EE123
Digital Signal Processing

Lecture 31
Generalized Linear Phase Systems

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Project

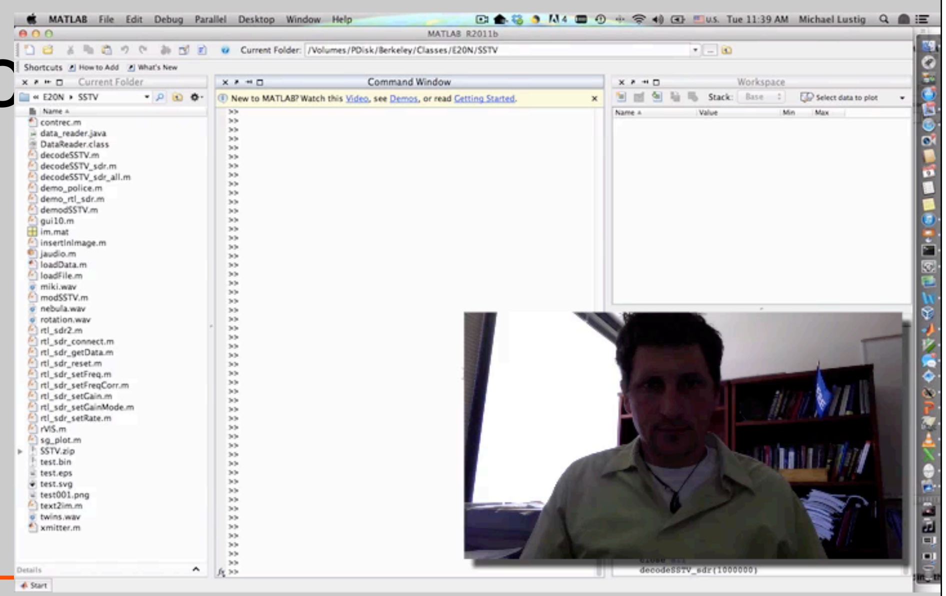
- Select a project by next Friday
  - Submit 2 paragraphs project proposal in bcourses
  - Includes partners and idea of approach
- Project Deliverables
  - Software
  - Demo
  - A few slides / Poster

Default Project

- Image communication
  - We will give you and image
  - You will need to transmit it with the best quallity over a limited amount of time (1min)
  - Evaluation is based on PICSNR and visual quality score
- You can use ANY method you write yourself
  - Compression
  - Filtering, image recovery....
  - Modulation (digital or analog), detection, ...

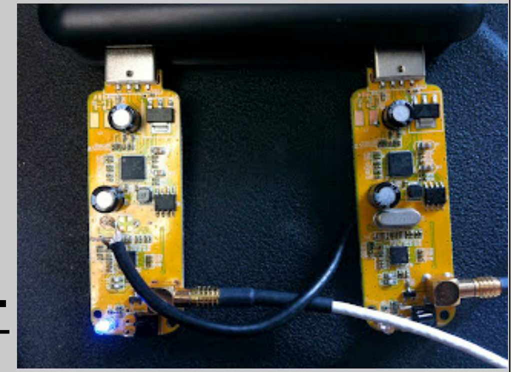
Default project

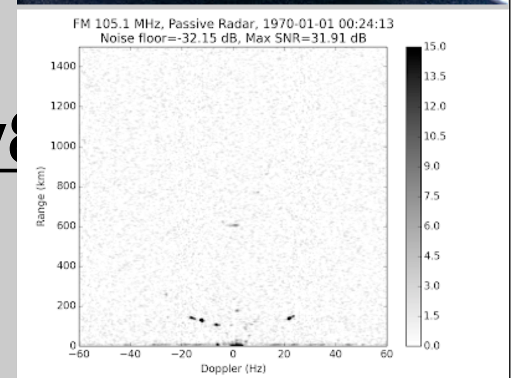
- Evaluation based on
  - Comparison to a baseline implementation with packet APRS -- slow and low res
  - Scope
  - Creativity
  - Presentation
- Winner gets a prize: radic

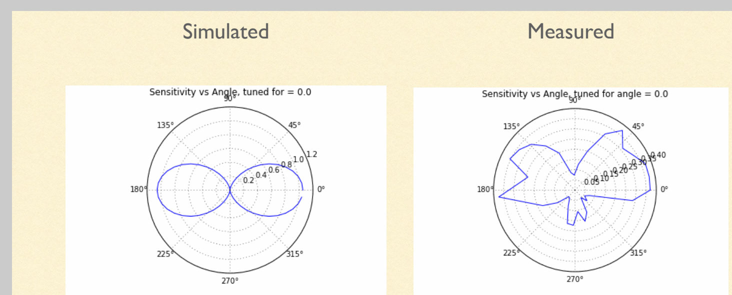


Other projects

- If you REALLY want to do something else...
- Phased array passive radar
  - –http://kaira.sgo.fi/2013/09/passive-radar-with-16-dual-coherent.html
  - –https://www.youtube.com/watch?v=6Wiv
    kA
- Electronically steerable antenna
  - – (Gabe Buckmater EE123 2014)







