# EE123
# Digital Signal Processing

Lecture 25
Filter Design



## Linear Filter Design

- Used to be an art
  - Now, lots of tools to design optimal filters
- For DSP there are two common classes
  - Infinite impulse response IIR
  - Finite impulse response FIR
- Both classes use finite order of parameters for design
- We will cover FIR designs, briefly mention IIR



## What is a linear filter

- Attenuates certain frequencies
- Passes certain frequencies
- Effects both **phase** and **magnitude**
- IIR
  - Mostly non-linear phase response
  - Could be linear over a range of frequencies
- FIR
  - Much easier to control the phase
  - Both non-linear and linear phase



## FIR Design by Windowing

- Given desired frequency response, $H_d(e^{j\omega})$ , find an impulse response

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

ideal

- Obtain the M<sup>th</sup> order causal FIR filter by truncating/windowing it

$$h[n] = \left\{ \begin{array}{ll} h_d[n]w[n] & 0 \le n \le M \\ 0 & \text{otherwise} \end{array} \right\}$$



## FIR Design by Windowing

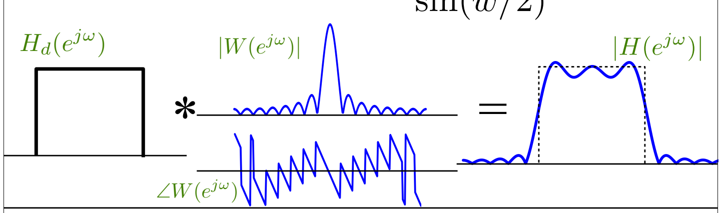
- We already saw that,

periodic

$$H(e^{j\omega}) = H_d(e^{j\omega}) * W(e^{j\omega})$$

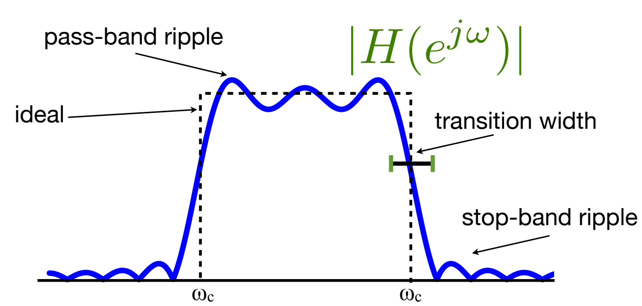
- For Boxcar (rectangular) window

$$W(e^{j\omega}) = e^{-j\omega\frac{M}{2}}\frac{\sin(w(M+1)/2)}{\sin(w/2)}$$

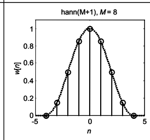
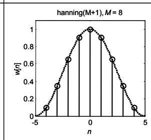
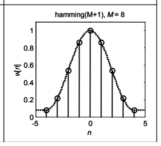





## FIR Design by Windowing

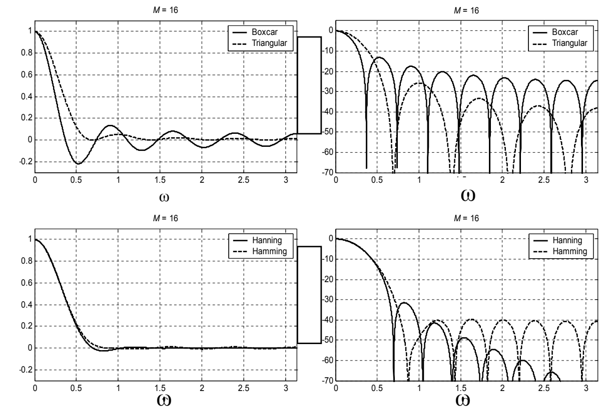


## Tapered Windows

| Name(s) | Definition | MATLAB Command | Graph ($M = 8$) |
|---|---|---|---|
| Hann | $w[n] = \begin{cases} \frac{1}{2}\left[1+\cos\left(\frac{\pi n}{M/2}\right)\right] & \|n\| \le M/2 \\ 0 & \|n\| > M/2 \end{cases}$ | `hann(M+1)` |  |
| Hanning | $w[n] = \begin{cases} \frac{1}{2}\left[1+\cos\left(\frac{\pi n}{M/2+1}\right)\right] & \|n\| \le M/2 \\ 0 & \|n\| > M/2 \end{cases}$ | `hanning(M+1)` |  |
| Hamming | $w[n] = \begin{cases} 0.54+0.46\cos\left(\frac{\pi n}{M/2}\right) & \|n\| \le M/2 \\ 0 & \|n\| > M/2 \end{cases}$ | `hamming(M+1)` |  |



