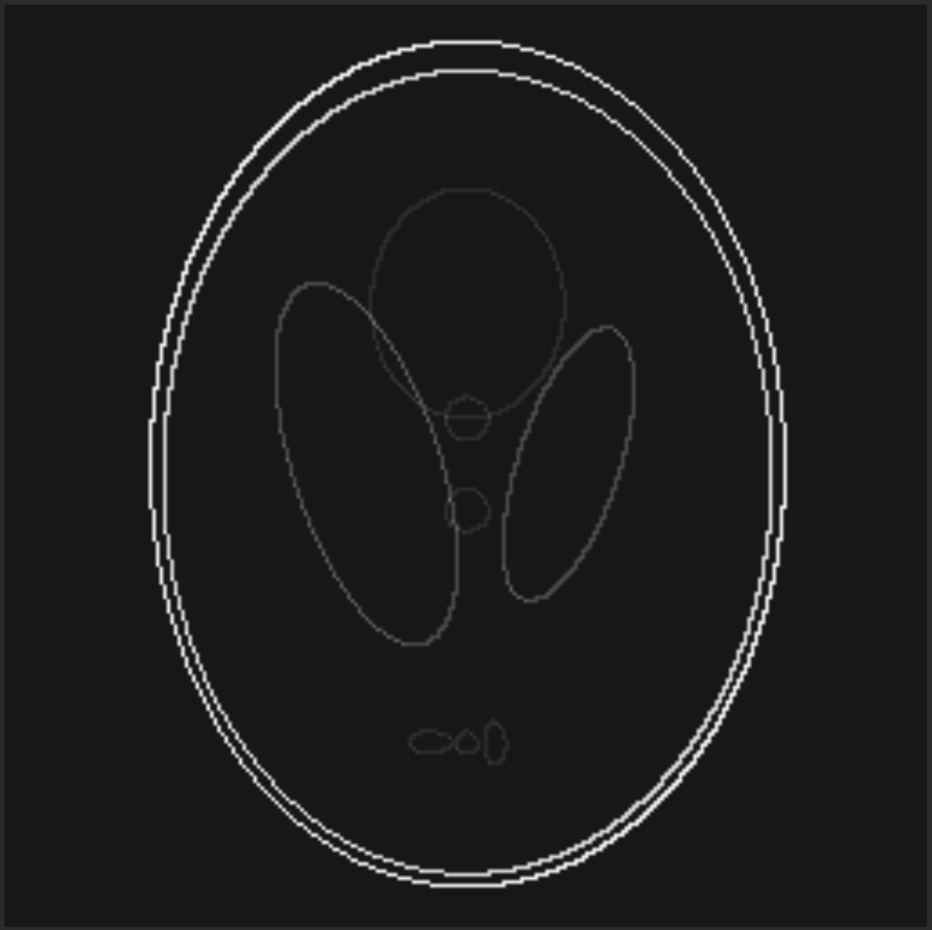
*image courtesy of J. Trzasko


THIS SPECTRUM IS NOISY
CAN YOU GIVE ME A HAND?
AHA
SURE!
ON THE COUNT OF THREE
THRESHOLD
YEP, JUST A THRESHOLD
LET'S GET OUT OF HERE BEFOR SOMEONE SEES US
THAT WAS EASY
IT'S LIGHT TOO
SPARSITY MAKES. IT EASY TO SEPARATE SIGNAL FROM NOISE

