EE123
Digital Signal Processing

Lecture 29
All-Pass and Minimum Phase

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* Beautiful handwritten figures by Prof. Murat Arcak

Announcements

- Lab 5 due today
- Lab 6 (prelab and lab) will be out today
- HW8 Optional Due Friday
- HW9 Due Friday -- not optional
- I'll talk about lab 6 and Project on Wednesday
- Midterm 2 is graded, grades will post tomorrow 8:00am
- Don't use simplex channels for labs!
  - and identify yourself!!!!!!

A general~~l~~ all-pass system:

Real, complex

$$H_{ap}(z) = \prod_{k=1}^{M_r} \frac{z^{-1} - d_k}{1 - d_k z^{-1}} \cdot \prod_{k=1}^{M_c} \frac{z^{-1} - e_k^*}{1 - e_k z^{-1}} \cdot \frac{z^{-1} - e_k}{1 - e_k^* z^{-1}}$$

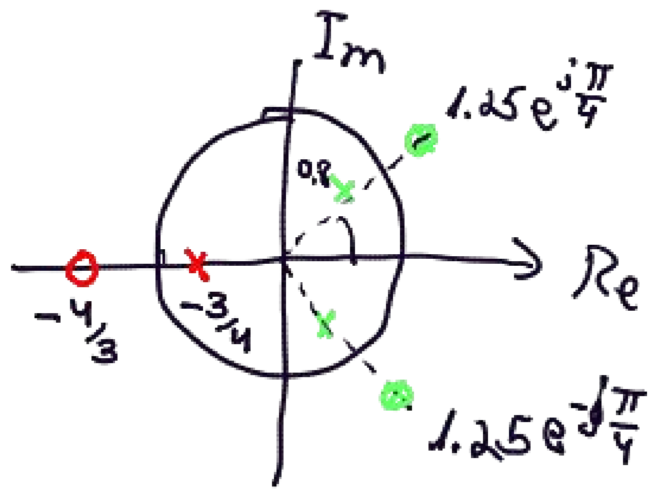
d_k: real Poles

e_k: complex poles paired w/ conjugate e_k^*

Text

$$|H_{ap}(e^{j\omega})| \equiv 1$$

Example



<u>Claim</u>: for a stable ap system $H_{ap}(z)$: ⑥

(i) $\text{grd}\left[H_{ap}(e^{j\omega})\right] > 0$

(ii) $\arg\left[H_{ap}(e^{j\omega})\right] \leq 0$

Delay positive → causal
phase negative → phase lag.

proof in book.

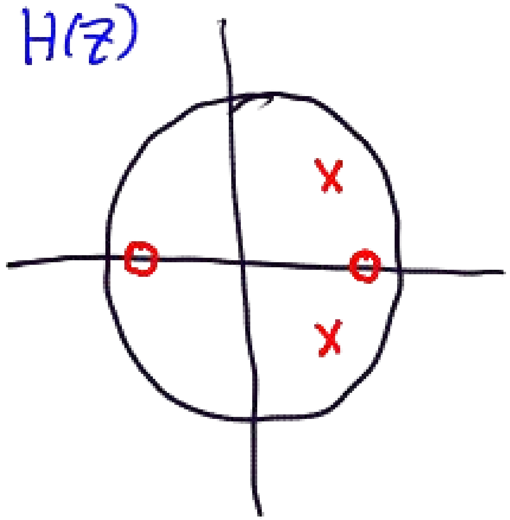
Minimum-Phase Systems

Definition: a $\underbrace{\text{stable and causal}}_{\text{poles inside unit circle}}$ system $H(z)$

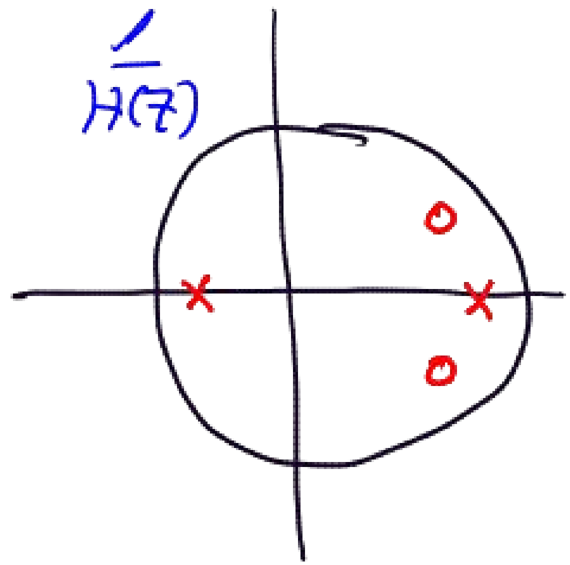
who'se inverse $\frac{1}{H(z)}$ is also stable & causal

zeros are inside unit circle.

