EE123
Digital Signal Processing

Lecture 24
Compressed Sensing III

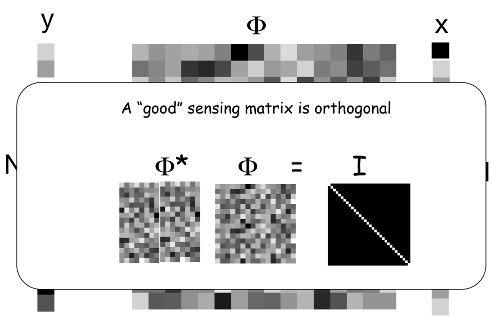
RADIOS

- https://inst.eecs.berkeley.edu/~ee123/sp15/radio.html
- Interfaces and radios on Wednesday -- please come to pick up
- Midterm II this Friday -- same deal - open everything covers everything including 2D

Traditional Sensing

Arbitrary sensing

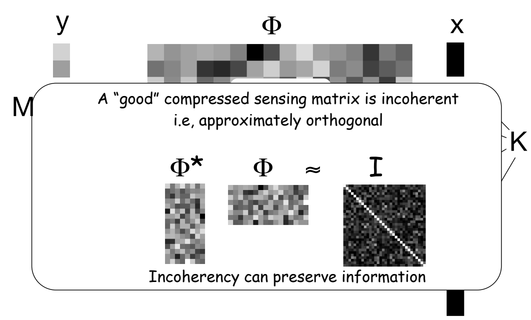
- $x \in \Re^N$ is a signal
- Make N linear measurements



Compressed Sensing

(Candes, Romber, Tao 2006; Donoho 2006)

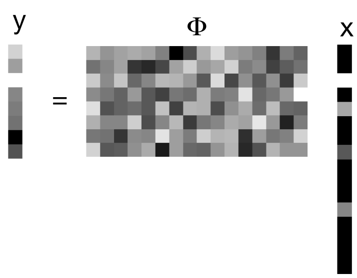
- $x \in \Re^N$ is a K-sparse signal (K<<N)
- Make M (K<M<<N) incoherent linear projections



CS recovery

- Given $y = \Phi x$
  find x

  Under-determined
- But there's hope, x is sparse!



CS recovery

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  Under-determined
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CS recovery

- Given $y = \Phi x$, find x } Under-determined
- But there's hope, x is sparse!

$$\text{minimize } \|x\|_2 \quad \text{s.t. } y = \Phi x$$

WRONG!

CS recovery

- Given $y = \Phi x$, find x } Under-determined
- But there's hope, x is sparse!

$$\text{minimize } \|x\|_0 \quad \text{s.t. } y = \Phi x$$

HARD!

CS recovery

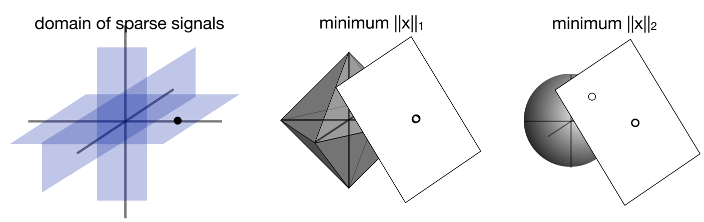
- Given $y = \Phi x$, find x } Under-determined
- But there's hope, x is sparse!

$$\text{minimize } \|x\|_1 \quad \text{s.t. } y = \Phi x$$

need $M \approx K \log(N) \ll N$

Solved by linear-programming

Geometric Interpretation



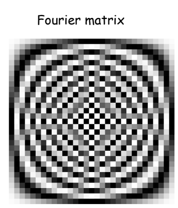
$$\begin{bmatrix}0\\3\\0\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} \qquad \begin{bmatrix}a_1 & a_2 & a_3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = [y_1]$$

A non-linear sampling theorem

- $f \in C^N$ supported on a set Ω in Fourier
- Shannon:
  - Ω is known connected set, size B
  - Exact recovery from B equispaced time samples
  - Linear reconstruction by sinc interpolation
- Non-linear sampling theorem
  - Ω is an arbitrary, unknown set of size B
  - Exact recovery from ~ B logN (almost) arbitrary placed samples
  - Nonlinear reconstruction by convex programming

Practicality of CS

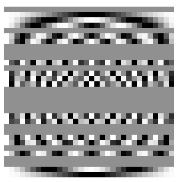
- Can such sensing system exist in practice?