Transform Sparsity

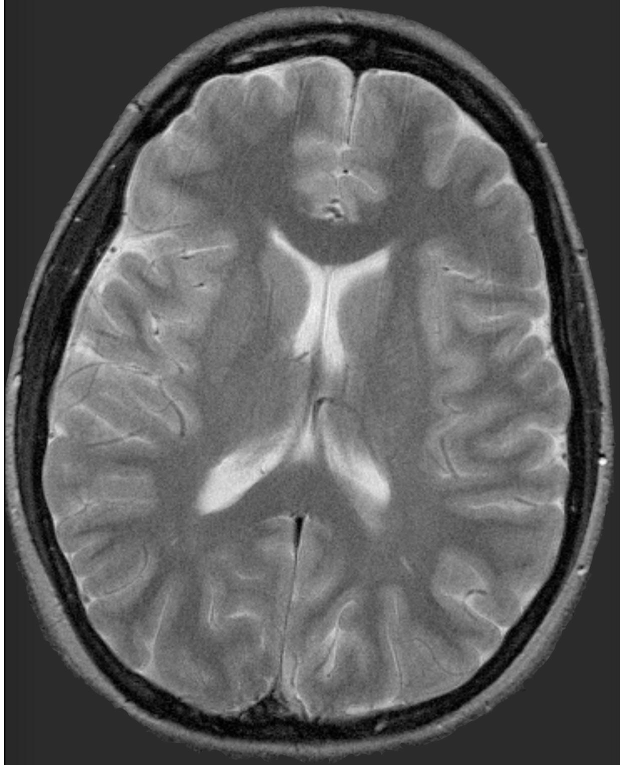
not sparse

Sparse Edges



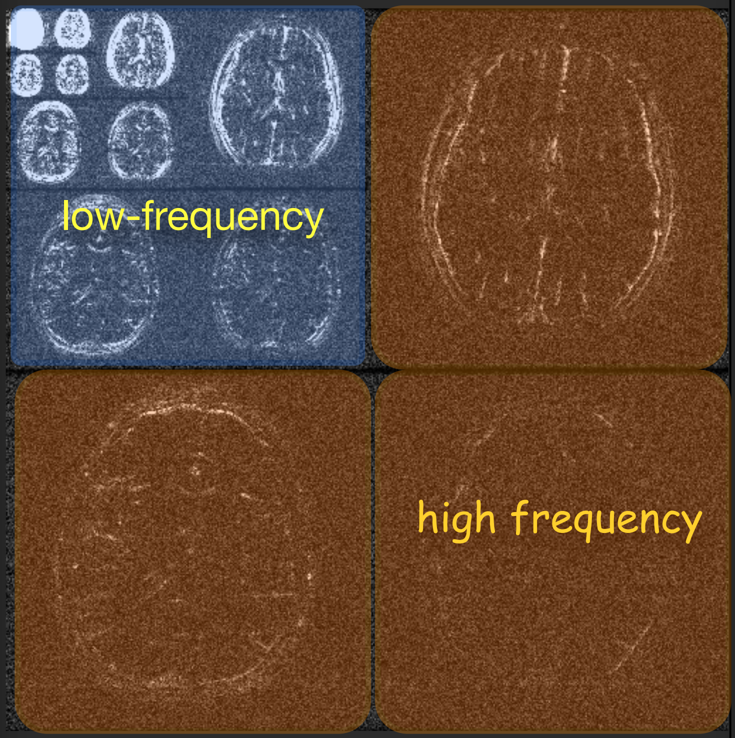
Transform Sparsity and Denoising

not sparse



sparse

wavelet transform

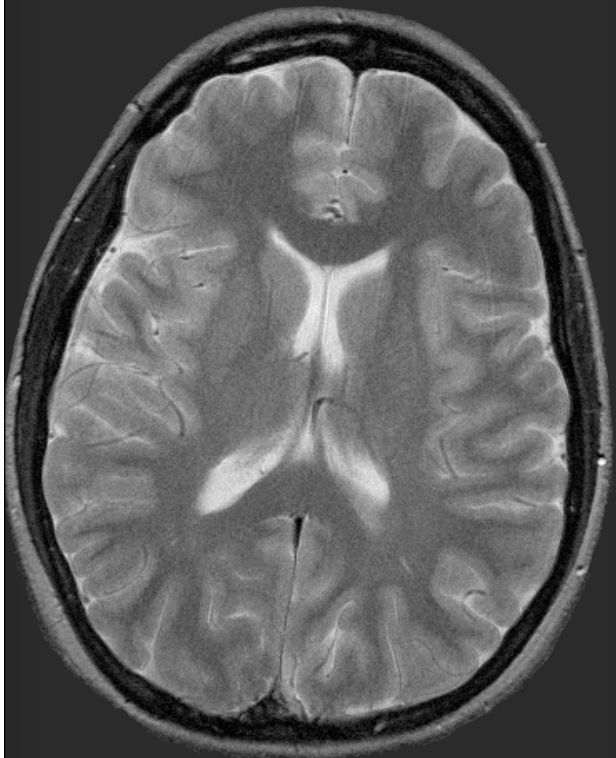


denoised

DL Donoho, I Johnstone Biometrika 1994;81(3):425-55

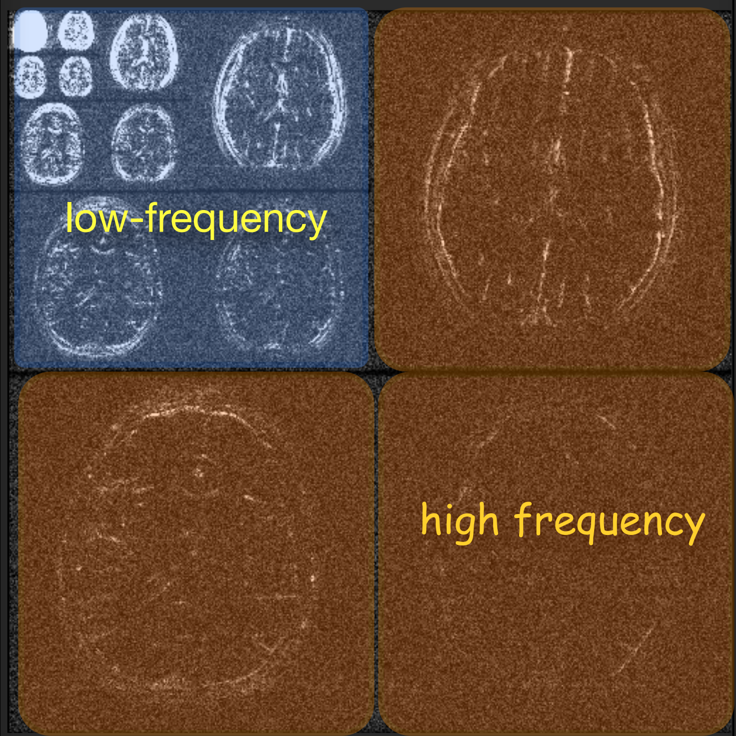
