EE123
Digital Signal Processing

Lecture 27
Transform Analysis of LTI Systems

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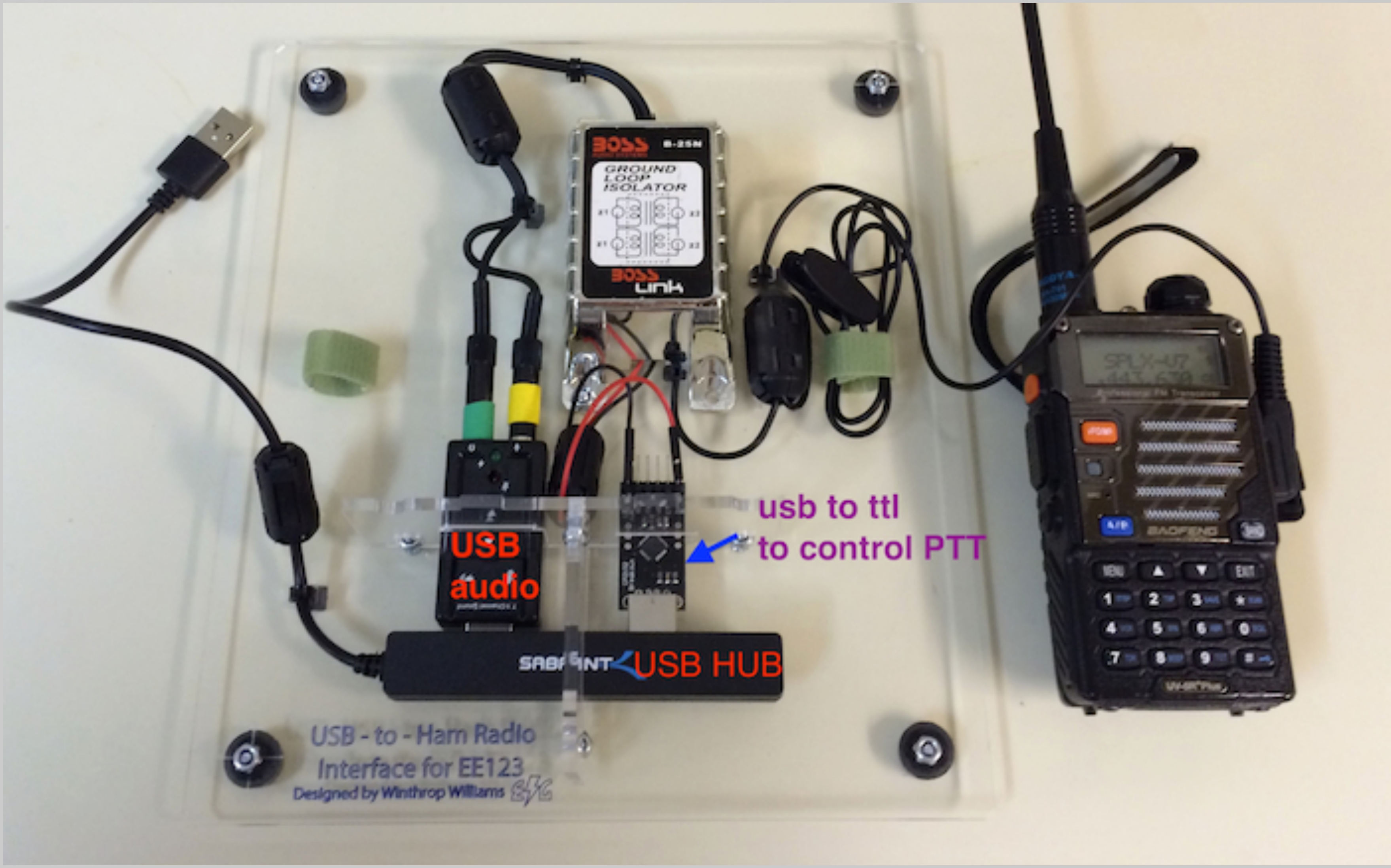
* Beautiful handwritten figures by Prof. Murat Arcak