$H(z)$



$\frac{1}{H(z)}$



AP-Min-Phase decomposition:

8

stable, causal system can be decomposed to:

$$H(z) = \underbrace{H_{min}(z)}_{\text{min phase}} \cdot \underbrace{H_{ap}(z)}_{\text{all pass}}$$

Approach: ① first construct H_{ap} with all zeros outside unit circle

② compute

$$H_{min}(z) = \frac{H(z)}{H_{ap}(z)}$$

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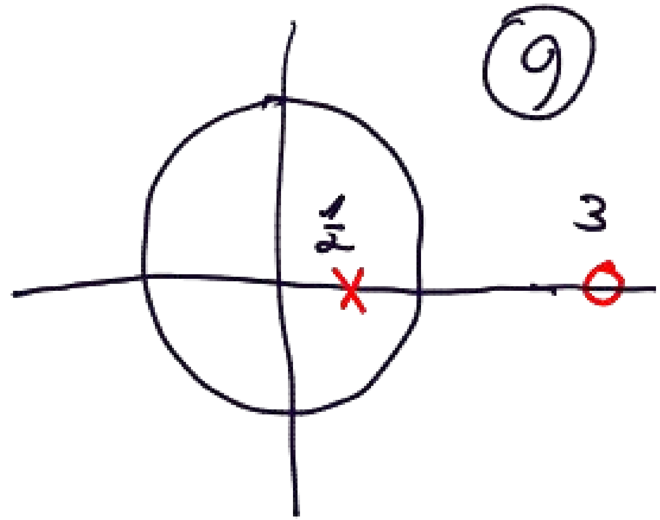
Example

$$H(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}}$$



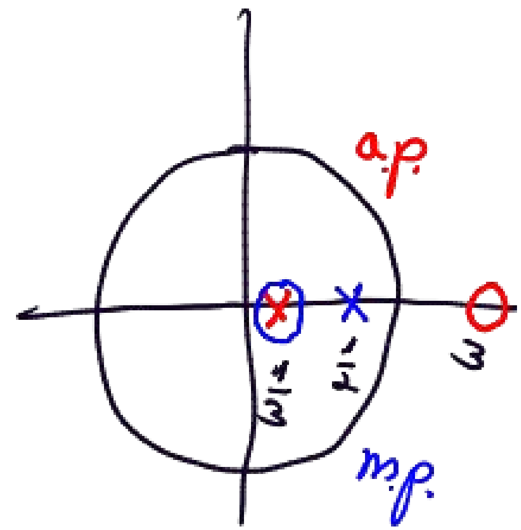
Sol:

$$H_{ap} = \frac{z^{-1}-\frac{1}{3}}{1-\frac{1}{3}z^{-1}}$$

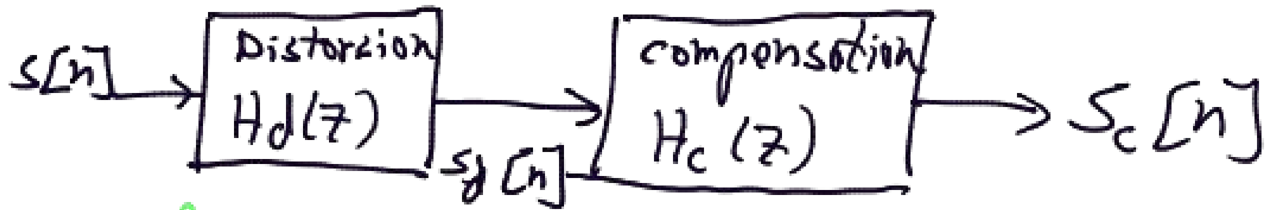
$-3 =$

$$H_{min}(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}} \cdot \frac{1-\frac{1}{3}z^{-1}}{z^{-1}-\frac{1}{3}} =$$

$$= -3\,\frac{1-\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$$



why m.p. property important?



for ex. communication chan.

If $H_d(z)$ is minimum phase, design

$$H_c(z) = \frac{1}{H_d(z)}$$ (stable!)

If not m.p., decompose: $H_d(z) = H_{d,mp}(z) \cdot H_{d,ap}(z)$

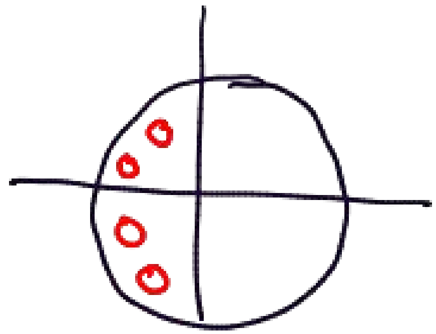
$$H_c(z) = \frac{1}{H_{d,min}(z)} \Rightarrow H_d H_c = H_{d,ap}(z)$$

only compensate for mag.

