EE123 Spring 2015
Discussion Section 10

Giulia Fanti (slides by Frank Ong)

Pole-zero

	Magnitude	Phase	Group delay
Poles	push up	go down	delay (positive)
Zeros	push down	go up	advance (negative)

Problem 1

Consider $$h_1[n] = \begin{cases} 1 & |n| \leq 1 \\ 0 & else \end{cases}$$

- Compute its z-transform and plot the poles and zeros
- Sketch its magnitude response

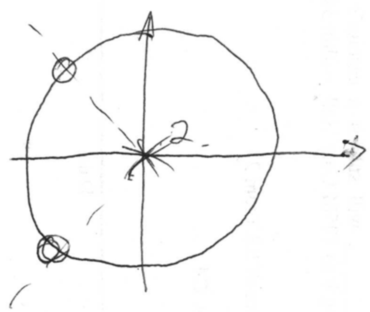
What about the pole zero diagram for

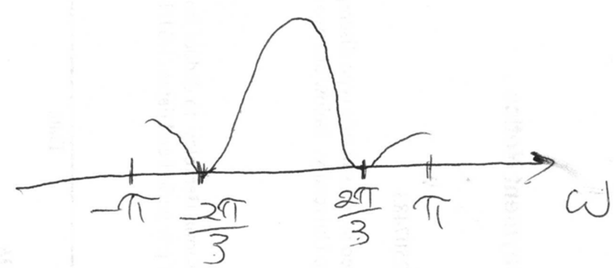
$$h_1[n] = \begin{cases} 1 & |n| \leq 3 \\ 0 & else \end{cases}$$

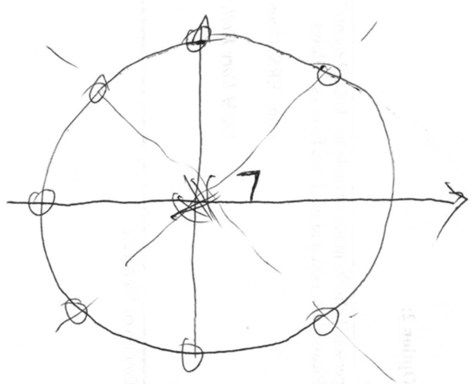
Solution 1

$$H_3(z) = z + 1 + z^{-1} = z(1 + z^{-1} + z^{-2})$$

$$= z\,\frac{(1 - z^{-3})}{(1 - z^{-1})}$$







What is wrong with these pole-zero diagrams?

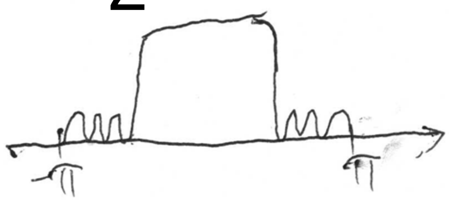
There should only be 1 and 3 poles at z=0, respectively

