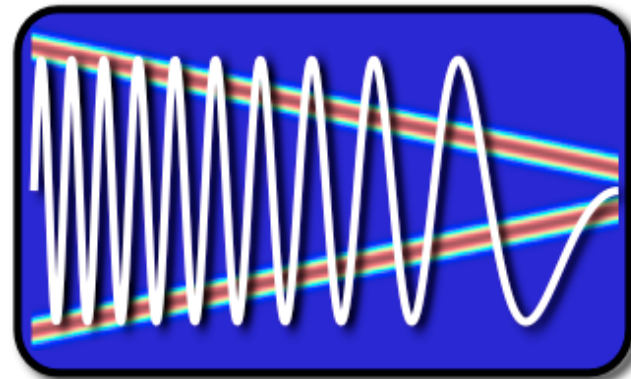


EE123



Digital Signal Processing

Lecture 25
Filter Design

Linear Filter Design


- Used to be an art
 - Now, lots of tools to design optimal filters
- For DSP there are two common classes
 - Infinite impulse response IIR
 - Finite impulse response FIR
- Both classes use finite order of parameters for design
- We will cover FIR designs, briefly mention IIR

What is a linear filter

- Attenuates certain frequencies
- Passes certain frequencies
- Effects both **phase** and **magnitude**
- IIR
 - Mostly non-linear phase response
 - Could be linear over a range of frequencies
- FIR
 - Much easier to control the phase
 - Both non-linear and linear phase

FIR Design by Windowing

- Given desired frequency response, $H_d(e^{j\omega})$, find an impulse response

$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$


- Obtain the M^{th} order causal FIR filter by truncating/windowing it

$$h[n] = \left\{ \begin{array}{ll} h_d[n]w[n] & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{array} \right\}$$