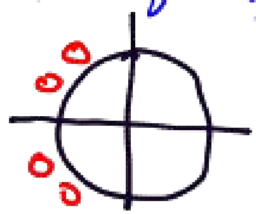
Why called "minimum phase"?

Different systems can have same mag. response.

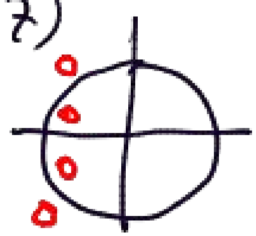
$H_1(z)$ min phase:



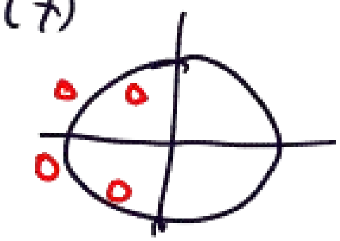
$H_2(z)$ (max phase)



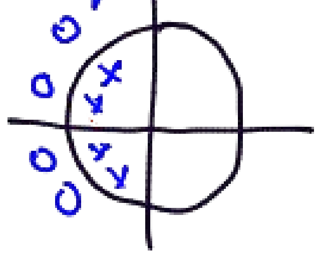
$H_3(z)$



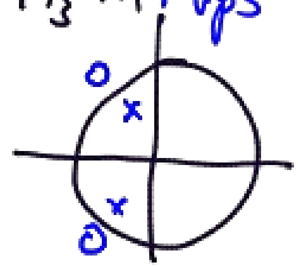
$H_4(z)$



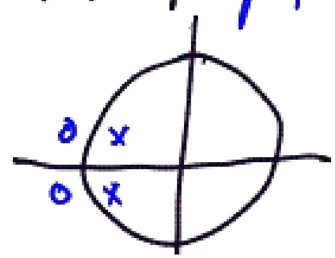
$H_2 = H_1 H_{op,2}$



$H_3 = H_1 H_{op3}$



$H_4 = H_1 \cdot H_{op4}$



of all, $H_1(z)$ has minimum phase lag (12)

because:

$$\arg\left[H_i(e^{j\omega})\right] = \arg\left[H_1(e^{j\omega})\right] + \arg\left[H_{ap_i}\right]$$

$$\arg\left[H_{ap_i}\right] \leq 0$$

other properties:

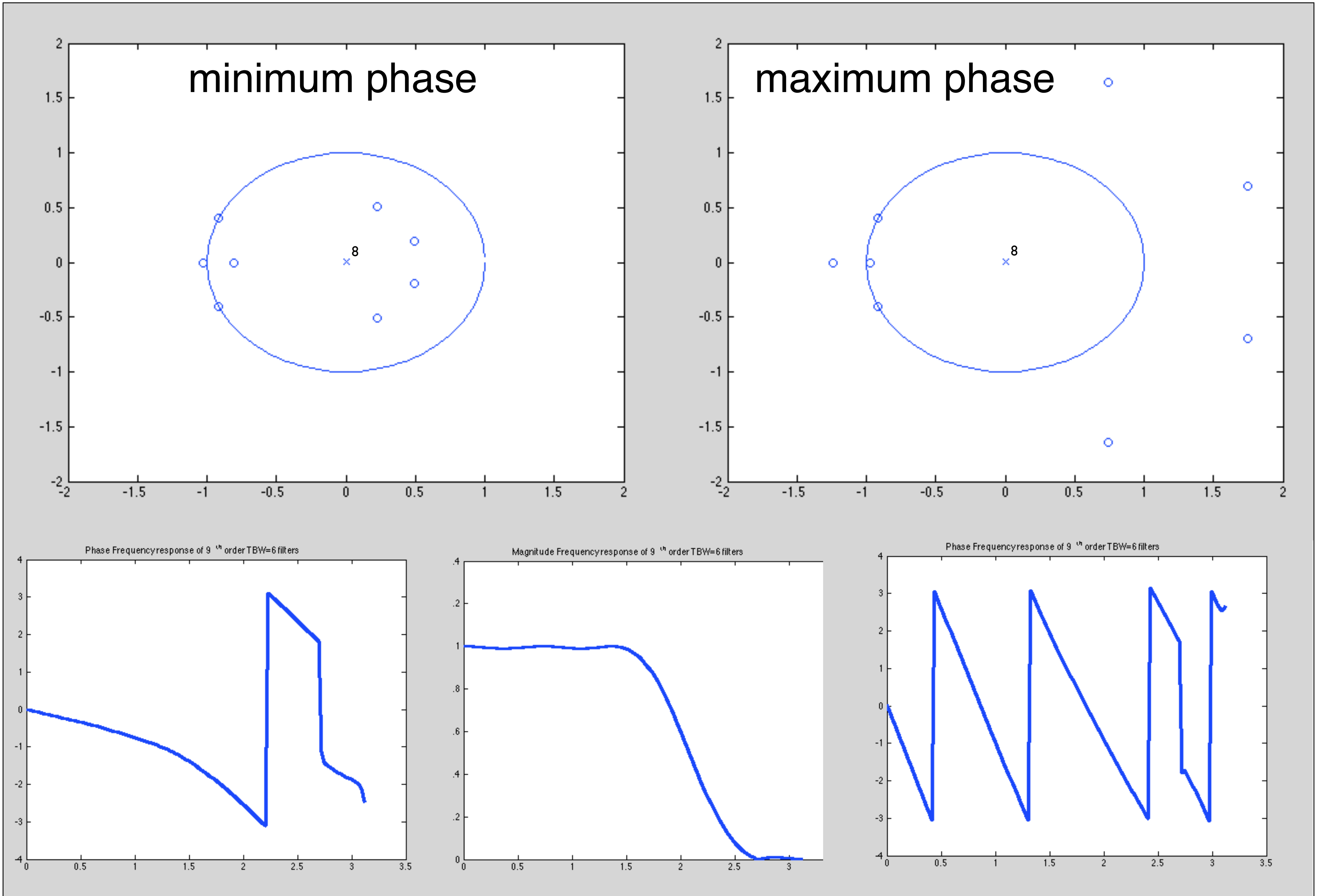
<u>minimum group delay:</u>

$$grd\left[H(e^{j\omega})\right] = grd\left[H_{min}\right] + grd\left[H_{ap}\right]$$