Practicality of CS

- Can such sensing system exist in practice?

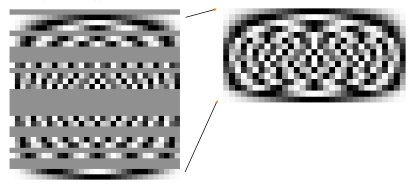
Fourier matrix



Practicality of CS

- Can such sensing system exist in practice?

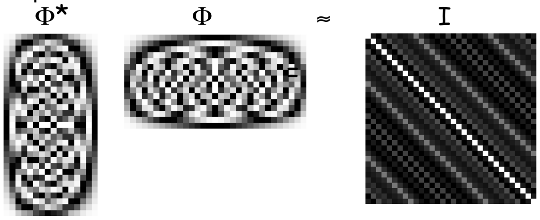
Fourier matrix

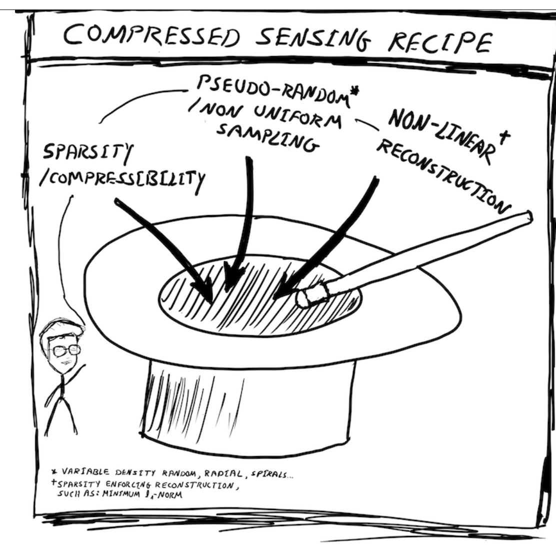


Practicality of CS

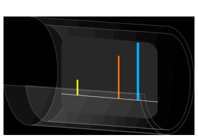
- Can such sensing system exist in practice?
- Randomly undersampled Fourier is incoherent
- MRI samples in the Fourier domain!

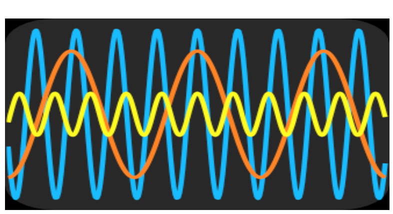
$$\Phi^* \quad \Phi \quad \approx \quad I$$





Intuitive example of CS





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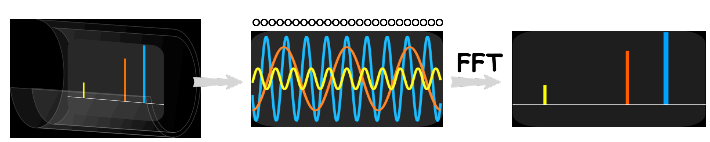