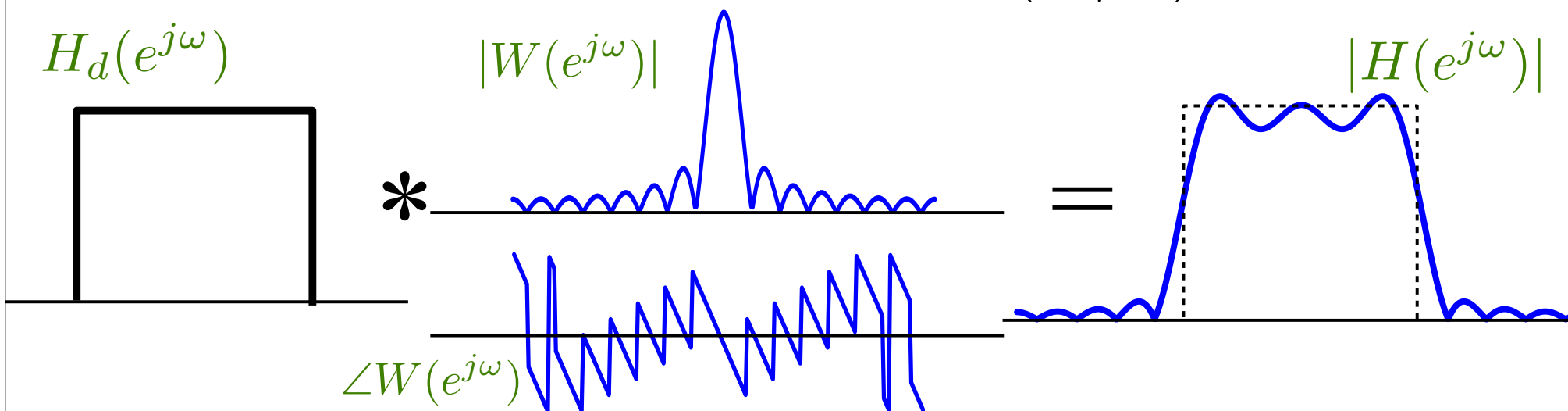
FIR Design by Windowing

- We already saw that,

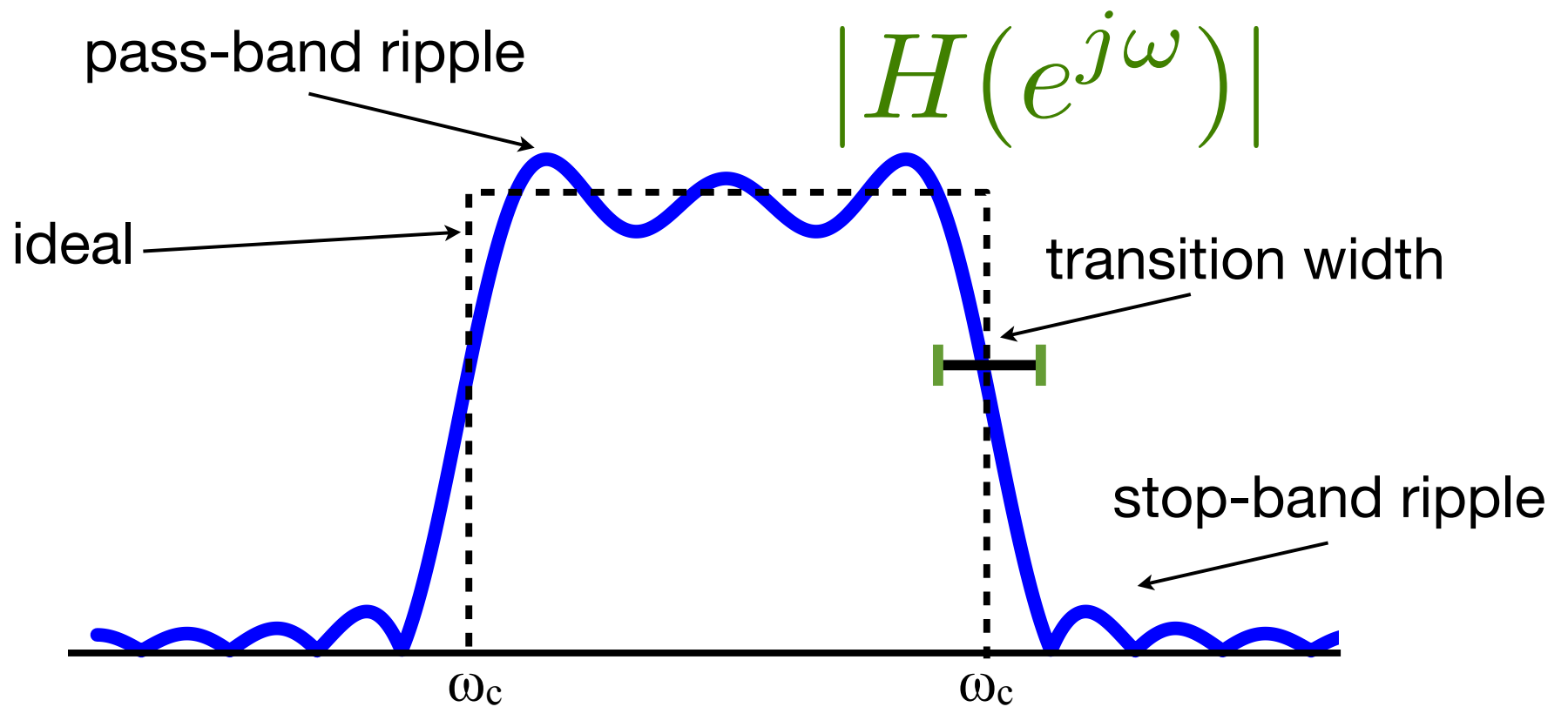
$$H(e^{j\omega}) = H_d(e^{j\omega}) * \overset{\text{periodic}}{W}(e^{j\omega})$$