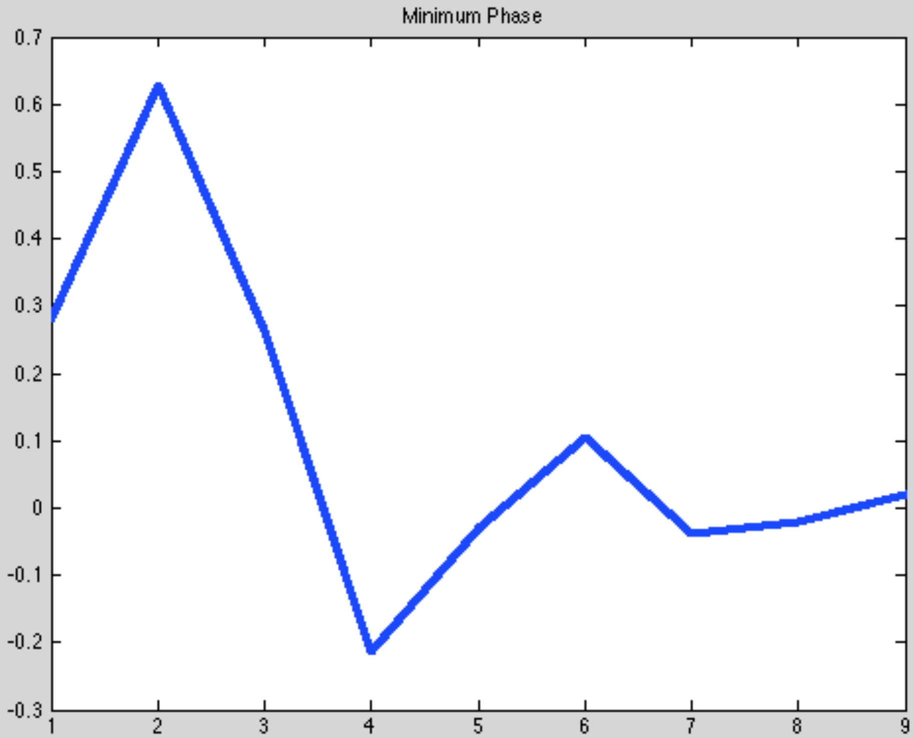
$$grd\left[H_{ap}\right] > 0$$

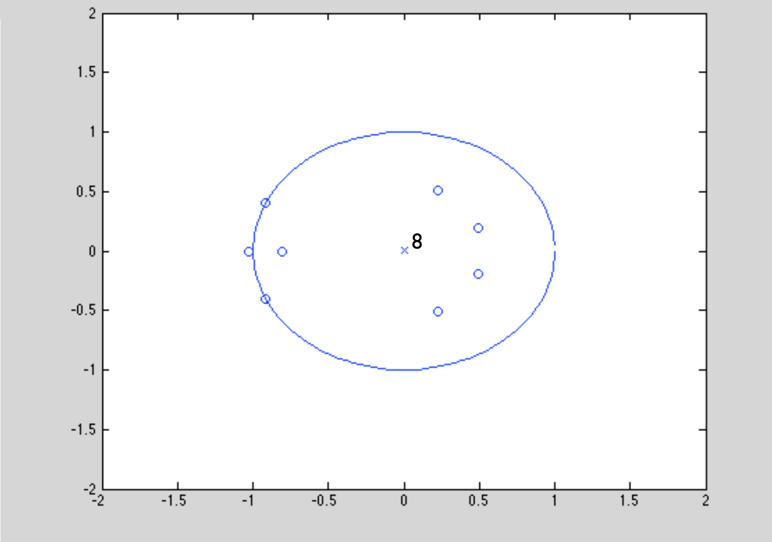
<u>minimum energy delay:</u>

Problem 5.72

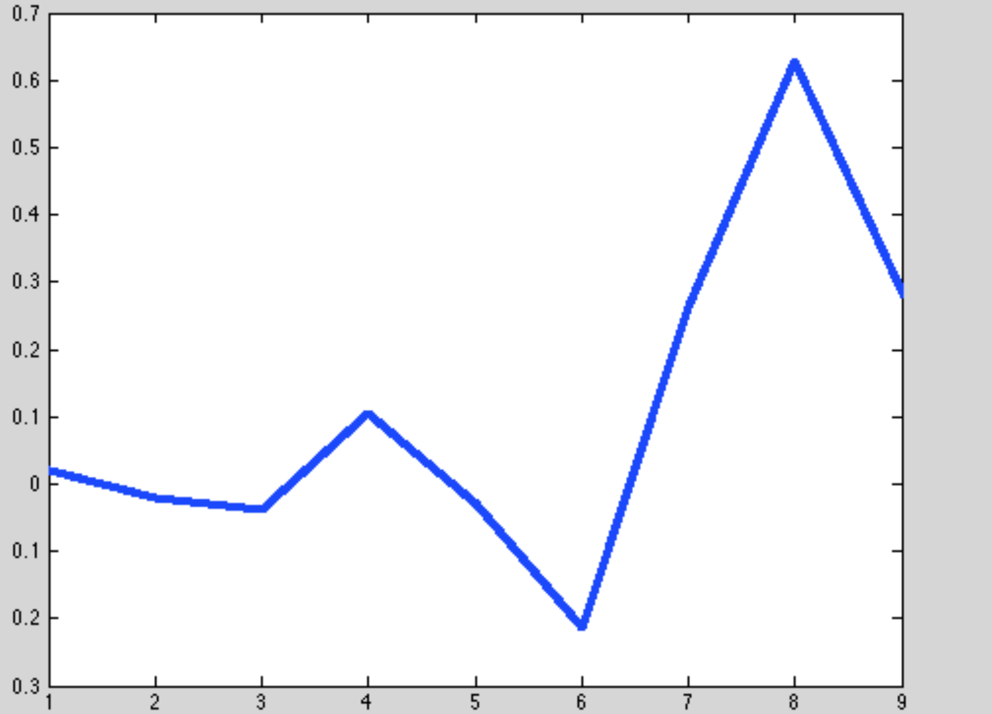

minimum phase
8
maximum phase
8
Phase Frequency response of 9 th order TBW=6 filters
Magnitude Frequency response of 9 th order TBW=6 filters
Phase Frequency response of 9 th order TBW=6 filters