Transform Sparsity and Denoising

not sparse

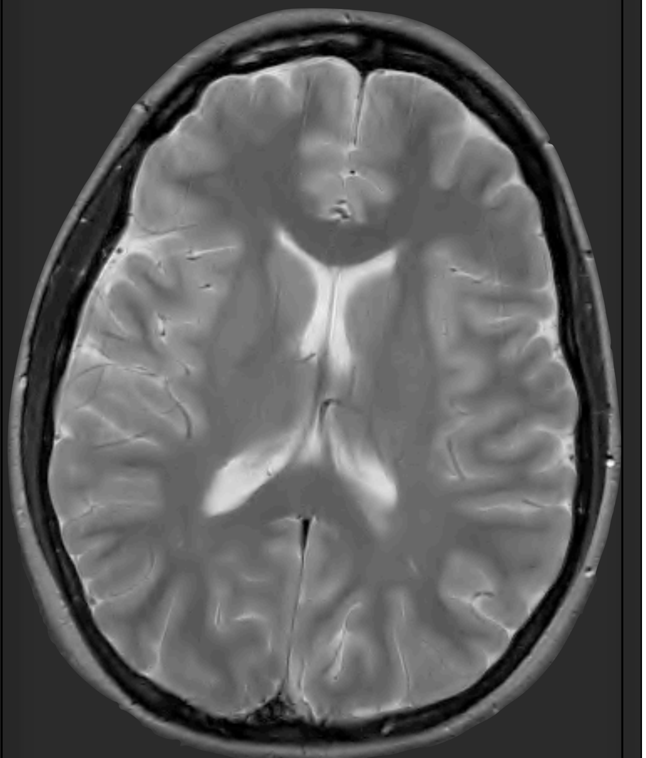


sparse

wavelet transform

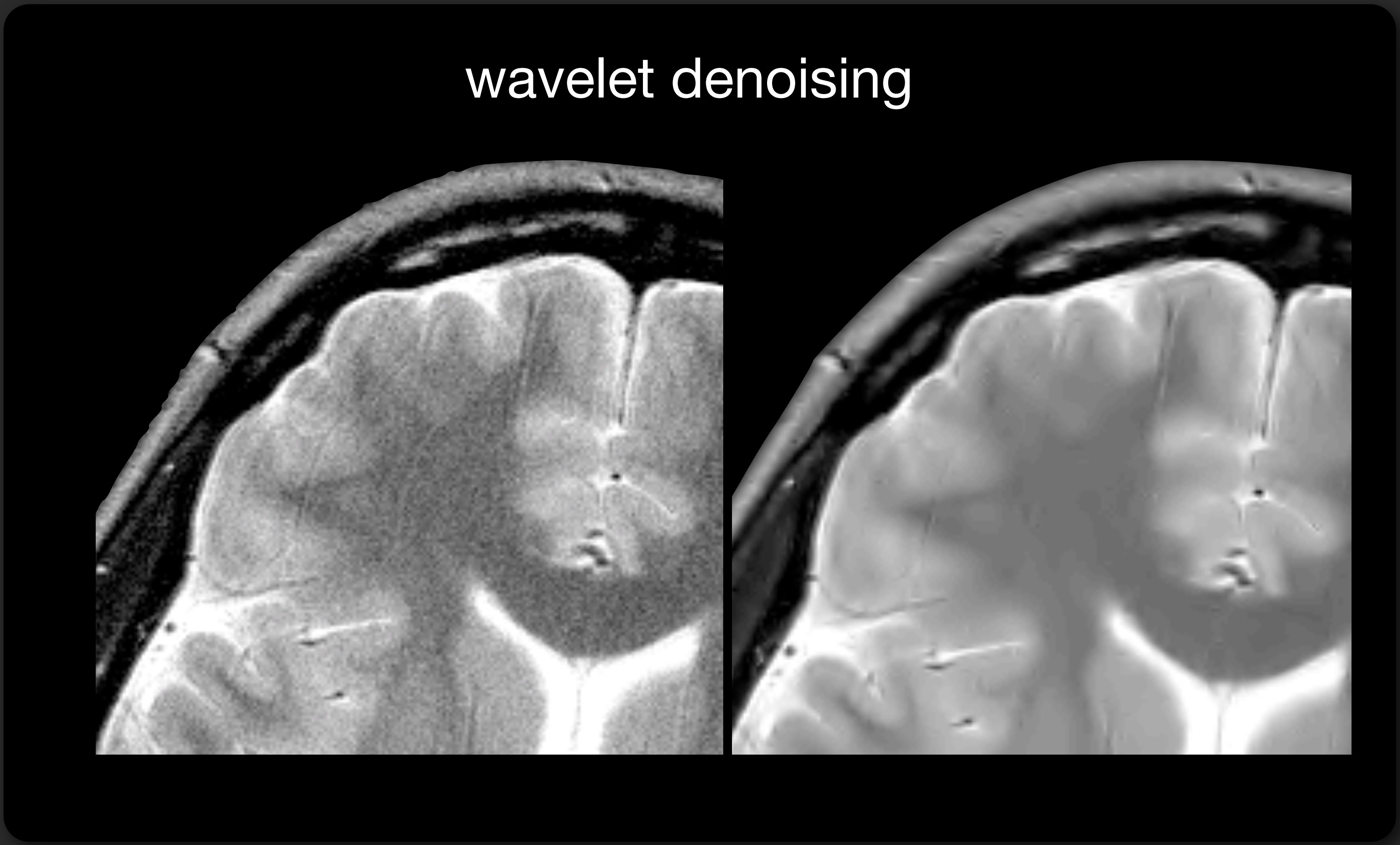


denoised



DL Donoho, I Johnstone Biometrika 1994;81(3):425-55

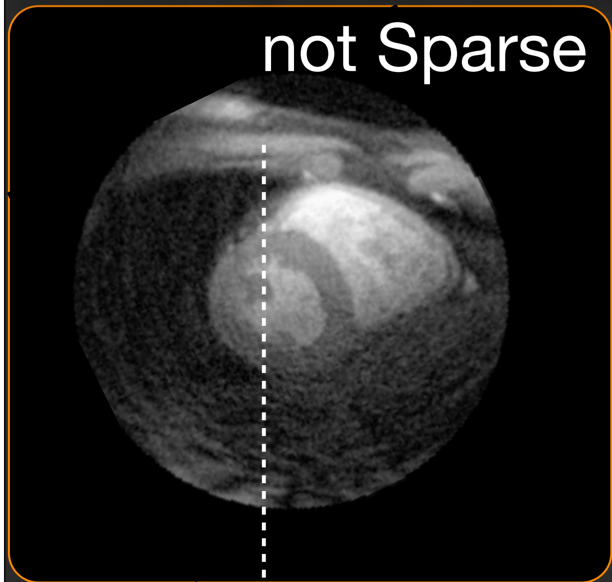
Transform Sparsity and Denoising