Announcements

- To Make the CS HW worthwhile we will use the score to replace your worst HW AND your worst lab score.
- Projects: Start thinking about it.
  – It's should 1 week of work.
  – Groups of 2 minimum, 3 possible depending on the scope
  – There will be a default project based on lab 6
  – Custom projects: I'm hoping you use either SDR or radios or both -- but open for other suggestions

Lab 5

- Test the interface



Problems the could occur

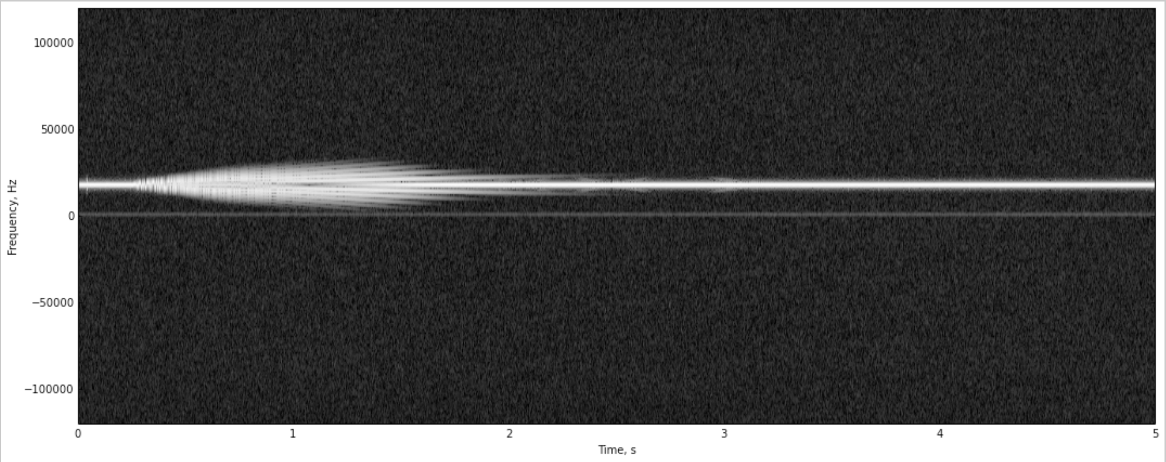
*****Windows users: Check the com port of your usb-to-serial device and use the right one in the code¶**

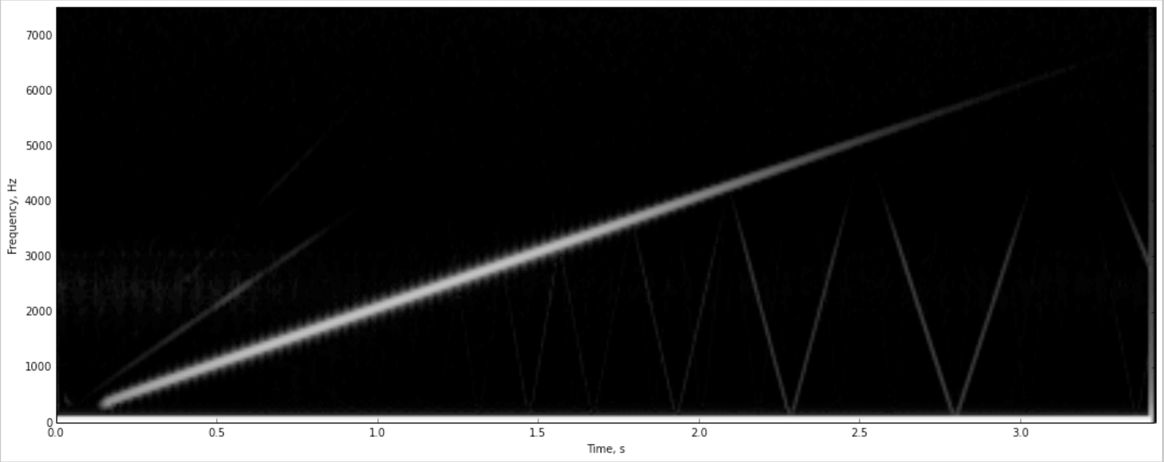
In [ ]:
```
if sys.platform == 'darwin':  # Mac
    s = serial.Serial(port='/dev/tty.SLAB_USBtoUART')
else:                         #windows
    s = serial.Serial(port='COM1') ##### CHANGE !!!!!!
s.setDTR(0)

for n in range(0,10):
    s.setDTR(1)
    time.sleep(0.25)
    s.setDTR(0)
    time.sleep(0.25)
```

```
def audioDevNumbers(p):
    # din, dout, dusb = audioDevNumbers(p)
    # The function takes a pyaudio object
    # The function searches for the device numbers for built-in mic and
    # speaker and the USB audio interface
    # some devices will have the name “Generic USB Audio Device”. In that case, replace it with the the right name.
```