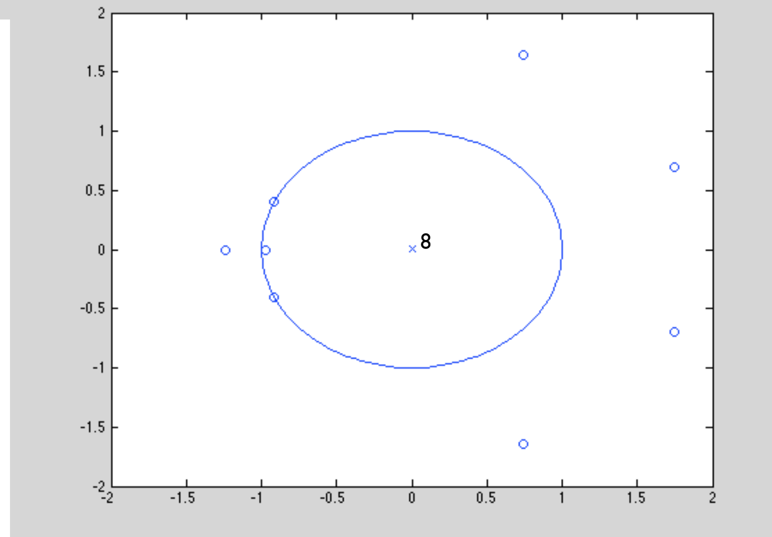
minimum phase

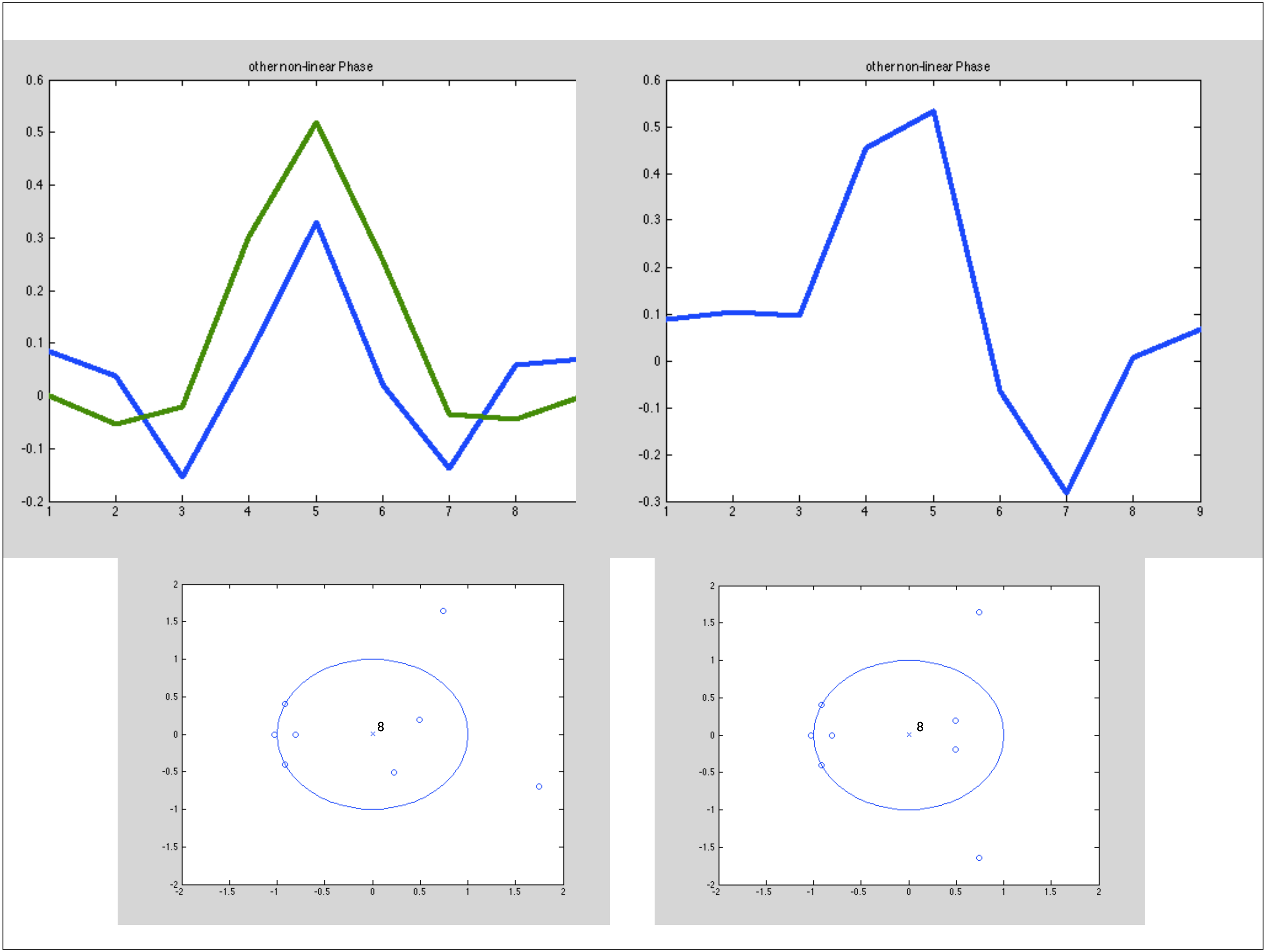




maximum phase



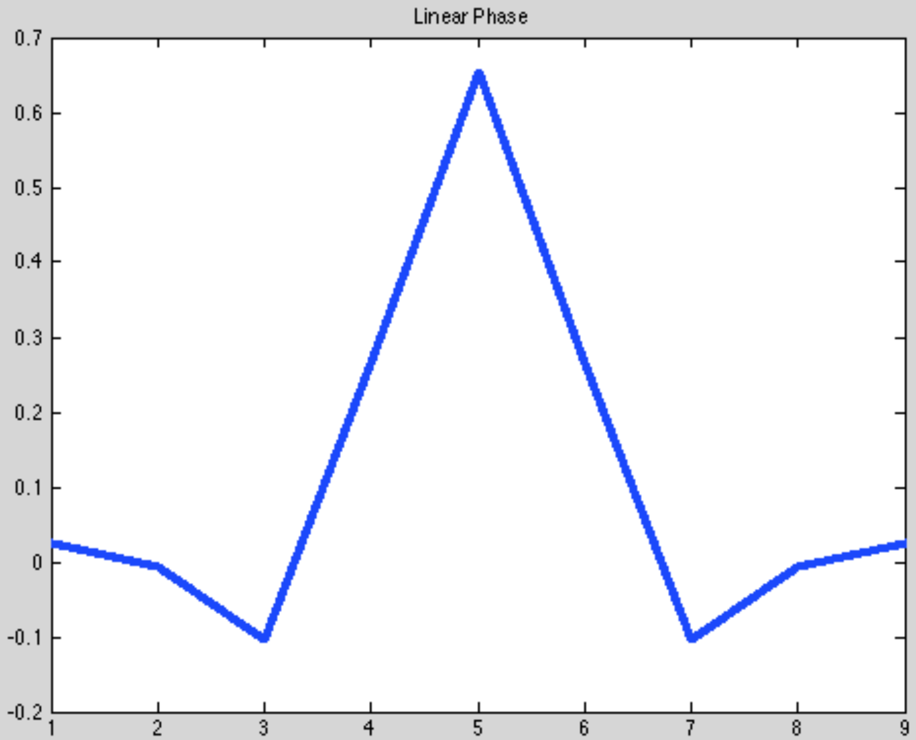


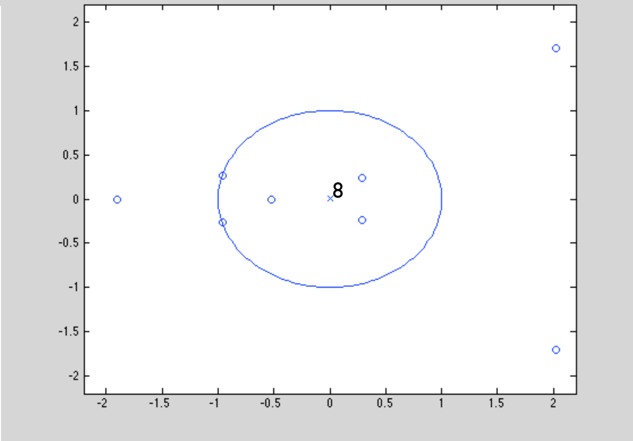