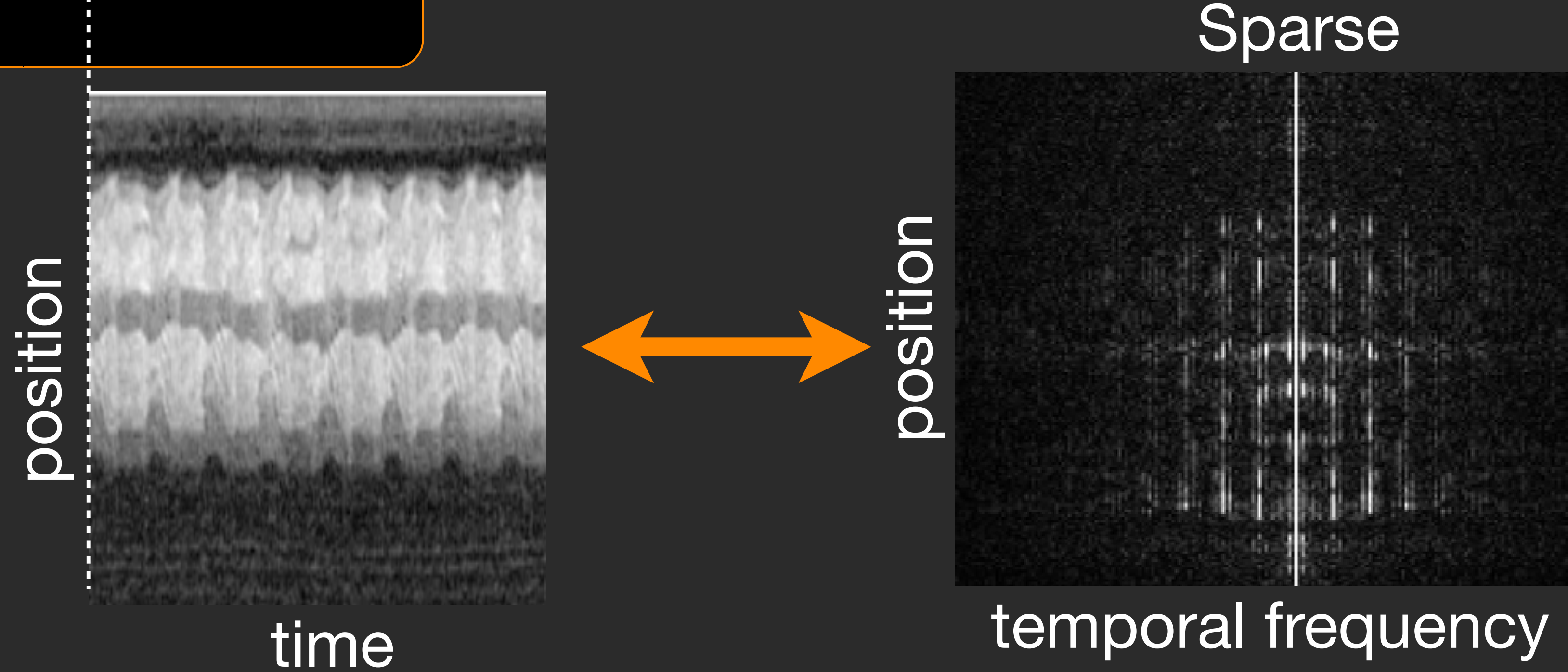


DL Donoho, I Johnstone Biometrika 1994;81(3):425-55

More Sparse Transforms

*Video courtesy of Juan Santos, Heart Vista





Sparsity and Compression

- Only need to store non-zeros

From Samples to Measurements

- Shanon-Nyquist sampling
  - Worst case for ANY bandlimited data
- Compressive sampling (CS)