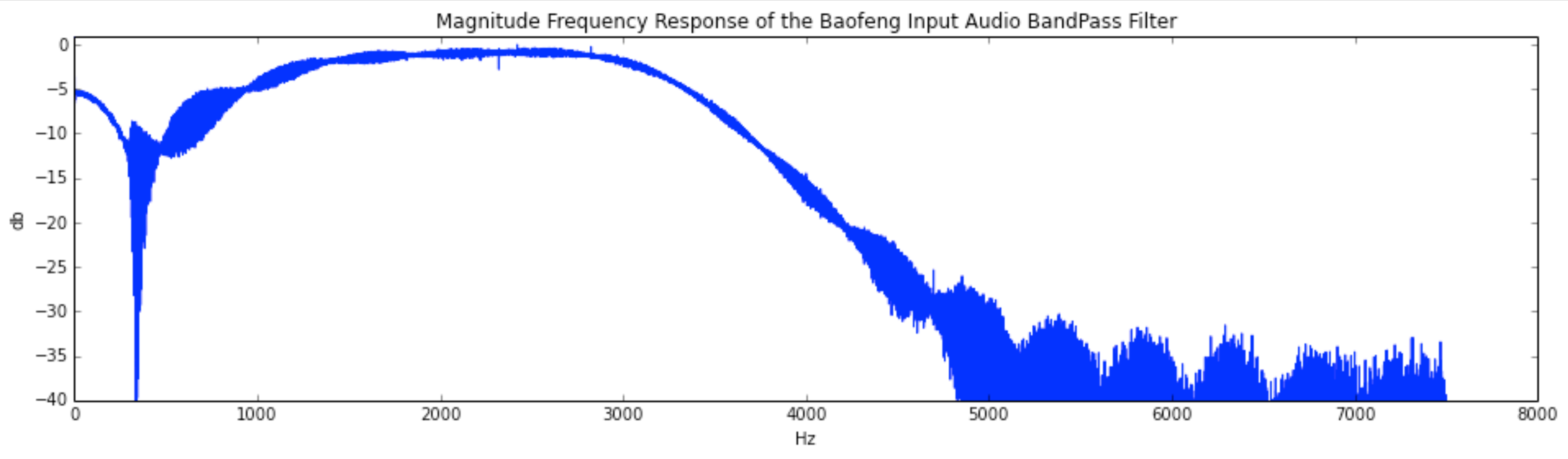
Measure Frequency Response

- Of the Radio Audio input
  - Play a chirp
  - Listen using SDR
  - Demodulate



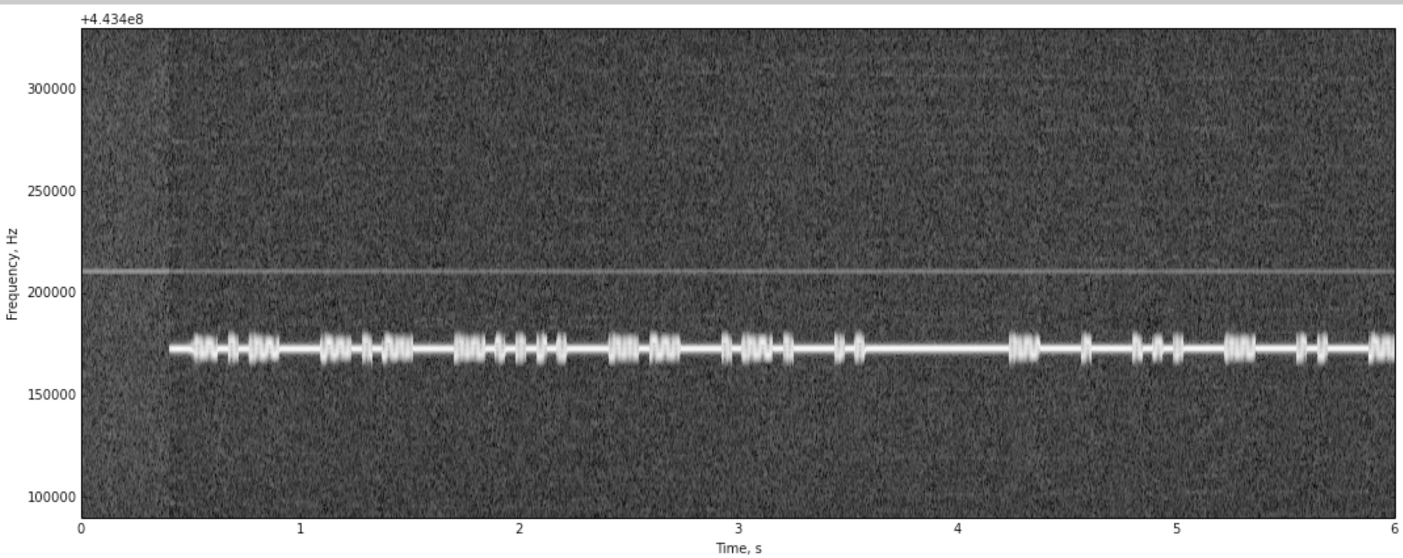


Frequency Response



Morse Code Function

- Write a function
  - Converts text to morse
  - Transmit and receive



Rational system response

Linear difference equations :

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

Example: $y[n] = x[n] + 0.1y[n-1]$

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \ldots + a_N z^{-N}} = \frac{b_0}{a_0} \frac{\prod_{k=1}^{M}(1 - c_k z^{-1})}{\prod_{k=1}^{N}(1 - d_k z^{-1})}$$

IIR Design

- Historically
  - – Continuous IIR design was advanced
  - – Use results from C.T to D.T
  - – C.T IIT designs have closed form, easy to use
  - – Easy to control Magnitude, not easy to control phase
- Common Types:
  - –Butterworth - monotonic, no ripple
  - –Chebyshev - Type I, pass band ripple, Type II stop band ripple
  - – Elliptic - Ripples in both bands

Design of D.T IIR Filters from Analog

- Discretize by one of many techniques
- $H_c(s) \Rightarrow H(z)$
- Must satisfy:
  - Imaginary axis is mapped to unit circle
  - stability of $H_c(s)$ should result in stable $H(z)$
- Two methods:
  - Impulse invariance - match impulse response
  - Bilinear transformation