othernon-linear Phase
othernon-linear Phase
8
8

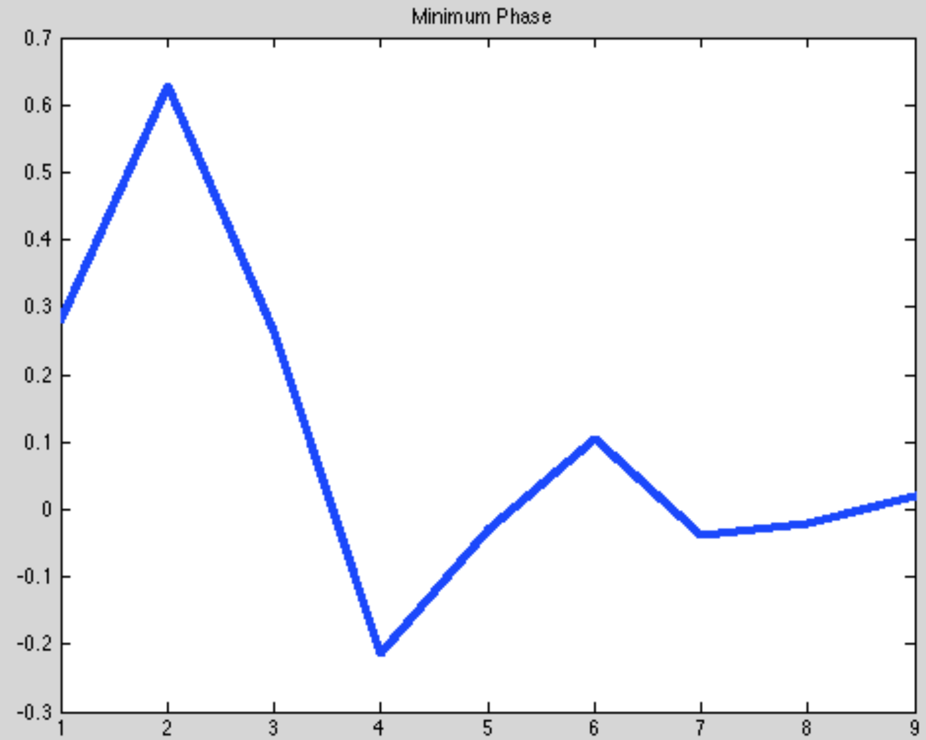
Minimum-phase VS Linear Phase

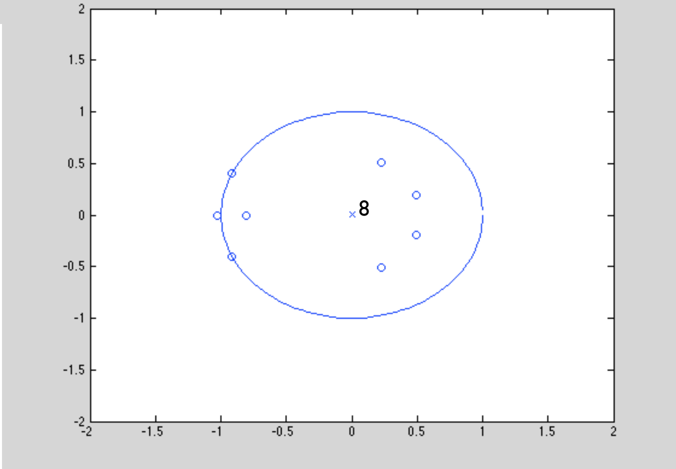
linear phase