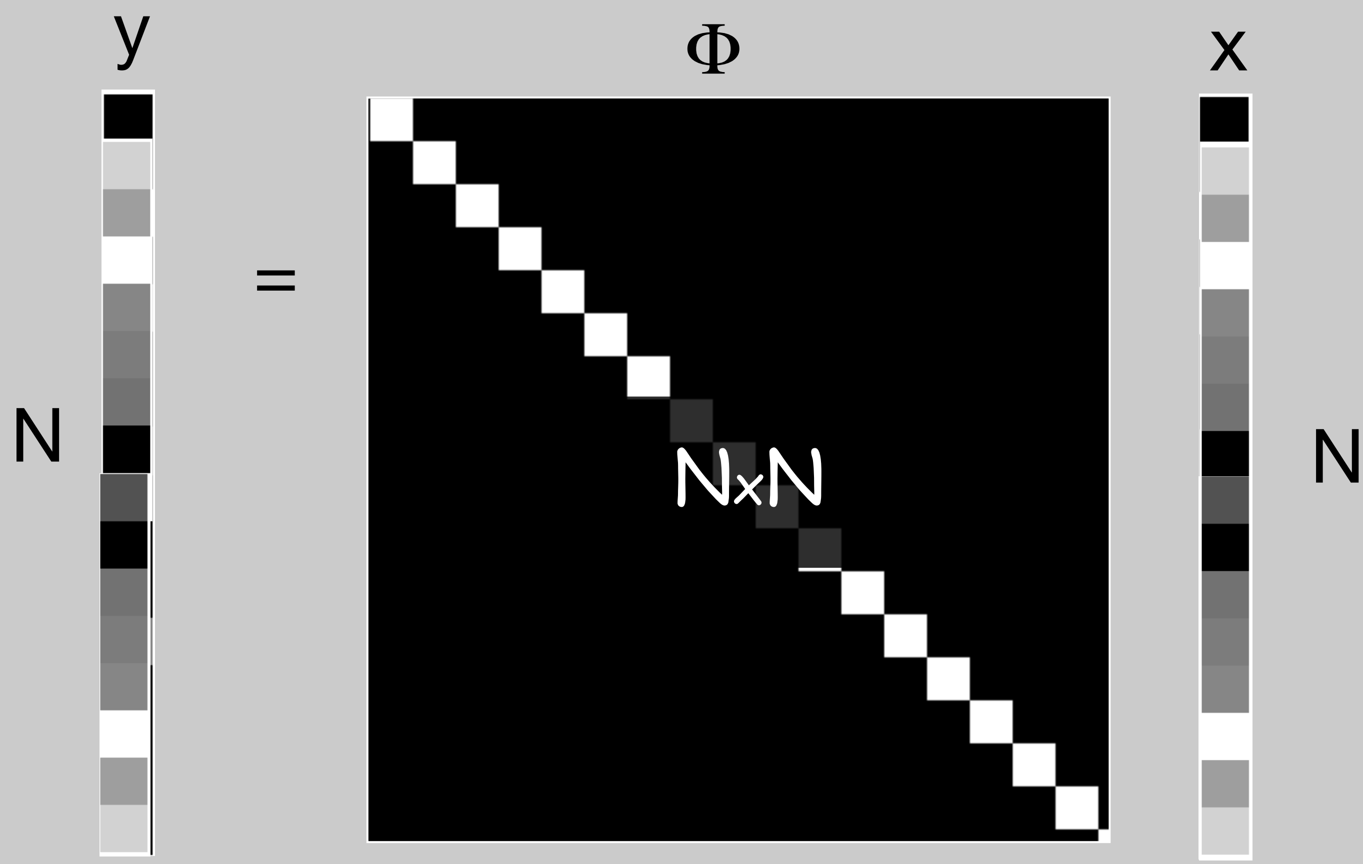
  "Sparse signals statistics can be recovered from a small number of non-adaptive linear measurements"

  - Integrated sensing, compression and processing.
  - Based on concepts of incoherency between signal and measurements