Other projects

- Weak Signal communications with OOK and incoherent codes
  - –Inspired by JT65
  - –Used for telemetry low-rate
- http://physics.princeton.edu/pulsar/K1JT/JT65.pdf

1,0,0,1,1,0,0,0,1,1,1,1,1,1,0,1,0,1,0,0,0,1,0,1,1,0,0,1,0,0,
0,1,1,1,0,0,1,1,1,1,0,1,1,0,1,1,1,1,0,0,0,1,1,0,1,0,1,0,1,1,
0,0,1,1,0,1,0,1,0,1,0,0,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,1,1,
0,1,0,0,1,0,1,1,0,1,0,1,0,1,0,0,1,1,0,0,1,0,0,1,0,0,0,0,1,1,
1,1,1,1,1,1

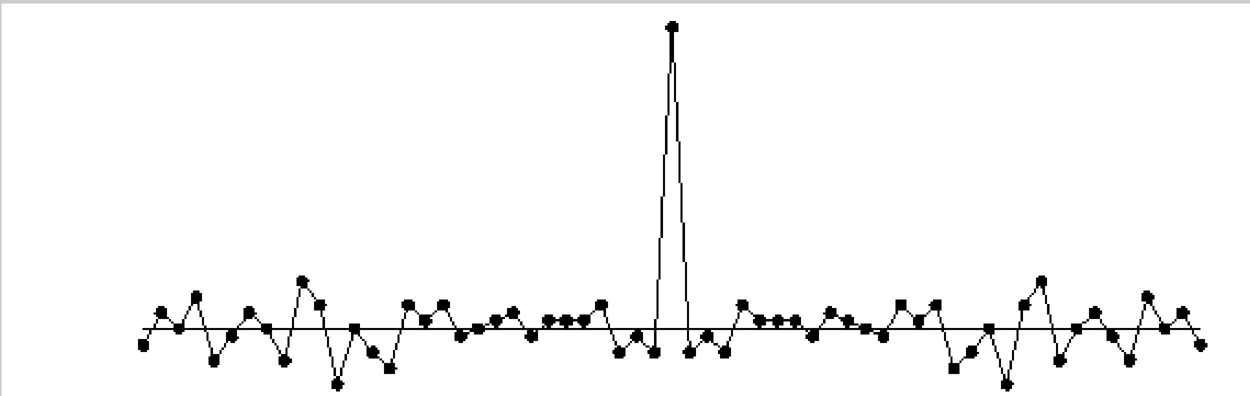
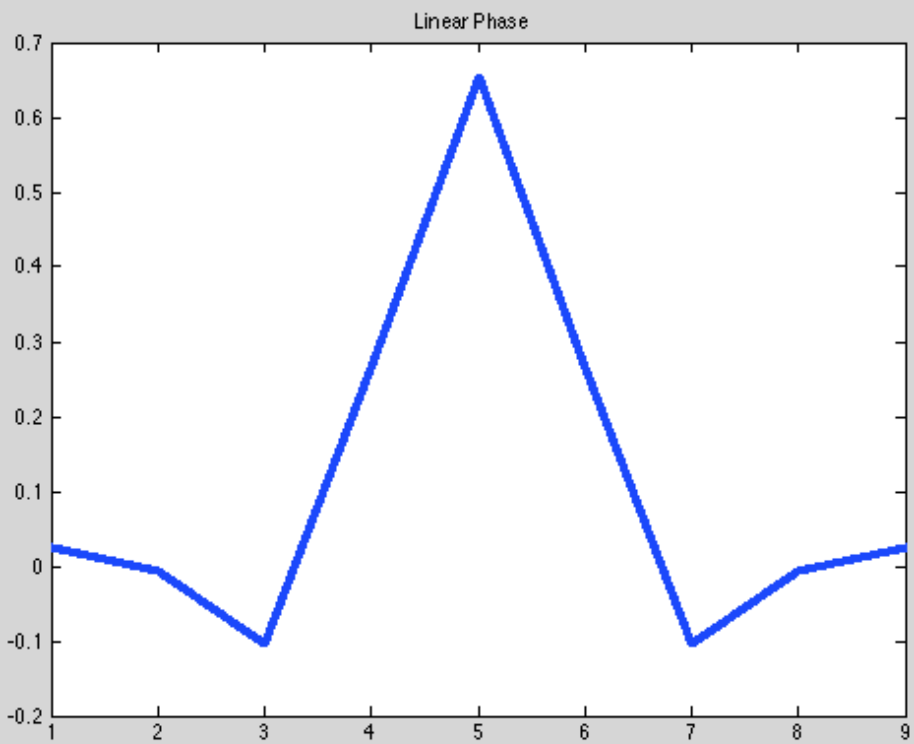
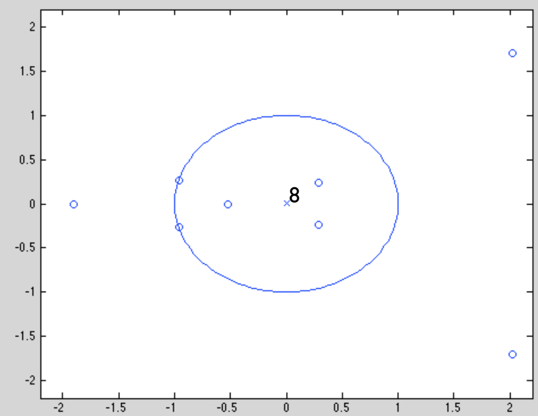


Fig. 3. – The pseudo-random sequence used in JT65 as a "synchronizing vector," and a graphical representation of its autocorrelation function. The isolated central correlation spike serves to synchronize time and frequency between transmitting and receiving stations.