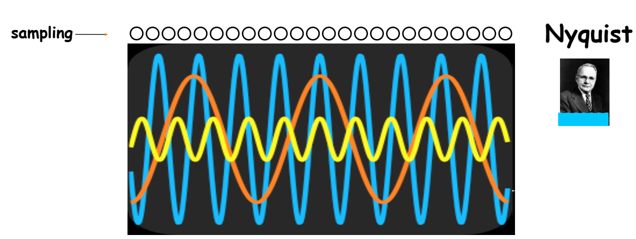

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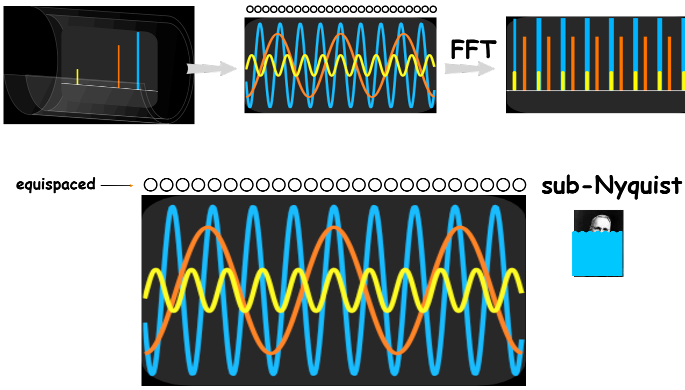

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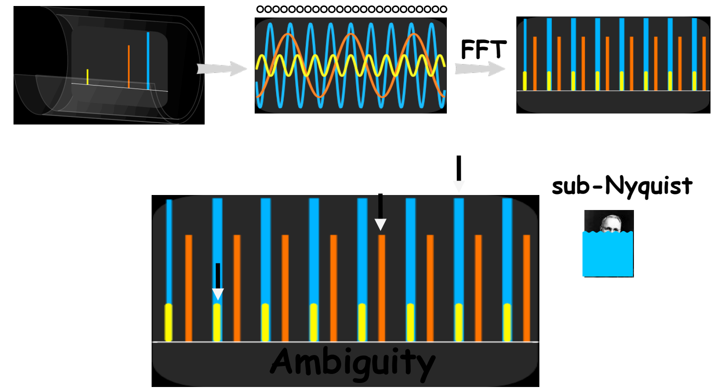

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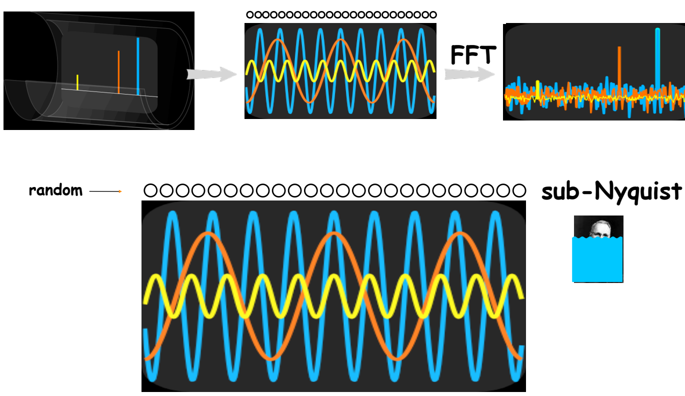