Traditional Sensing

Desktop scanner/ digital camera sensing

- $x \in \Re^N$ is a signal
- Make N linear measurements

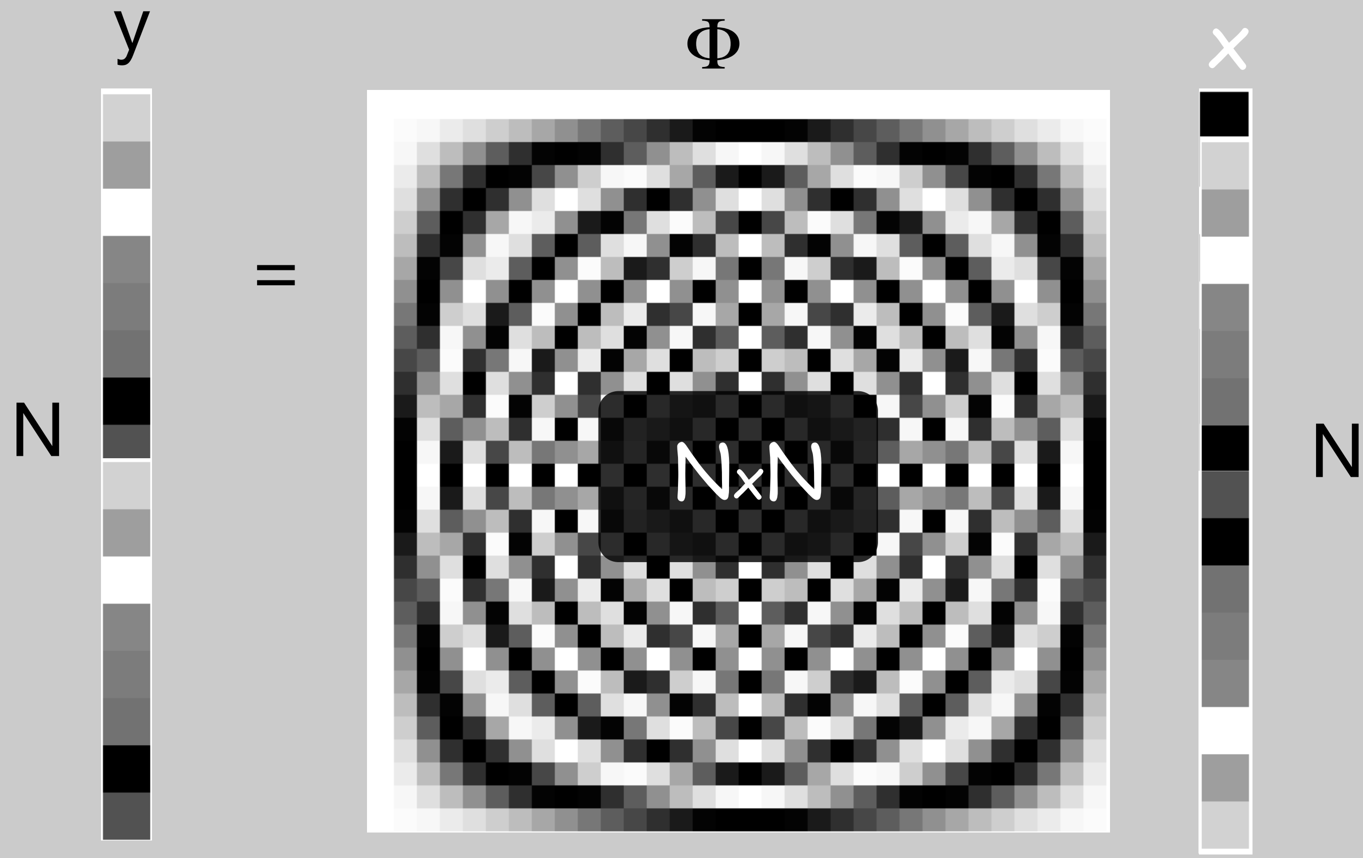


sensing matrix

Traditional Sensing

MRI Fourier Imaging

- $x \in \Re^N$ is a signal
- Make N linear measurements

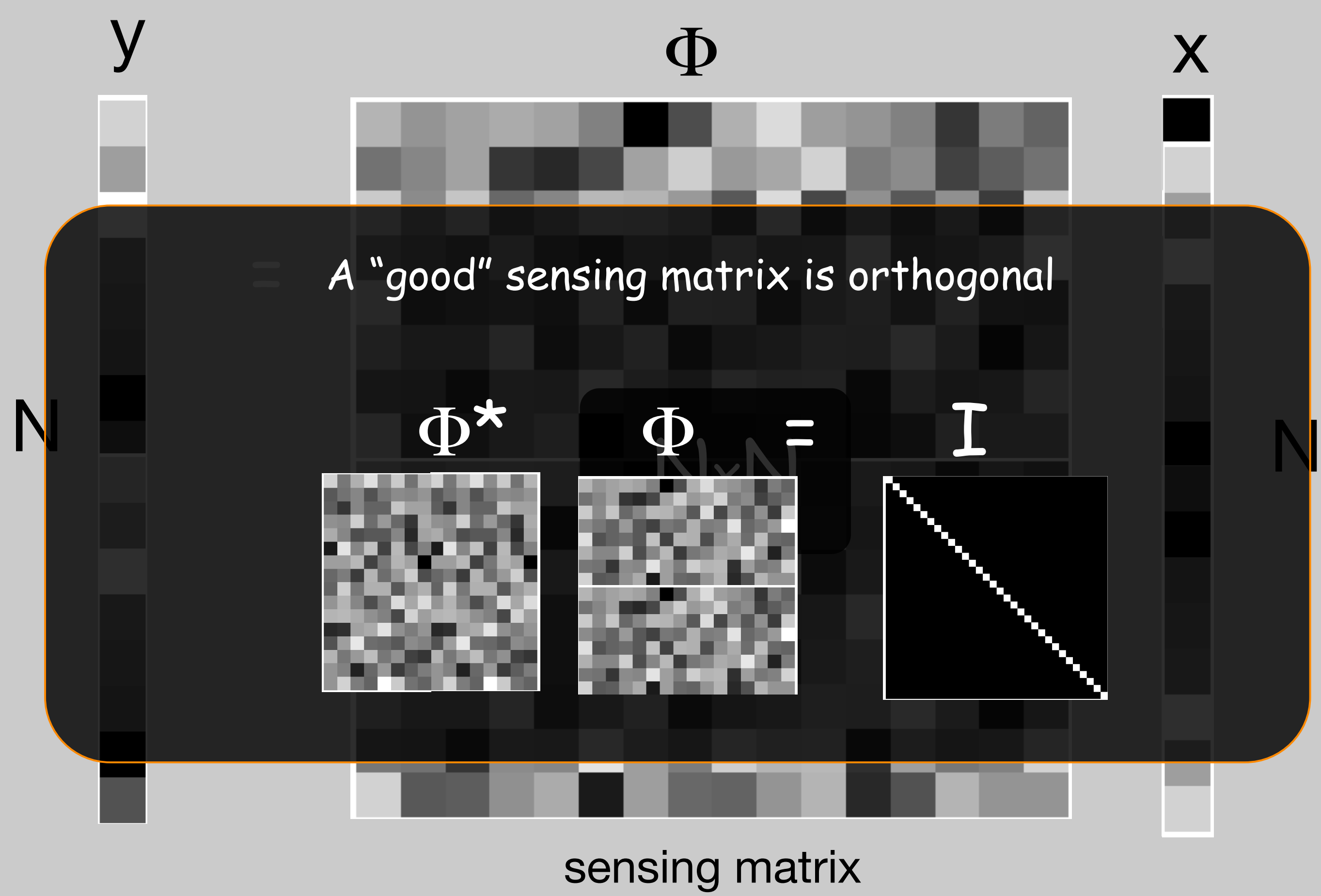


sensing matrix