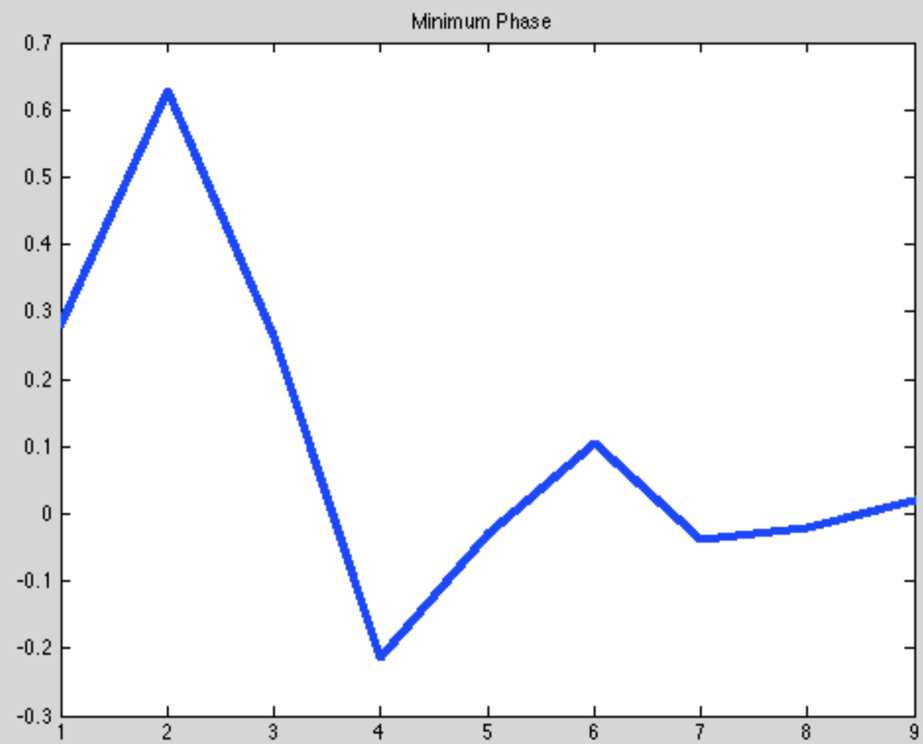
Minimum-phase VS Linear Phase

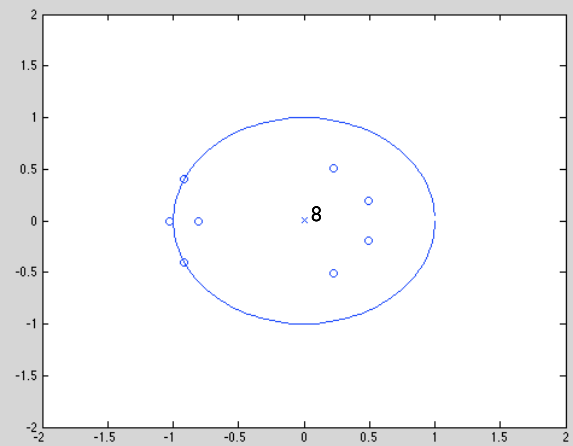
linear phase

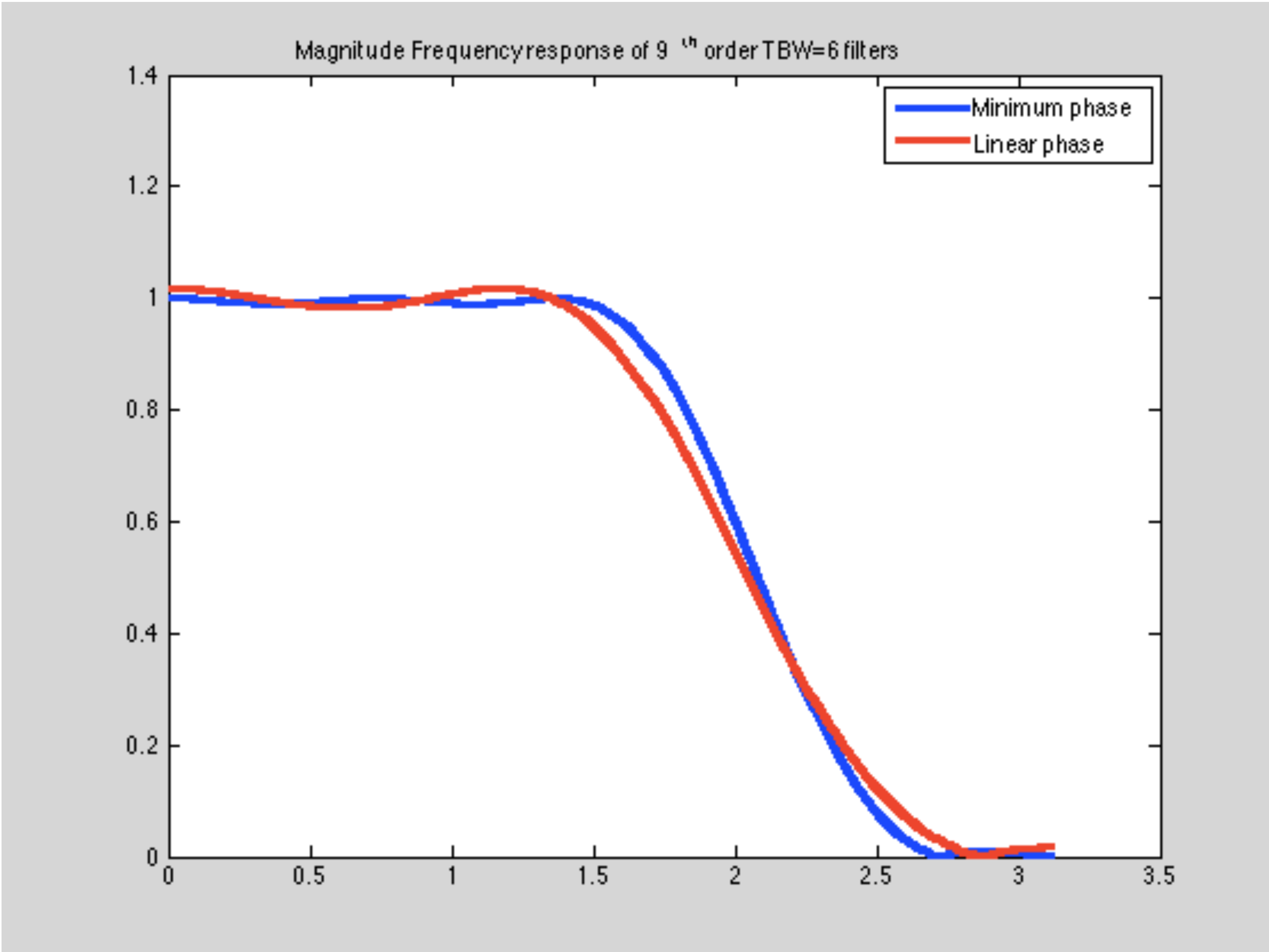




minimum phase







Magnitude Frequency response of 9 th order TBW=6 filters
Minimum phase
Linear phase
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5

Generalized linear-phase systems

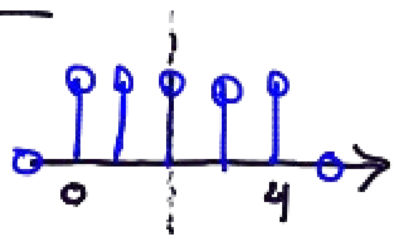
$$H(e^{j\omega}) = \underbrace{A(e^{j\omega})}_{\text{Real, allow sign change}} e^{-j\alpha\omega + j\beta}$$