- For Boxcar (rectangular) window

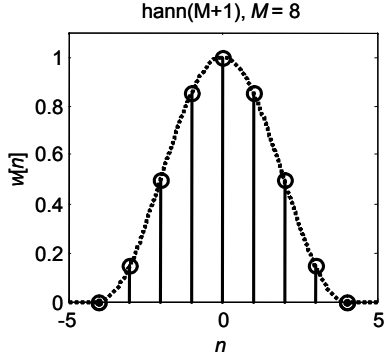
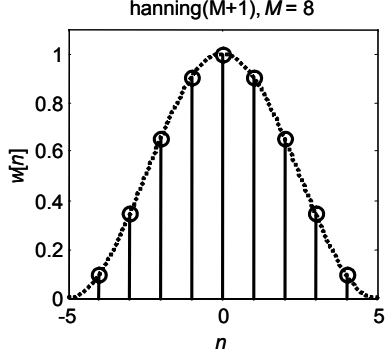
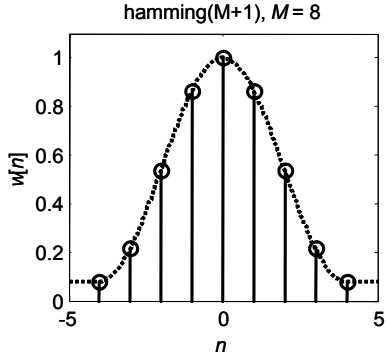
$$W(e^{j\omega}) = e^{-j\omega \frac{M}{2}} \frac{\sin(\omega(M+1)/2)}{\sin(\omega/2)}$$