## Tradeoff - Ripple vs Transition Width




Python: scipy.filter.firwin



## FIR Filter Design

- Choose a desired frequency response $H_d(e^{j\omega})$
  - non causal (zero-delay), and infinite imp. response
  - If derived from C.T, choose T and use:

$$H_d(e^{j\omega}) = H_c(j\frac{\Omega}{T})$$

- Window:
  - Length M+1 ⇔ effect transition width
  - Type of window ⇔ transition-width/ ripple
  - Modulate to shift impulse response

$$H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}$$



## FIR Filter Design

- Determine truncated impulse response $h_1[n]$

$$h_1[n] = \begin{cases} \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}e^{j\omega n} & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

- Apply window

$$h_w[n] = w[n]h_1[n]$$

- Check:
  - Compute $H_w(e^{j\omega})$, if does not meet specs increase M or change window



## Example: FIR Low-Pass Filter Design

$$H_d(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Choose M ⇒ Window length and set

$$H_1(e^{j\omega}) = H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}$$

h1[n]

$$H_1[n] = \begin{cases} \frac{\sin(\omega_c(n-M/2))}{\pi(n-M/2)} & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\omega_c}{\pi}\text{sinc}(\frac{\omega_c}{\pi}(n-M/2))$$



## Example: FIR Low-Pass Filter Design

- The result is a windowed sinc function

$$h_w[n] = w[n]h_1[n]$$

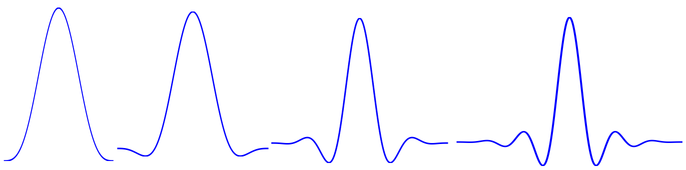
$$\frac{\omega_c}{\pi}\text{sinc}(\frac{\omega_c}{\pi}(n-M/2))$$

- High Pass Design:
  - Design low pass $h_w[n]$
  - Transform to $h_w[n](-1)^n$
- General bandpass
  - Transform to $2h_w[n]\cos(\omega_0 n)$

## Characterization of Filter Shape

Time-Bandwidth Product, a unitless measure

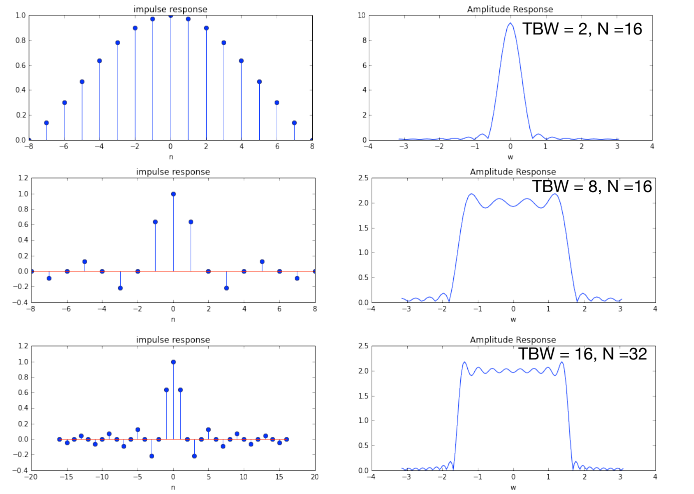
$T(BW) = (M+1)\omega/2\pi$ ⇒ also, total # of zero crossings




Larger TBW ⇒ More of the "sinc" function

hence, frequency response looks more like a rect function

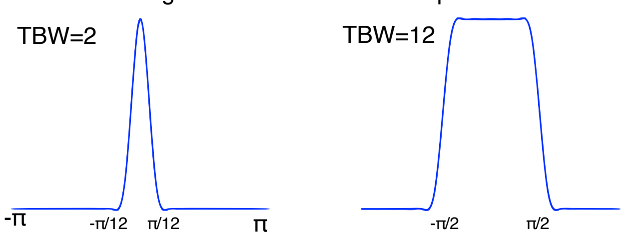







## Frequency Response Profile

Q: What are the lengths of these filters in samples?




$2 = (M+1)*(\pi/6) / (2\pi) \Rightarrow M=23$    $12= (M+1)*(\pi) / (2\pi) \Rightarrow M=23$

Note that transition is the same!



## Optimal Filter Design

- Window method
  - Design Filters heuristically using windowed sinc functions
- Optimal design
  - Design a filter h[n] with $H(e^{j\omega})$
  - Approximate $H_d(e^{j\omega})$ with some optimality criteria - or satisfies specs.