$$\text{grd}\left[H(e^{j\omega})\right] = \alpha \quad \left(\text{except when } A(e^{j\omega}) \text{ changes sign}\right)$$

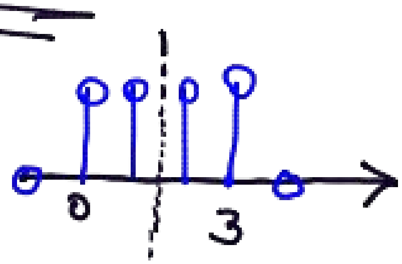
GLP for FIR → MUST have symmetry

$h[n]=h[M-n]$:

Type I (M even)



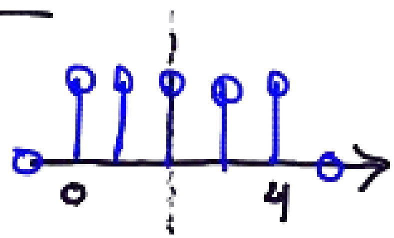
Type II (M odd)

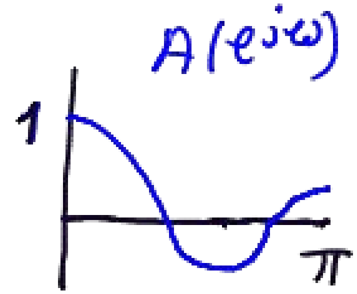


GLP for FIR $\rightarrow$ must have symmetry

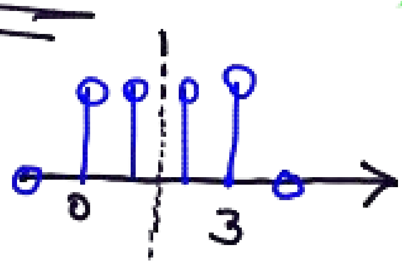
$h[n] = h[M-n]$:

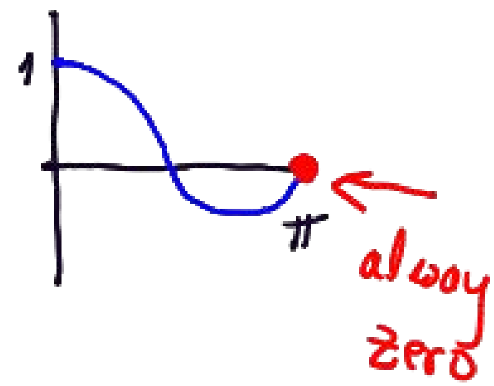
Type I (M even)





Type II (M odd)





GLP for FIR → must have symmetry

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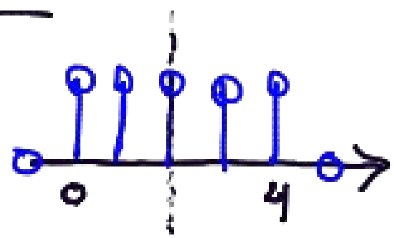
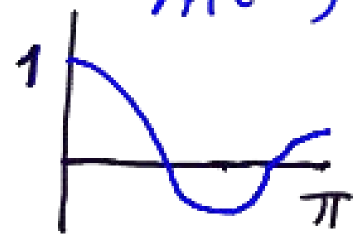
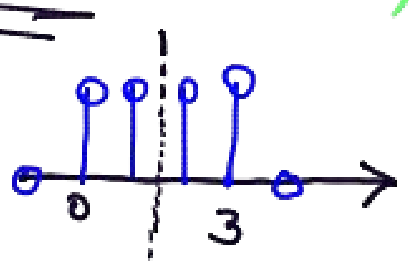
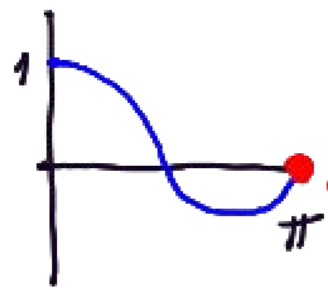




$$A(e^{j\omega}) = h\left[\tfrac{M}{2}\right] + 2\sum_{k=1}^{M/2} h\left[\tfrac{M}{2}-k\right]\cos(\omega k)$$

Type II (M odd)





alway zero

$A(e^{j\omega})$ = In the text

Least Squares

$$\text{argmin}_{\tilde{h}} \quad ||A\tilde{h} - b||^2$$

Solution:

$$\tilde{h} = (A^*A)^{-1}A^*b$$

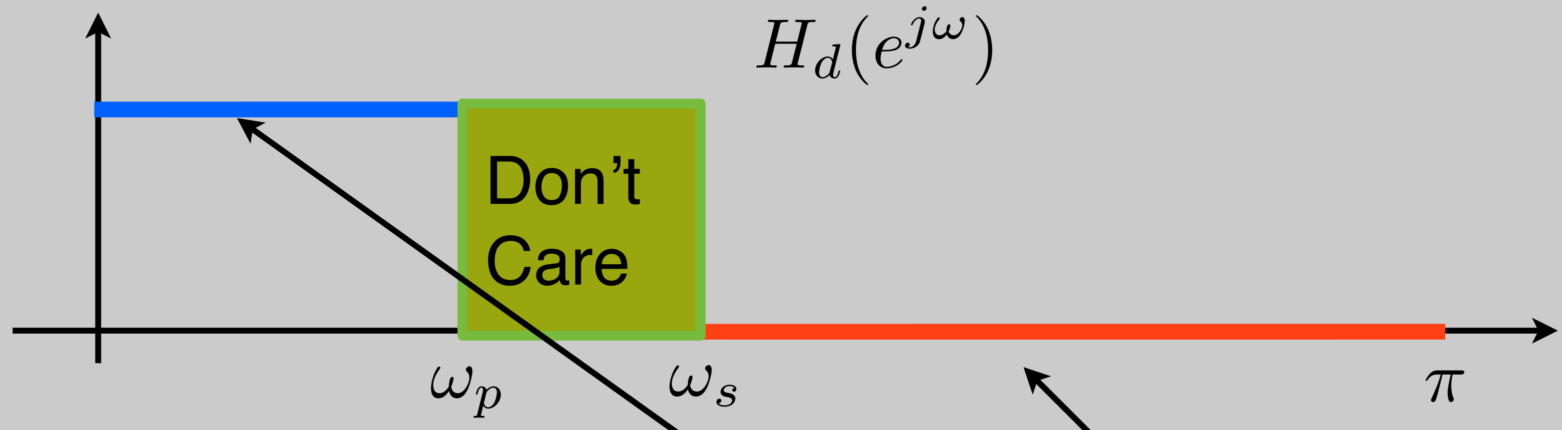
- Result will generally be non-symmetric and complex valued.
- However, if $\tilde{H}(e^{j\omega})$ is real, $\tilde{h}[n]$ should have symmetry!