Traditional Sensing

Arbitrary sensing

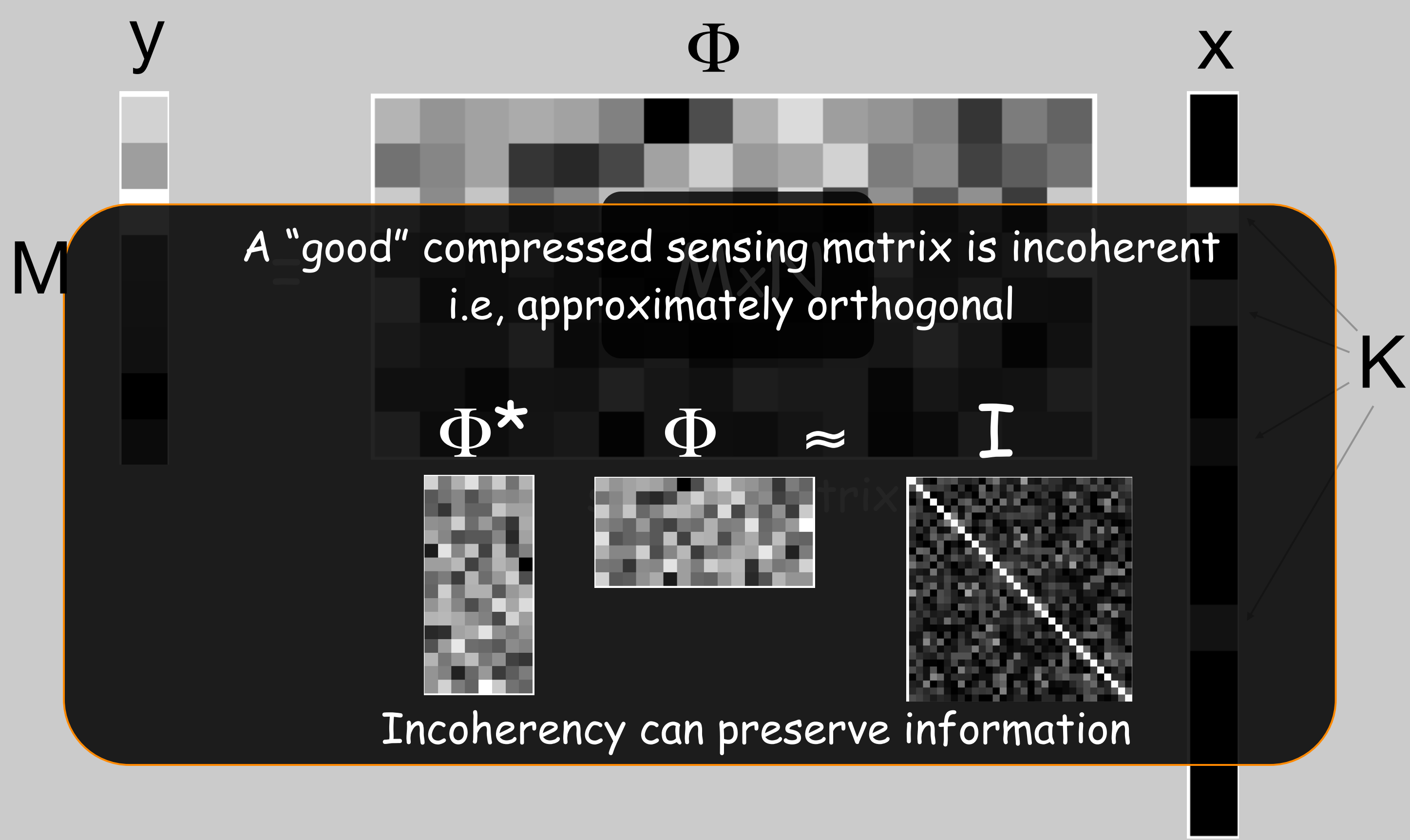
- $x \in \Re^N$ is a signal
- Make N linear measurements



Compressed Sensing

(Candes, Romber, Tao 2006; Donoho 2006)

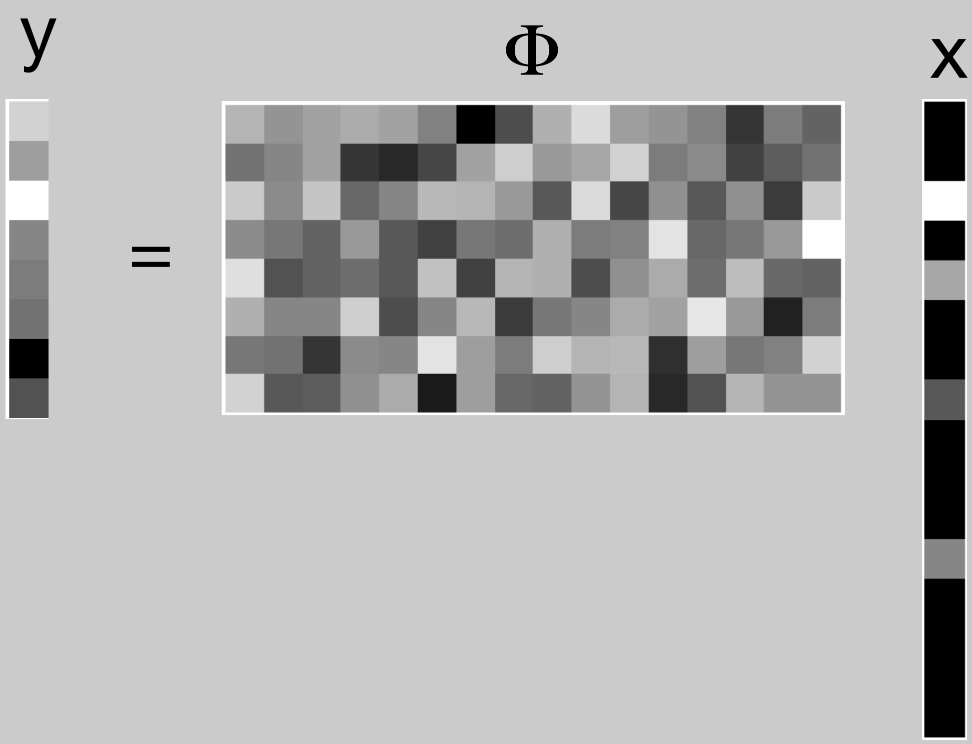
- $x \in \Re^N$ is a K-sparse signal (K<<N)
- Make M (K<M<<N) incoherent linear projections