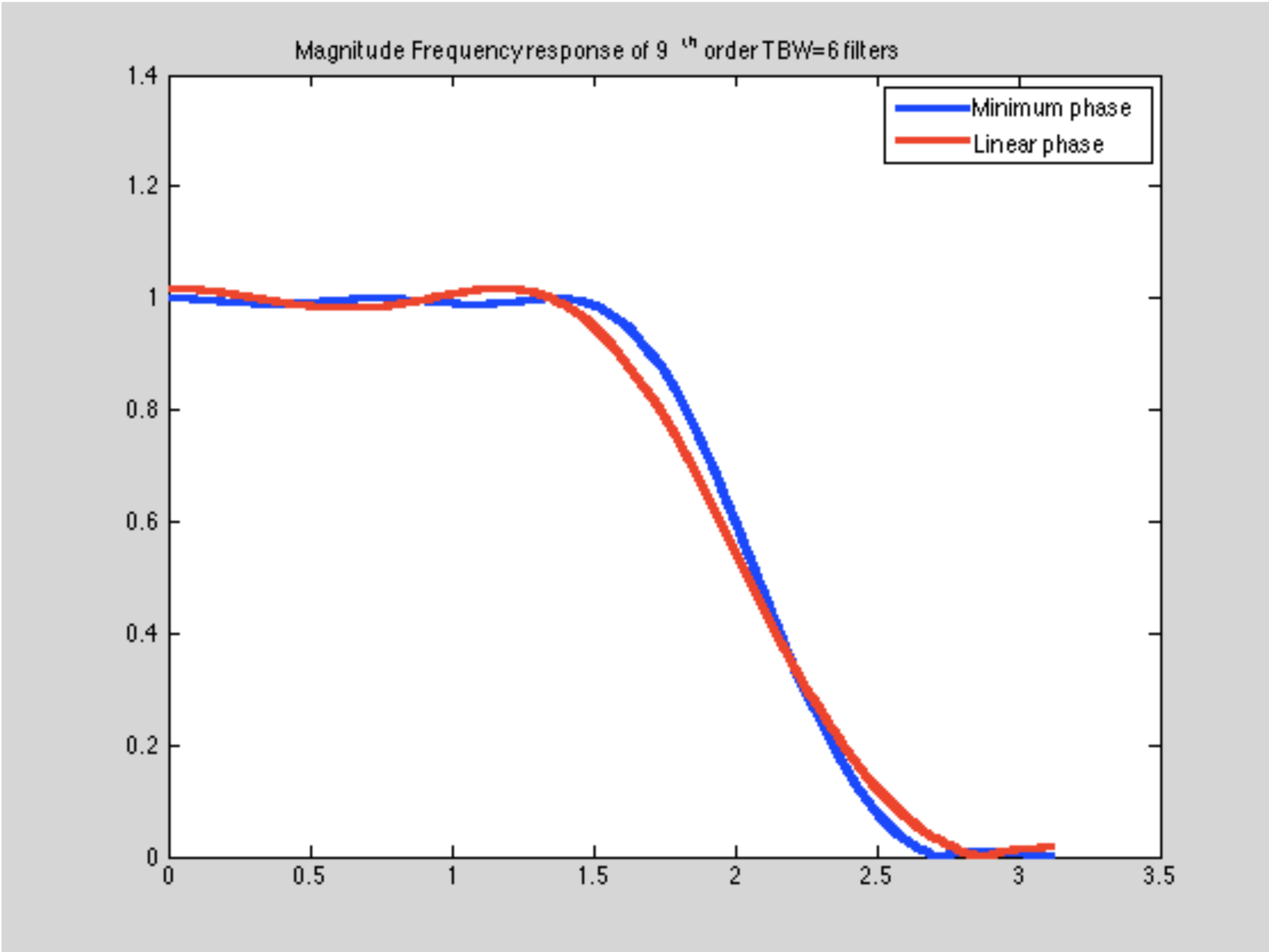




minimum phase






Magnitude Frequency response of 9 th order TBW=6 filters
Minimum phase
Linear phase
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3
3.5