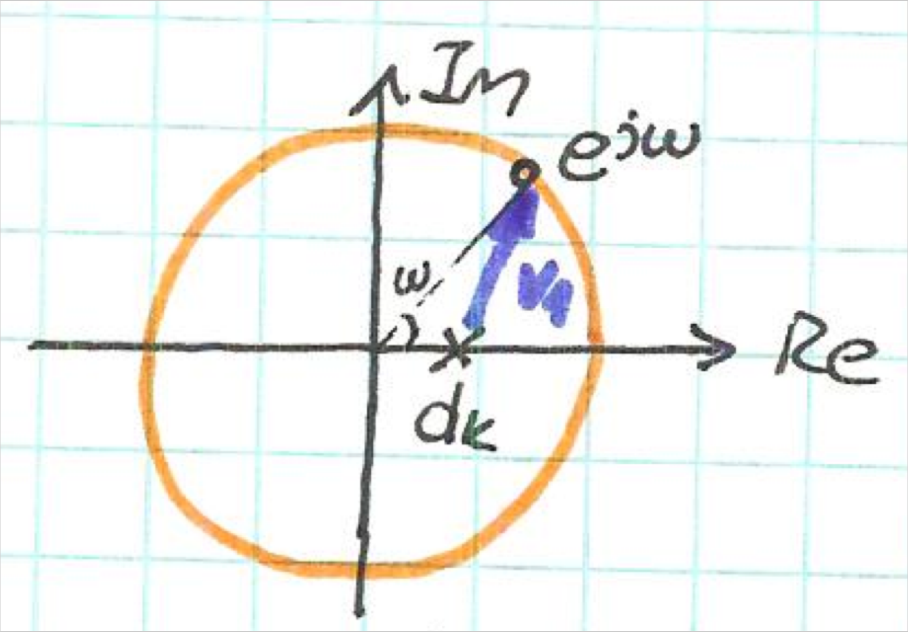
Magnitude Response

Magnitude of products is product of magnitudes

$$|H(e^{j\omega})| = |\frac{b_0}{a_0}| \cdot \frac{\prod_{k=0}^{M} |1 - c_k e^{-j\omega}|}{\prod_{k=0}^{N} |1 - d_k e^{-j\omega}|}$$

Consider one of the poles:

$$|1 - d_k e^{-j\omega}| = |e^{+j\omega} - d_k| \quad = |v_1|$$

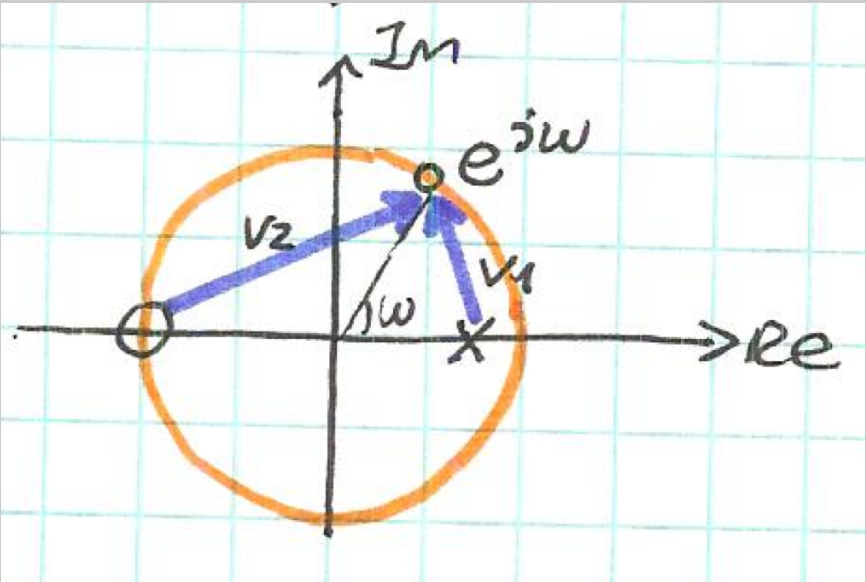


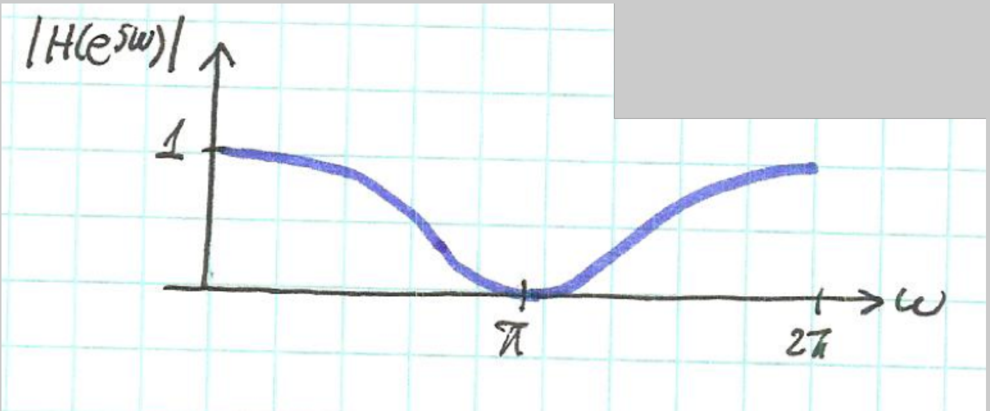
Magnitude Response Example

Example:

$$H(z) = 0.05\frac{1+z^{-1}}{1-0.9z^{-1}}$$

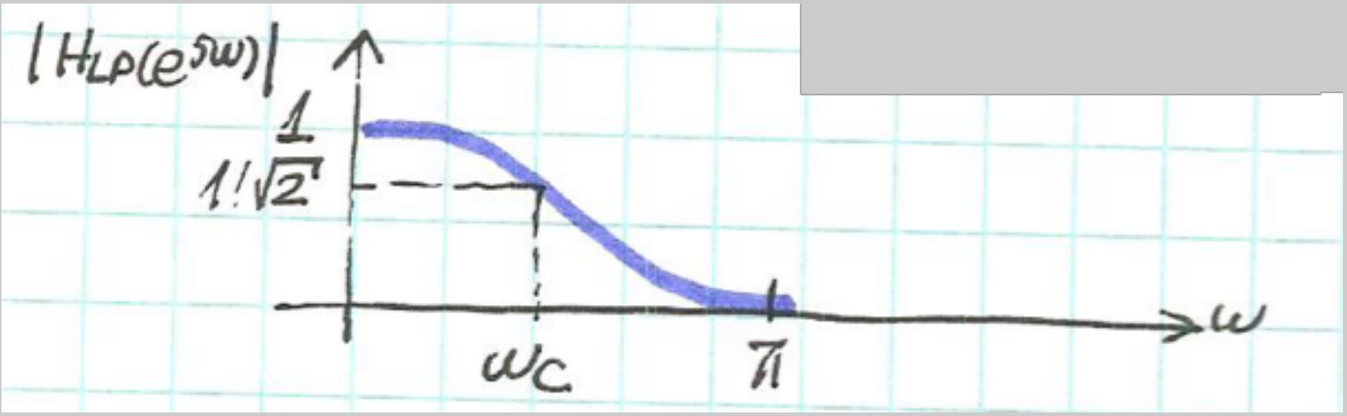
$$|H(z)| = 0.05\frac{|v_2|}{|v_1|}$$





Simple low pass filter

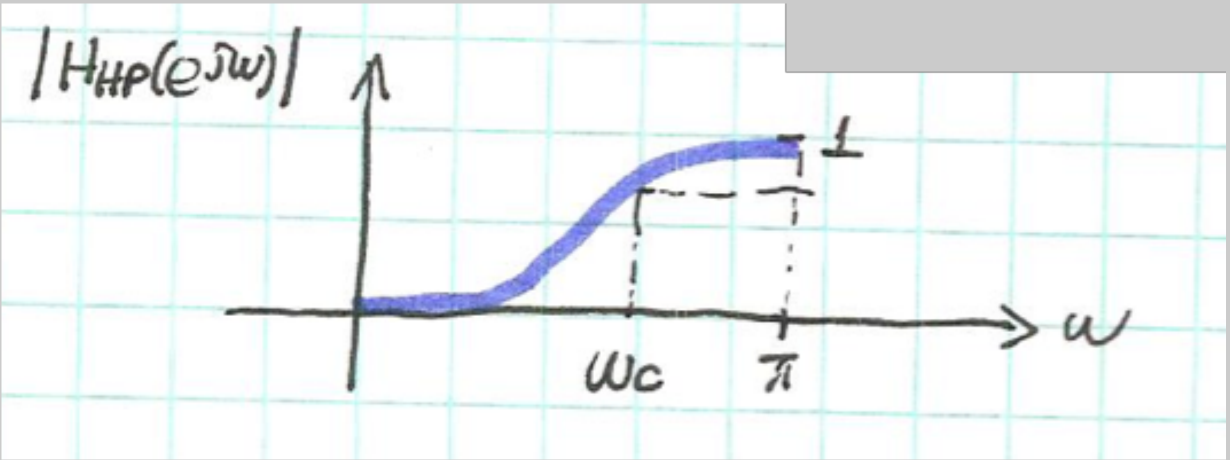
$$H_{LP}(z) = \frac{1-\alpha}{2}\frac{1+z^{-1}}{1-\alpha z^{-1}} \qquad |\alpha| < 1$$



w_c is the 3dB cutoff frequency $\qquad \alpha = \frac{1-\sin(\omega_c)}{\cos(\omega_c)}$