Design of Linear-Phase L.P Filter

- Suppose:
  - $\tilde{H}(e^{j\omega})$ is real-symmetric
  - M is even (M+1 taps)
- Then:
  - $\tilde{h}[n]$ is real-symmetric around midpoint
- So:

$$\begin{aligned}\tilde{H}(e^{j\omega}) &= \tilde{h}[0] + \tilde{h}[1]e^{-j\omega} + \tilde{h}[-1]e^{+j\omega} \\ &\quad +\tilde{h}[2]e^{-j2\omega} + \tilde{h}[-2]e^{+j2\omega} \dots \\ &= \tilde{h}[0] + 2\cos(\omega)\tilde{h}[1] + 2\cos(2\omega)\tilde{h}[2] + \cdots\end{aligned}$$

Least-Squares Linear-Phase Filter



Given M, ω_P, ω_s find the best LS filter:

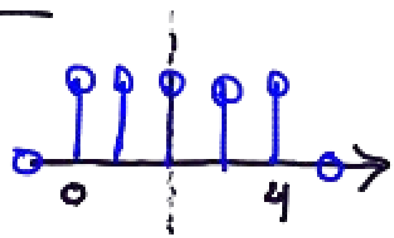
$$A = \begin{bmatrix} 1 & \cdots & 2\cos(\frac{M}{2}\omega_1) \\ \vdots & & \\ 1 & \cdots & 2\cos(\frac{M}{2}\omega_p) \\ 1 & \cdots & 2\cos(\frac{M}{2}\omega_s) \\ \vdots & & \\ 1 & \cdots & 2\cos(\frac{M}{2}\omega_P) \end{bmatrix}$$

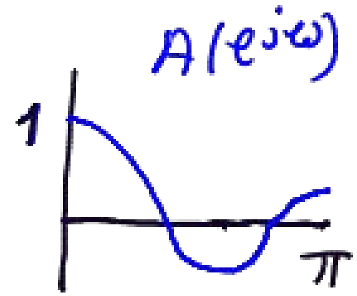
$$b = [1, 1, \cdots, 1, 0, 0, \cdots, 0]^T$$

GLP for FIR → must have symmetry

$h[n] = h[M-n]$:

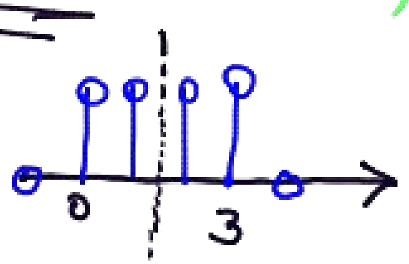
Type I (M even)

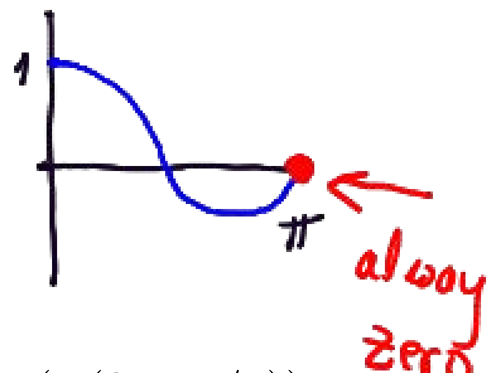




$$A(e^{j\omega}) = h[\tfrac{M}{2}] + 2\sum_{k=1}^{\frac{M}{2}} h[\tfrac{M}{2}-k]\cos(\omega k)$$

Type II (M odd)

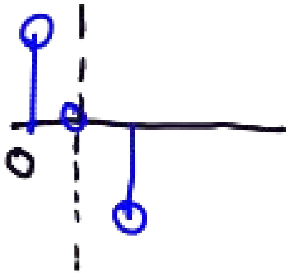


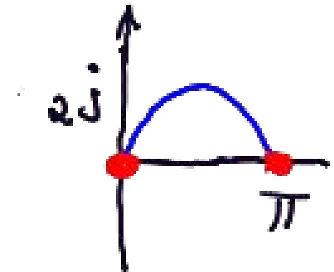


$$A(e^{j\omega}) = \sum_{k=1}^{(M+1)/2} 2h[(M+1)/2-k]\cos(\omega(k-1/2))$$

$$h[n] = -h[M-n]$$

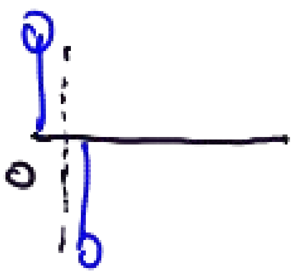
Type III (M even)

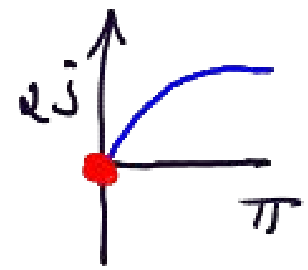




$$A(e^{j\omega}) = j\,2\sum_{k=1}^{M/2} h\left[\tfrac{M}{2}-k\right]\sin(\omega k)$$

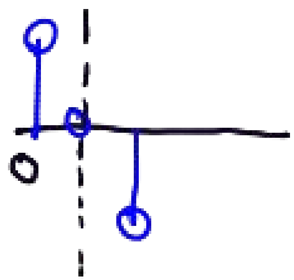
Type IV (M odd)