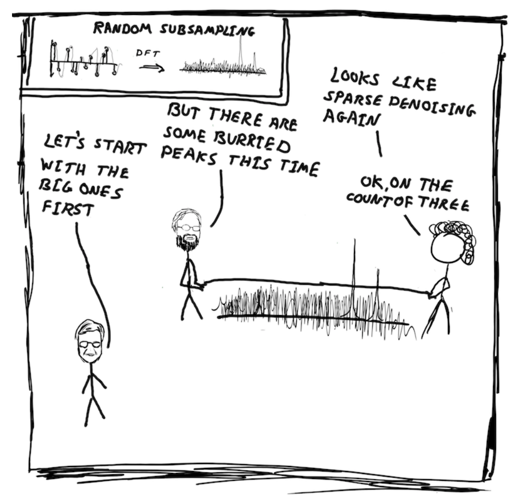

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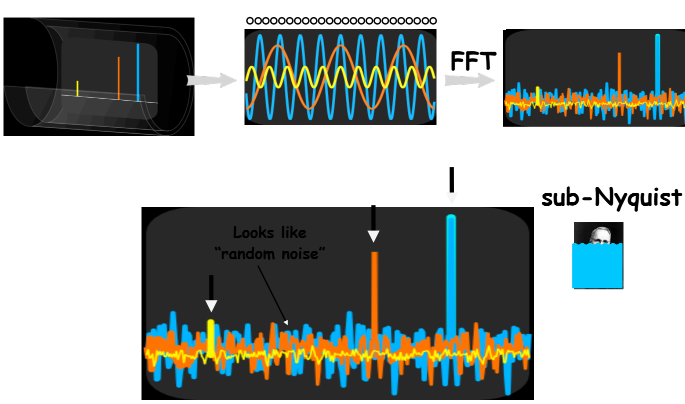

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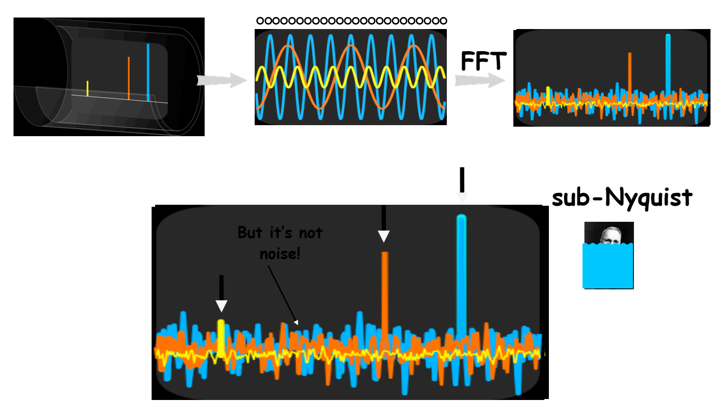