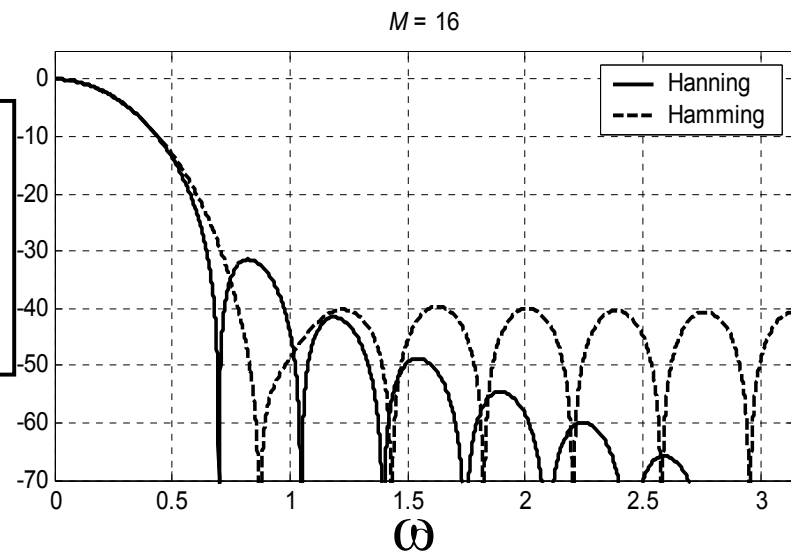
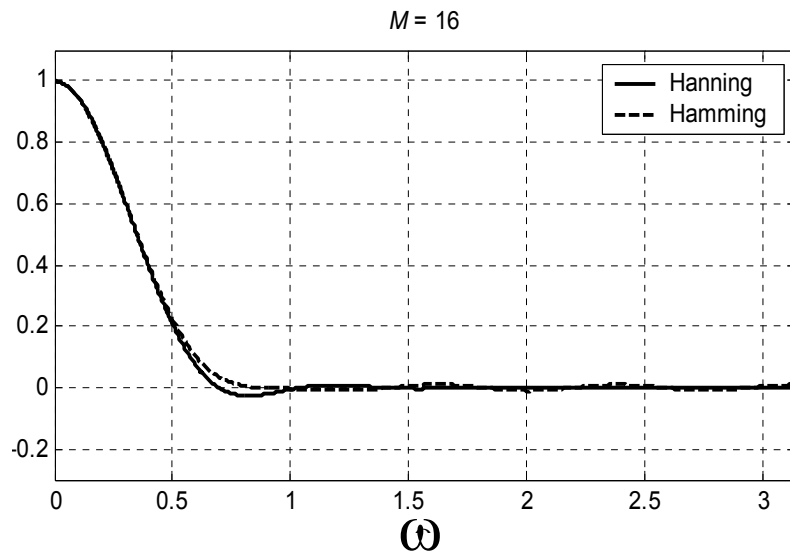
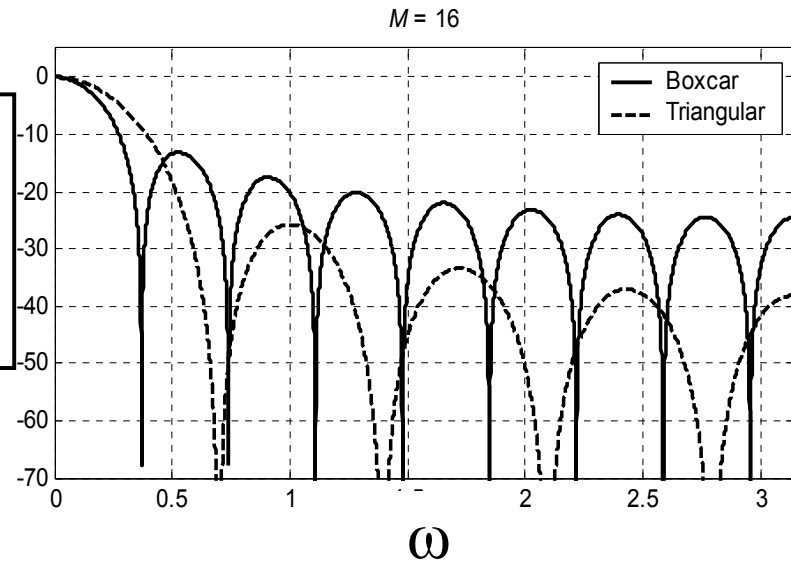
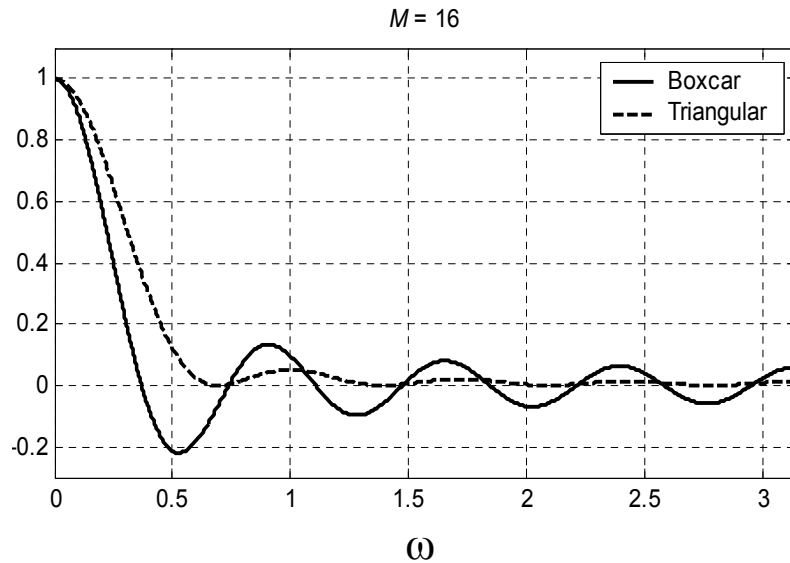
FIR Design by Windowing



Tapered Windows

Name(s)	Definition	MATLAB Command	Graph ($M = 8$)
Hann	$w[n] = \begin{cases} \frac{1}{2} \left[1 + \cos\left(\frac{\pi n}{M/2}\right) \right] & n \leq M/2 \\ 0 & n > M/2 \end{cases}$	<code>hann (M+1)</code>	
Hanning	$w[n] = \begin{cases} \frac{1}{2} \left[1 + \cos\left(\frac{\pi n}{M/2 + 1}\right) \right] & n \leq M/2 \\ 0 & n > M/2 \end{cases}$	<code>hanning (M+1)</code>	
Hamming	$w[n] = \begin{cases} 0.54 + 0.46 \cos\left(\frac{\pi n}{M/2}\right) & n \leq M/2 \\ 0 & n > M/2 \end{cases}$	<code>hamming (M+1)</code>	