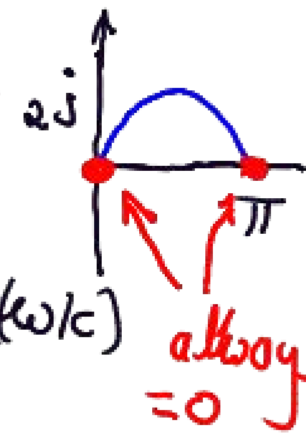




$h[n] = -h[M-n]$

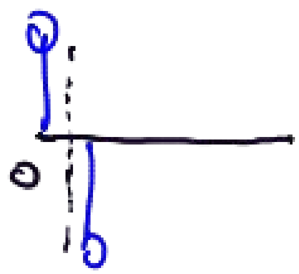
Type III (M even)

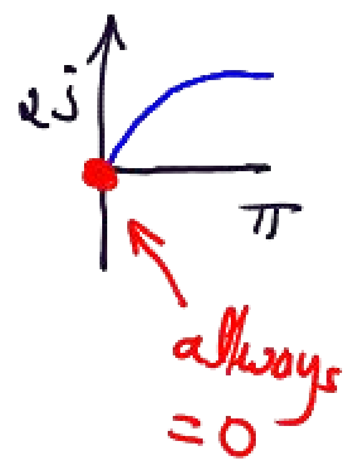




$$A(e^{j\omega}) = j\,2\sum_{k=1}^{M/2} h\left[\tfrac{M}{2}-k\right]\sin(\omega k)$$

Type IV (M odd)





$A(e^{j\omega}) =$ See text

Zeroes of GLP system

Type I, II: $h[n] = h[M-n]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

Zeroes of GLP system

Type I, II: $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

$$= \sum_{n=0}^{M} h[M-n] z^{-n} =$$

Zeroes of GLP system

⑥

Type I, II: $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

$$= \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} h[M-n] z^{M-n}$$

Zeroes of GLP system

⑥

Type I, II: $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

$$= \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} \underbrace{h[M-n]}_{\triangleq k} z^{\overbrace{M-n}^{\triangleq k}}$$

$$= z^{-M} \sum_{k=0}^{M} h[k] z^{k}$$

Zeroes of GLP system

Type I,II: $h[n] = h[n-M]$

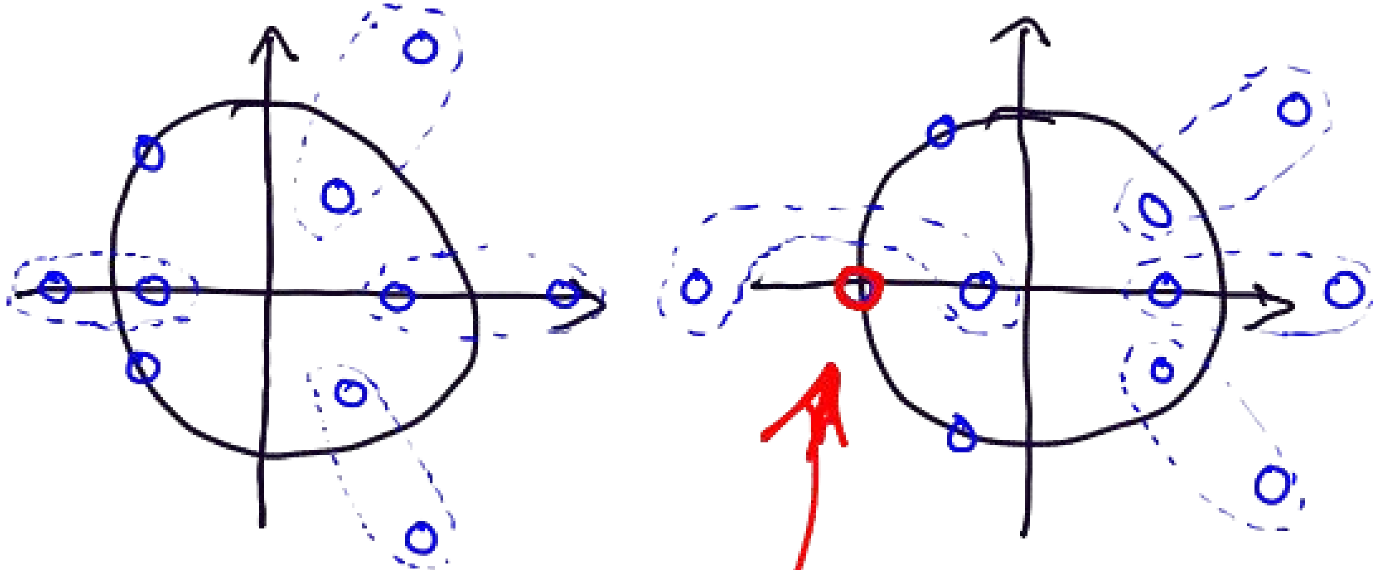
$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

$$= \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} h[\underbrace{M-n}_{\triangleq k}] z^{\underbrace{M-n}_{\triangleq k}}$$

$$= z^{-M} \sum_{k=0}^{M} h[k] z^{k}$$

$$\Longrightarrow \boxed{H(z) = z^{-M} H(z^{-1})}$$

$H(z) = z^{-M} H(z^{-1})$ Type I, II



$H(-1) = 0$ Type II (Never high-pass)