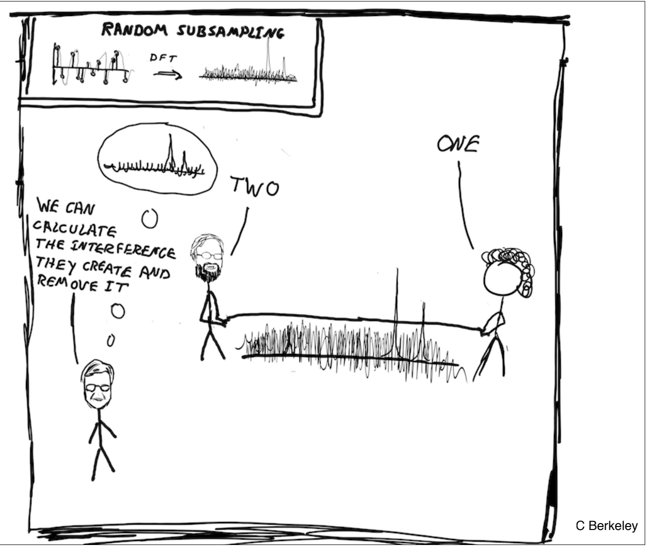

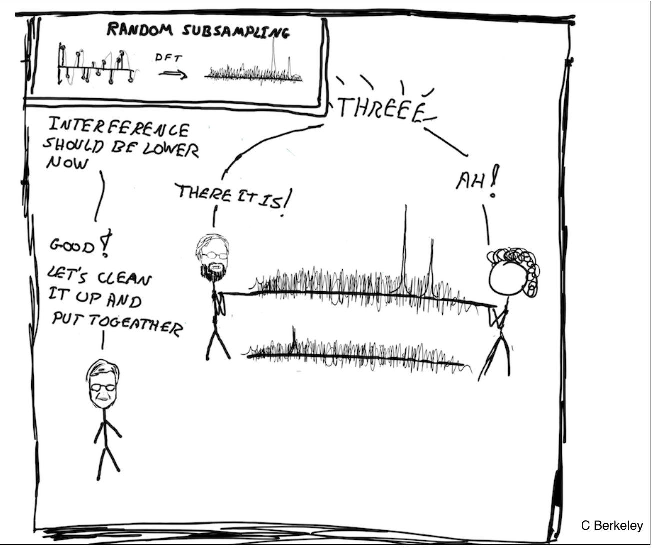

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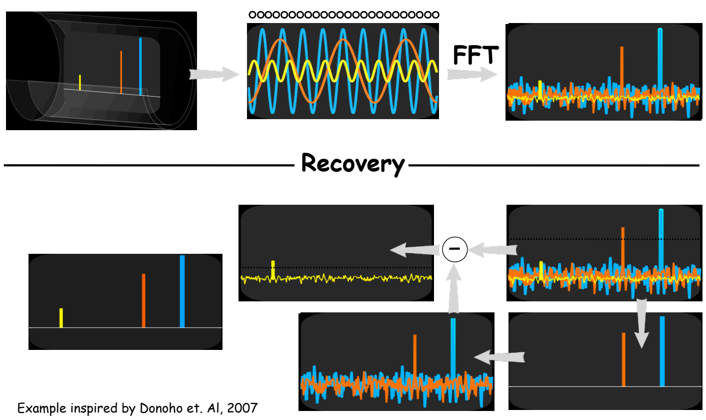
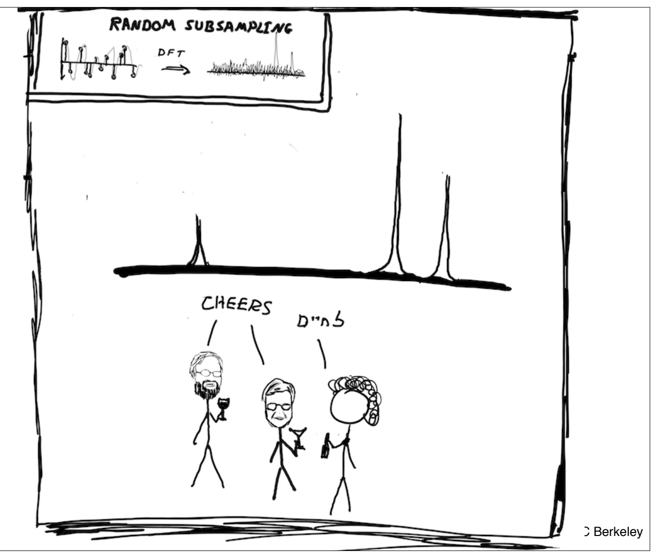
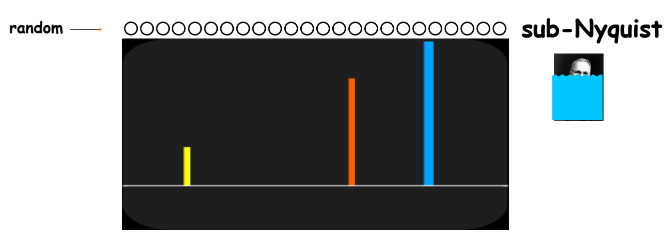


Example inspired by Donoho et. Al, 2007

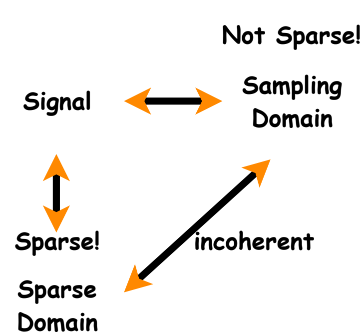


Question!

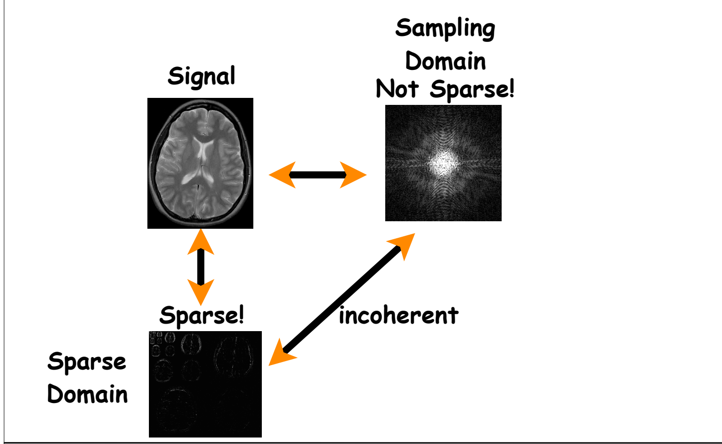
- What if this was the signal?
- Would CS still work?