Tradeoff - Ripple vs Transition Width



Python: `scipy.filter.firwin`

FIR Filter Design

- Choose a desired frequency response $H_d(e^{j\omega})$
 - non causal (zero-delay), and infinite imp. response
 - If derived from C.T, choose T and use:

$$H_d(e^{j\omega}) = H_c(j\frac{\Omega}{T})$$

- Window:
 - Length M+1 \Leftrightarrow effect transition width
 - Type of window \Leftrightarrow transition-width/ ripple
 - Modulate to shift impulse response

$$H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}$$

FIR Filter Design

- Determine truncated impulse response $h_1[n]$

$$h_1[n] = \begin{cases} \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{-j\omega \frac{M}{2}} e^{j\omega n} & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

- Apply window

$$h_w[n] = w[n]h_1[n]$$

- Check:

- Compute $H_w(e^{j\omega})$, if does not meet specs
increase M or change window

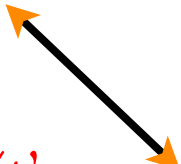
Example: FIR Low-Pass Filter Design

$$H_d(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Choose $M \Rightarrow$ Window length and set

$$H_1(e^{j\omega}) = H_d(e^{j\omega})e^{-j\omega \frac{M}{2}}$$

$$h_1[n] = \begin{cases} \frac{\sin(\omega_c(n-M/2))}{\pi(n-M/2)} & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$


$$\frac{\omega_c}{\pi} \operatorname{sinc}\left(\frac{\omega_c}{\pi}(n-M/2)\right)$$

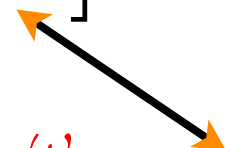
Example: FIR Low-Pass Filter Design

- The result is a windowed sinc function

$$h_w[n] = w[n]h_1[n]$$

- High Pass Design:

- Design low pass $h_w[n]$
- Transform to $h_w[n](-1)^n$


$$\frac{\omega_c}{\pi} \text{sinc}\left(\frac{\omega_c}{\pi}(n - M/2)\right)$$

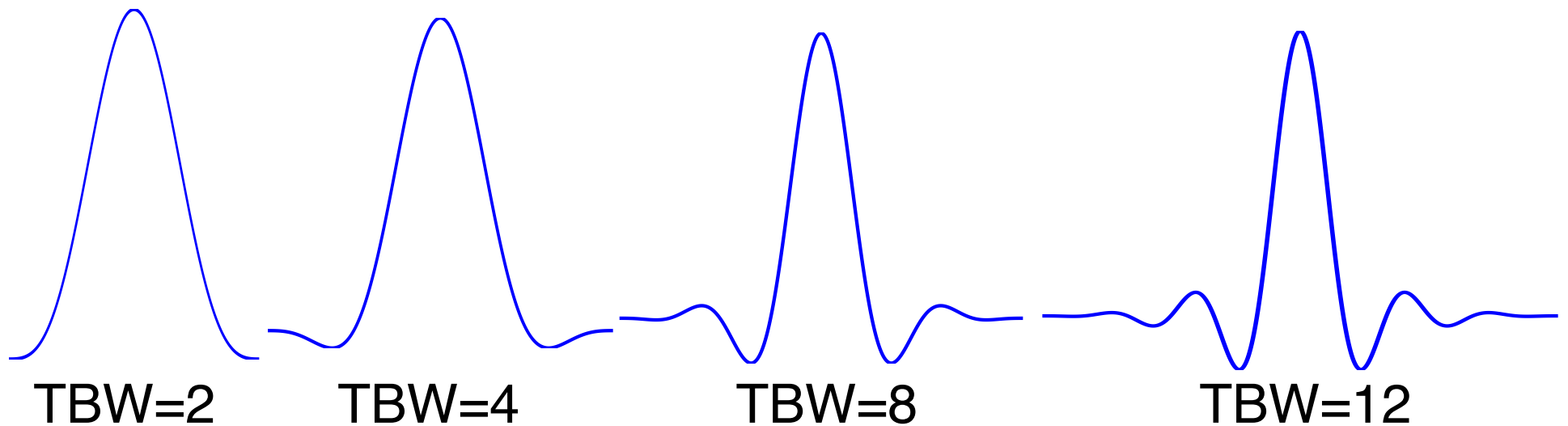
- General bandpass

- Transform to $2h_w[n]\cos(\omega_0 n)$

Characterization of Filter Shape

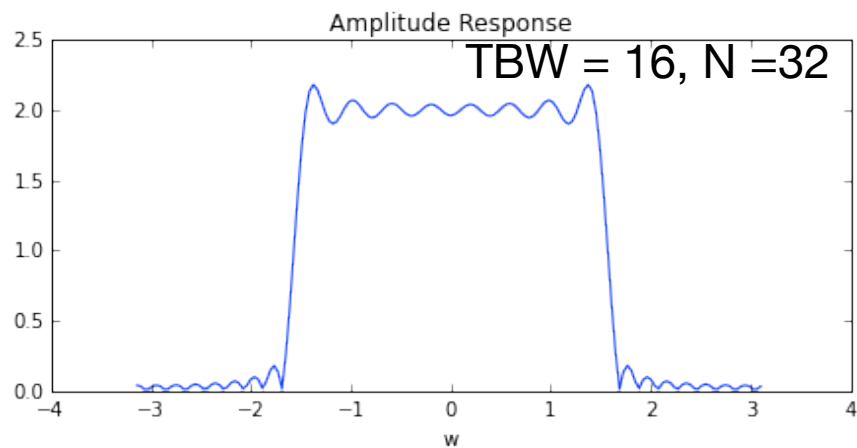