→ FOR GLP, IF $a = re^{j\theta}$ is a zero

$\frac{1}{a^*}$ is also a zero

Zeroes of GLP system

Type I, II: $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

$$= \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} \underbrace{h[M-n]}_{\hat{=}k} z^{\overbrace{M-n}^{\hat{=}k}}$$

$$= z^{-M} \sum_{k=0}^{M} h[k] z^{k}$$

$$\Rightarrow \boxed{H(z) = z^{-M} H(z^{-1})}$$

for type II:

$$H(-1) = (-1)^{\overset{\text{odd}}{M}} H(-1) = -H(-1) \Rightarrow \boxed{H(-1) = 0}$$

⑧

similarly, can show for

type III, IV

$$\boxed{H(z) = -z^{-M} H(z^{-1})}$$

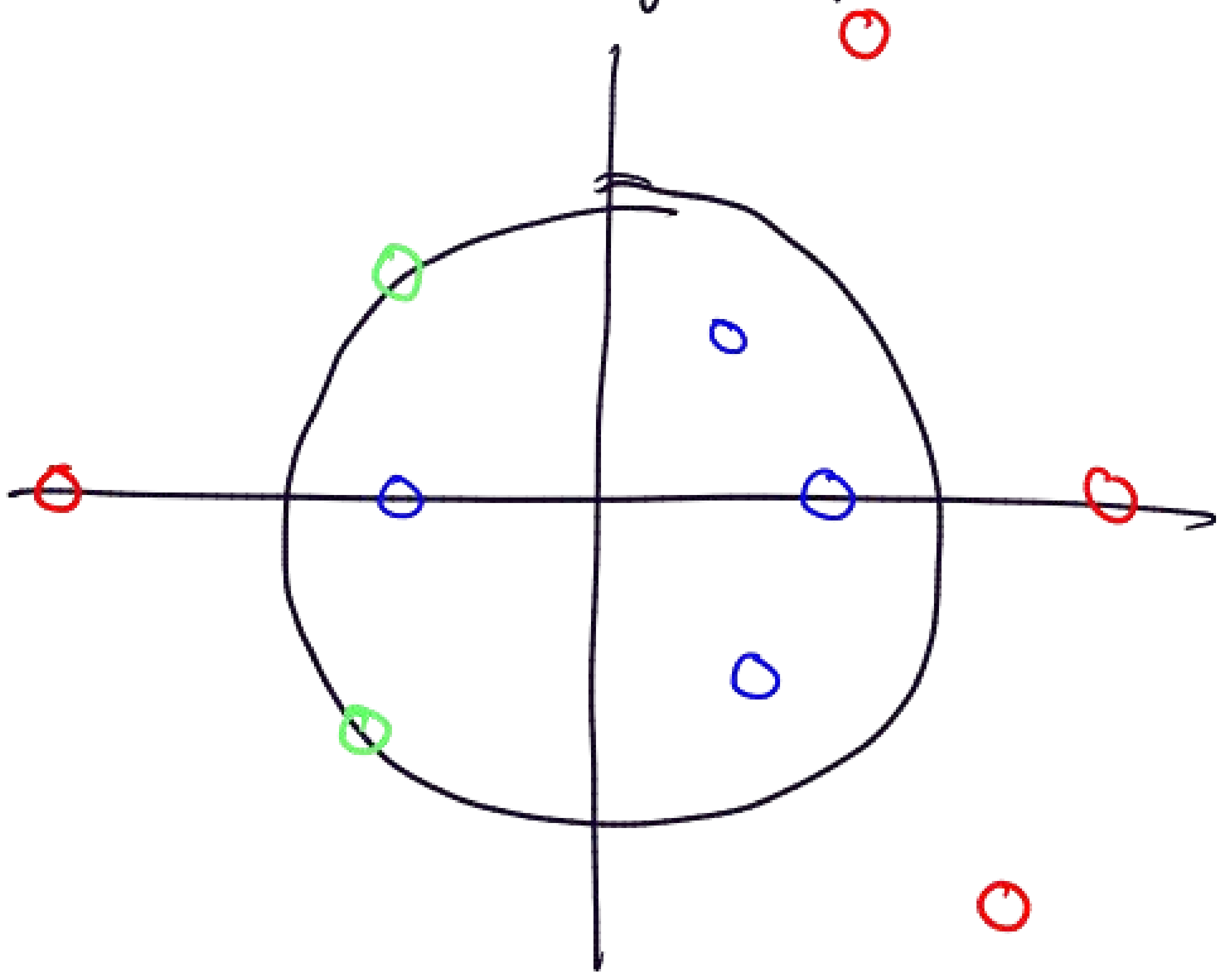
$\boxed{H(1) = 0}$ → Never low-pass

for type III

$\boxed{H(-1) = 0}$ only band pass

Relation of FIR GLP to min-phase systems

9



$$H(z) = H_{min}(z)\, H_{max}(z)\, H_{uc}(z)$$

H_{min}: minimum phase

H_{max}: maximum phase