CS recovery

- Given $y = \Phi x$
  find x

  Under-determined
- But there's hope, x is sparse!



CS recovery

- Given $y = \Phi x$
  find x

  } Under-determined

- But there's hope, x is sparse!

CS recovery

- Given $y = \Phi x$
  find x
  } Under-determined
- But there's hope, x is sparse!

minimize $||x||_2$

s.t. $y = \Phi x$

WRONG!

CS recovery

- Given $y = \Phi x$
  find x

  } Under-determined

- But there's hope, x is sparse!

$$\text{minimize } \|x\|_0$$
$$\text{s.t. } y = \Phi x$$

HARD!

CS recovery

- Given $y = \Phi x$
  find x

  } Under-determined

- But there's hope, x is sparse!

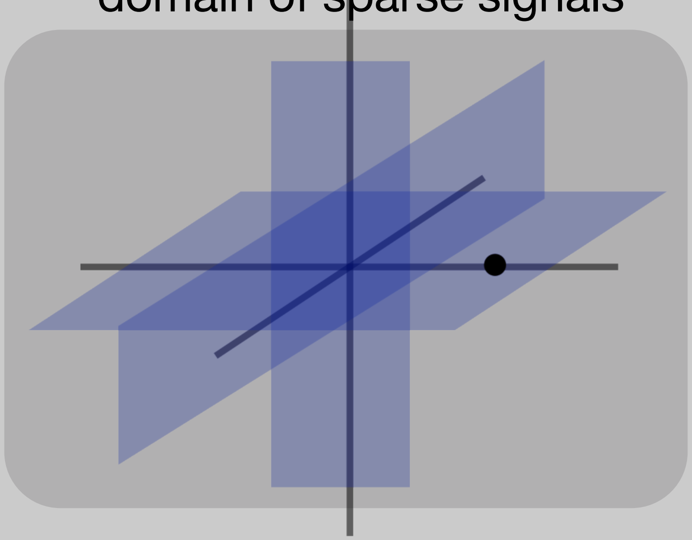
$$\text{minimize } ||x||_1$$
$$\text{s.t. } y = \Phi x$$

need $M \approx K \log(N) << N$

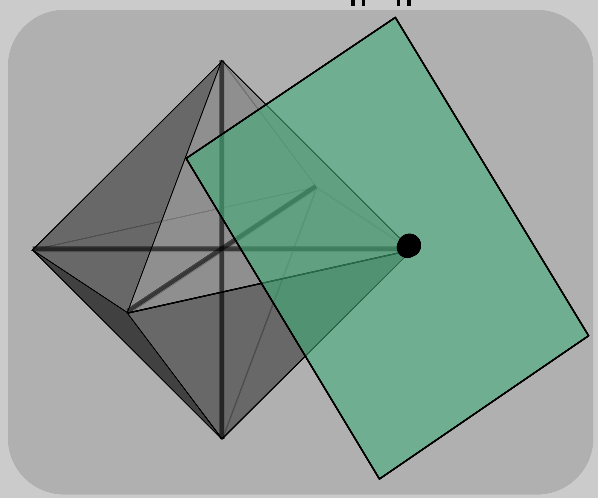
Solved by linear-programming

Geometric Interpretation

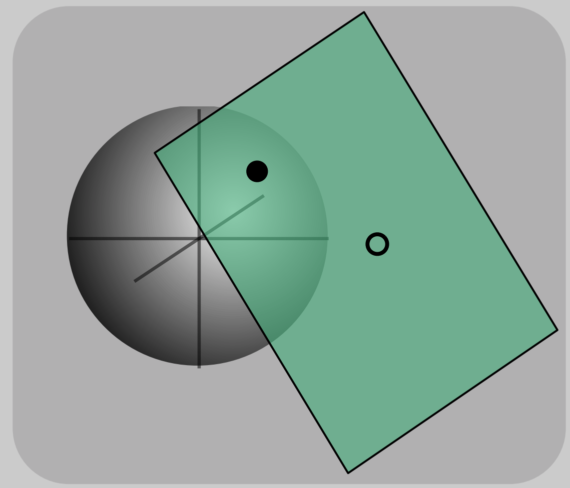
domain of sparse signals



minimum $||x||_1$



minimum $||x||_2$



$$\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [y_1]$$