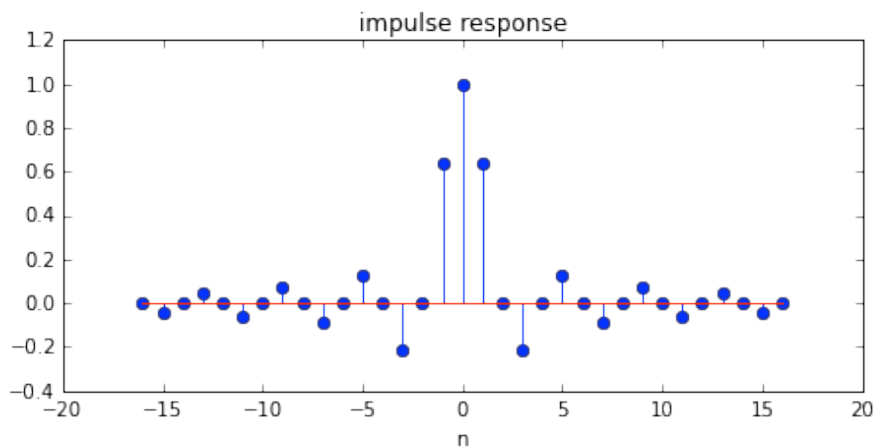
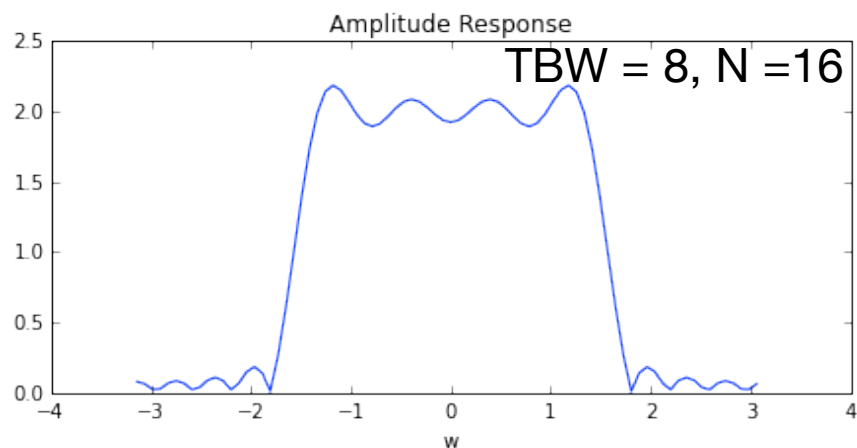
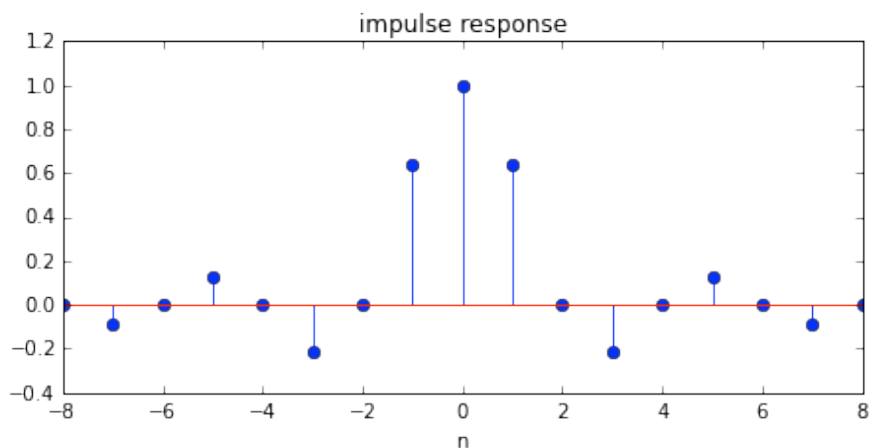
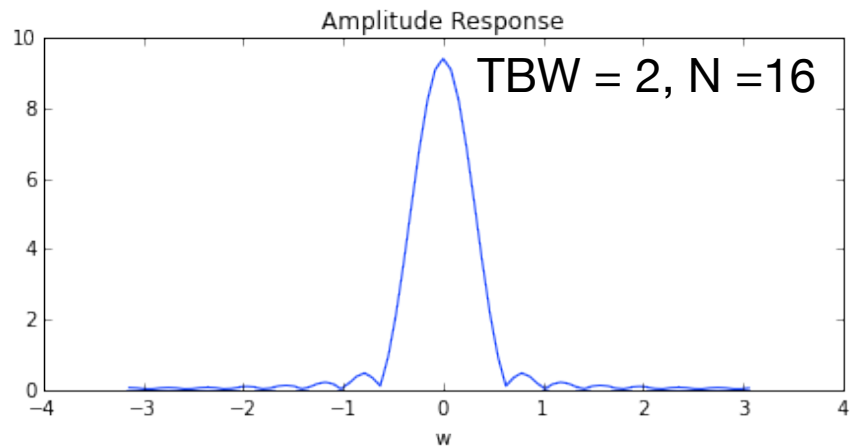
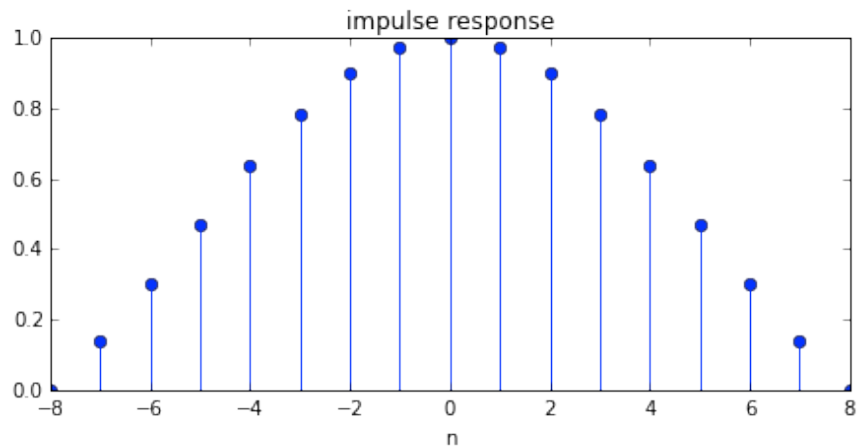
Time-Bandwidth Product, a unitless measure

$$T(\text{BW}) = (M+1)\omega/2\pi \quad \Rightarrow \text{also, total \# of zero crossings}$$