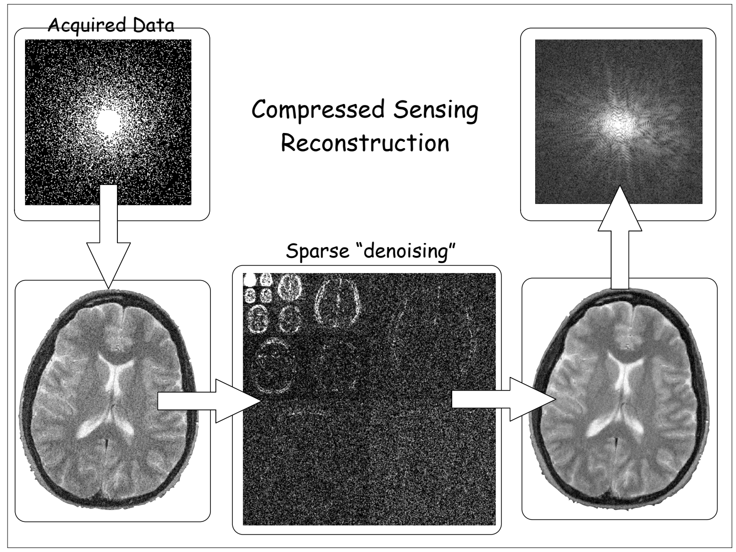


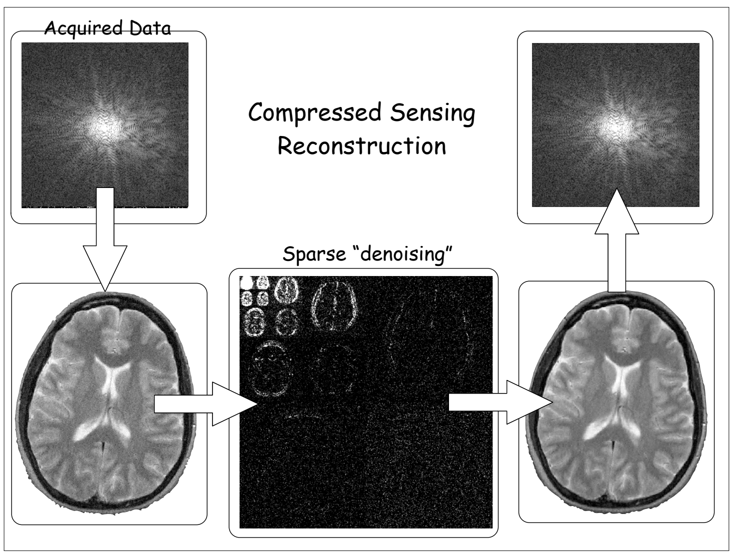
Domains in Compressed Sensing

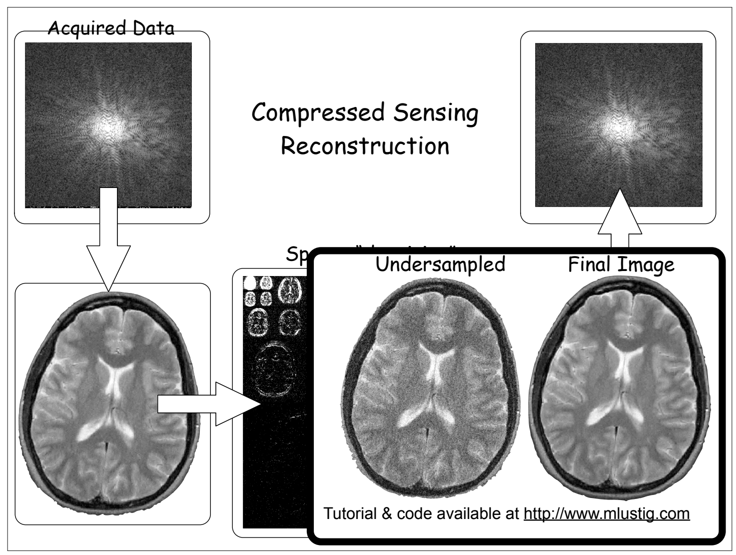


MRI

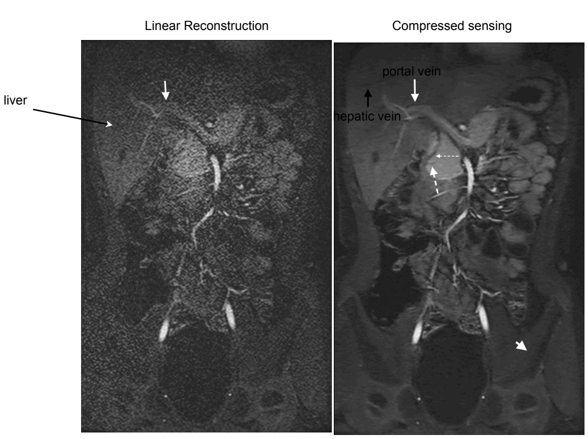








6 year old male abdomen. Fine structures (arrows) are buried in noise (artifactual + noise amplification) and are recovered by CS with L1-wavelets. x8 acceleration



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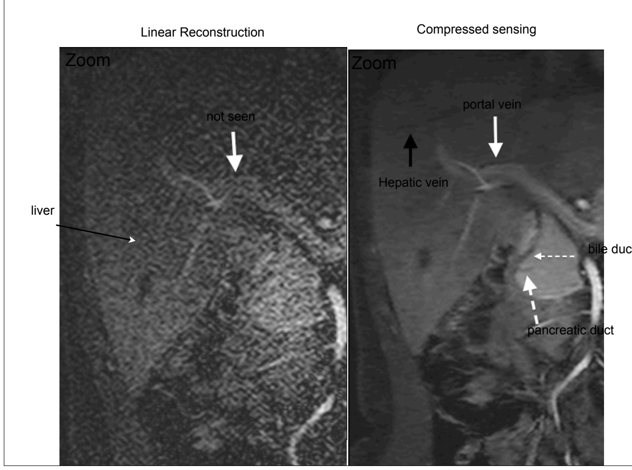
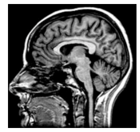
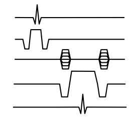


Back to Results

6 year old
8-fold acceleration
16 second scan
0.875 mm in-plane
1.6 slice thickness

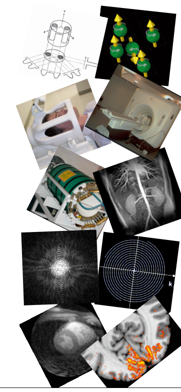


Principles of
Magnetic Resonance Imaging
EE c225E / BIOE c265



Spring 2016

Shameless Promotion



Other Applications

- Compressive Imaging
- Medical Imaging
- Analog to information conversion
- Biosensing
- Geophysical Data Analysis
- Compressive Radar
- Astronomy
- Communications
- More ......

Resources

- CS + parallel imaging matlab code, examples
  http://www.eecs.berkeley.edu/~mlustig/software/
- Rice University CS page: papers, tutorials, codes, ….
  http://www.dsp.ece.rice.edu/cs/
- IEEE Signal Processing Magazine, special issue on compressive sampling 2008;25(2)
- March 2010 Issue Wired Magazine: “Filling the Blanks”
- Igor Caron Blog: http://nuit-blanche.blogspot.com/

Thank you!
תודה רבה