Simple high pass filter

$$H_{HP}(z) = \frac{1+\alpha}{2} \frac{1+z^{-1}}{1-\alpha z^{-1}} \qquad |\alpha| < 1$$



w_c is the 3dB cutoff frequency $\qquad \alpha = \frac{1-\sin(\omega_c)}{\cos(\omega_c)}$

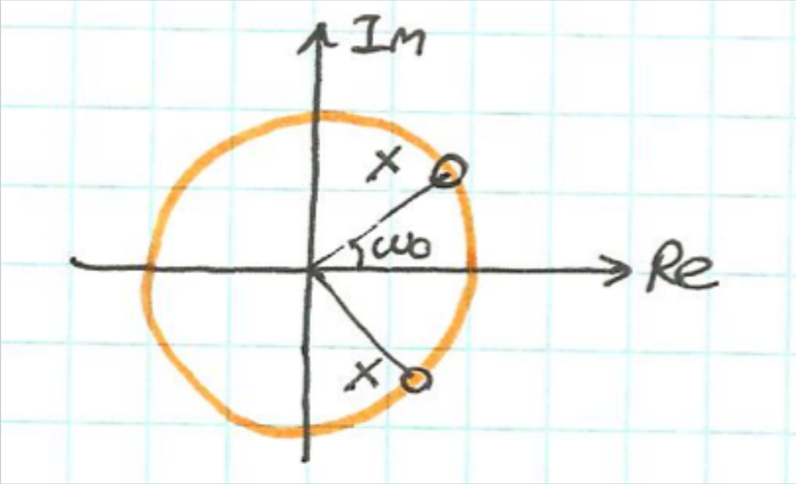
same as low pass

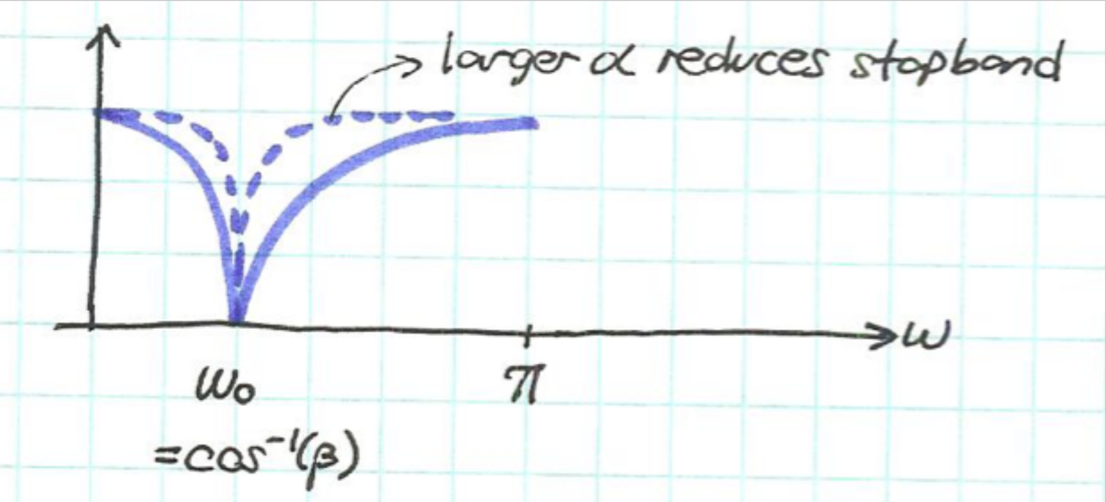
Simple band-stop (Notch) filter

$$H_{BS}(z) = \frac{1+\alpha}{2}\frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}} \qquad \begin{array}{l} |\alpha|<1 \\ |\beta|<1 \end{array}$$