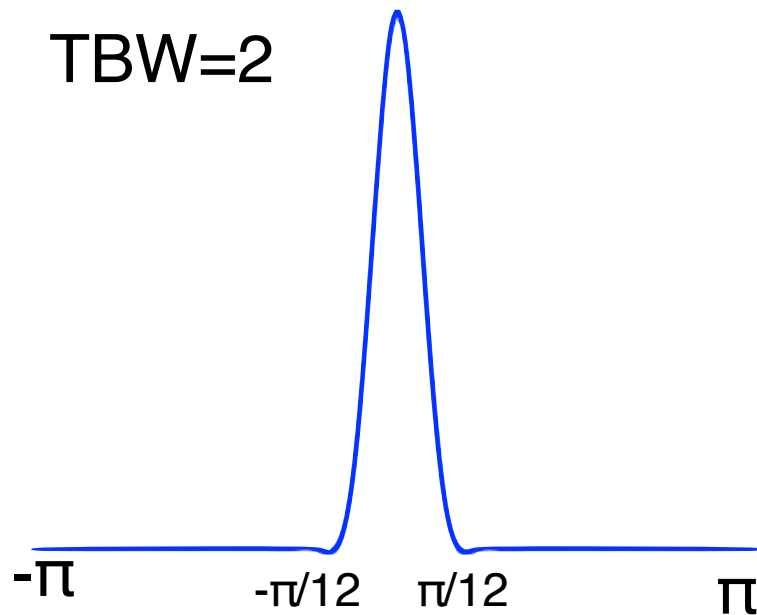
Larger TBW \Rightarrow More of the “sinc” function

hence, frequency response looks more like a rect function

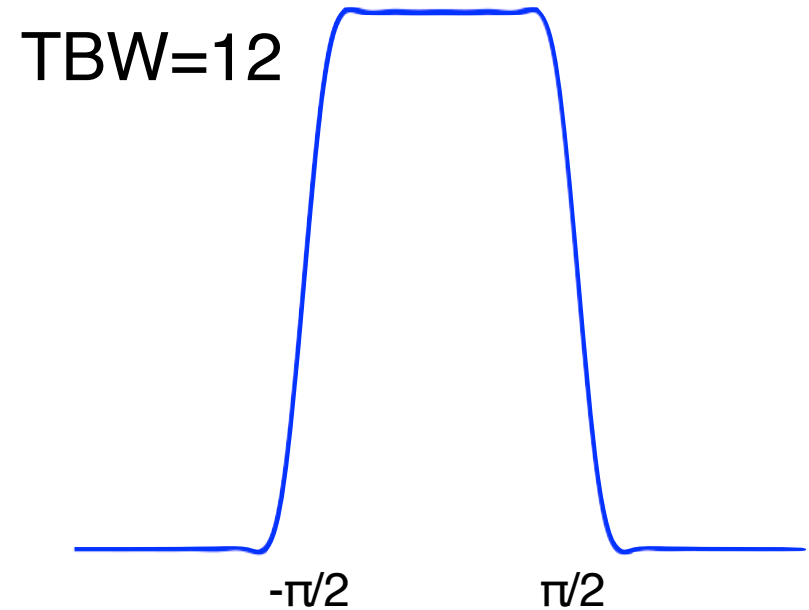


Frequency Response Profile

Q: What are the lengths of these filters in samples?



$$2 = (M+1) * (\pi/6) / (2\pi) \Rightarrow M=23$$



$$12 = (M+1) * (\pi) / (2\pi) \Rightarrow M=23$$

Note that transition is the same!

Optimal Filter Design

- Window method
 - Design Filters heuristically using windowed sinc functions
- Optimal design
 - Design a filter $h[n]$ with $H(e^{j\omega})$
 - Approximate $H_d(e^{j\omega})$ with some optimality criteria - or satisfies specs.