Note: $$1-2\beta z^{-1}+z^{-2} = (1-e^{j\omega_0}z^{-1})(1-e^{-j\omega_0}z^{-1})$$

$$\cos(\omega_0)=\beta$$



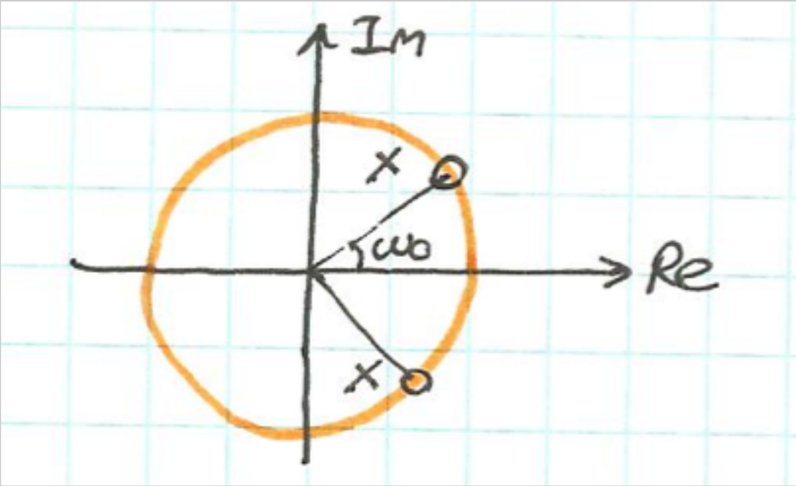


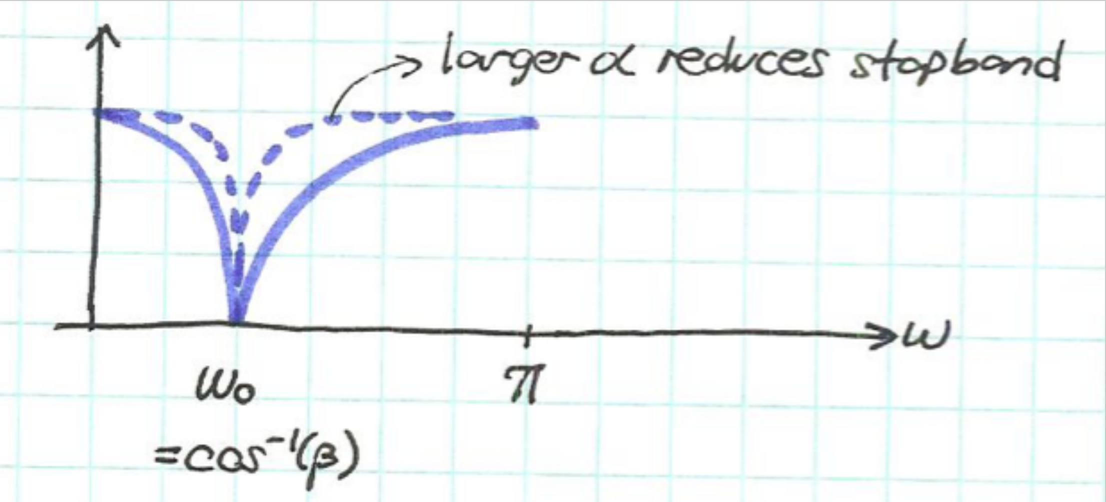
Simple band-stop (Notch) filter

$$H_{BS}(z) = \frac{1+\alpha}{2}\frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}} \qquad \begin{array}{l}|\alpha|<1\\|\beta|<1\end{array}$$

Note: As $\alpha \to 1$ poles approach zeros

$$H_{BS}(\mp 1) = \frac{1+\alpha}{2}\frac{2\pm 2\beta}{(1+\alpha)(1\pm\beta)} = 1$$





Simple band-pass filter

$$H_{BP}(z) = \frac{1-\alpha}{2}\frac{1-z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}} \qquad \begin{array}{l}|\alpha|<1\\ |\beta|<1\end{array}$$