Problem 2

Match the following magnitude response to their pole-zero plot

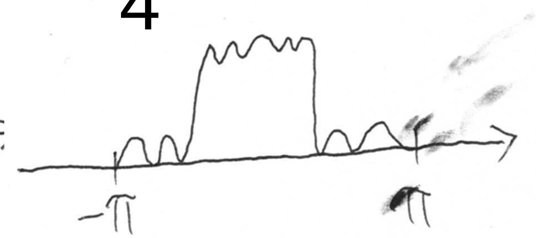
1

2

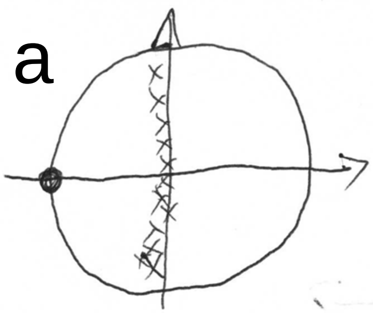


3

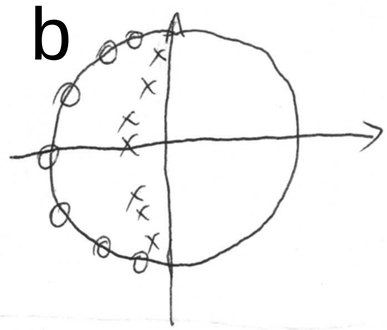
4



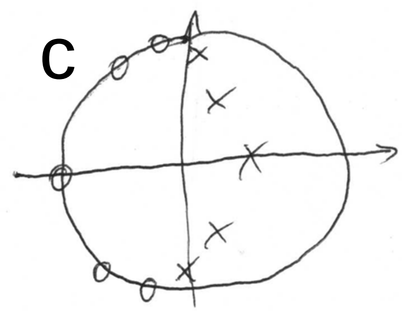
a



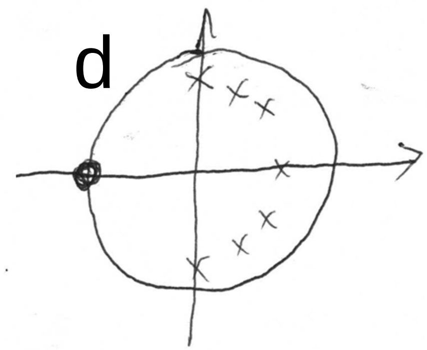
b



c



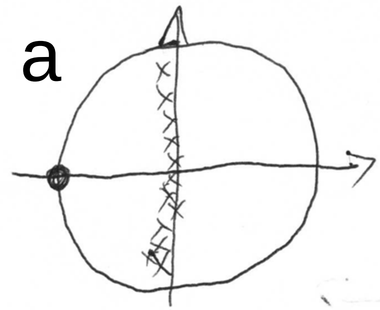
d



Solution 2

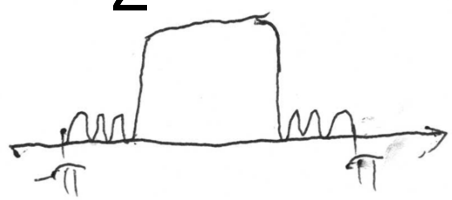
1

a

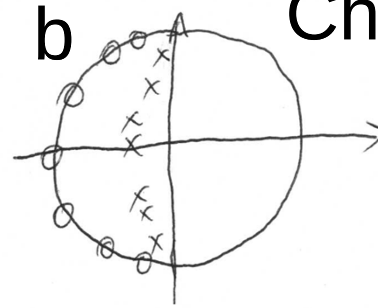


Butterworth

2



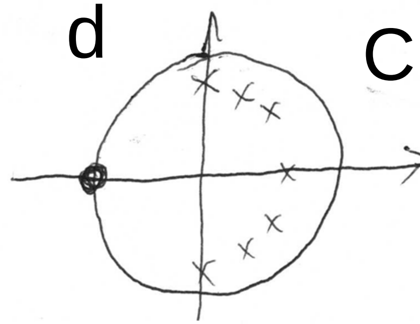
b



Chebyshev Type II

3

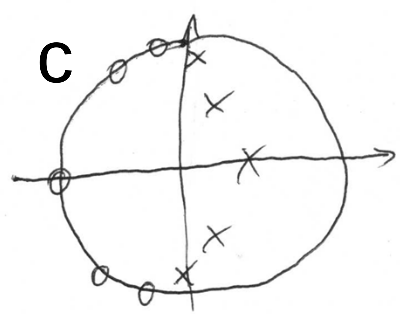
d



Chebyshev Type I

4

c



Elliptic

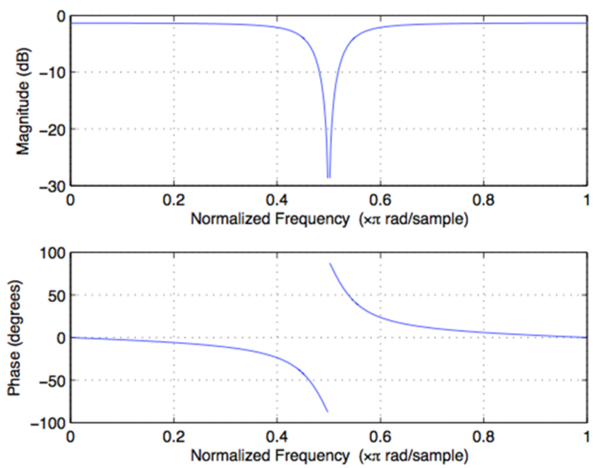
Problem 3

b) (10 points) Match the frequency responses below with the transfer functions:

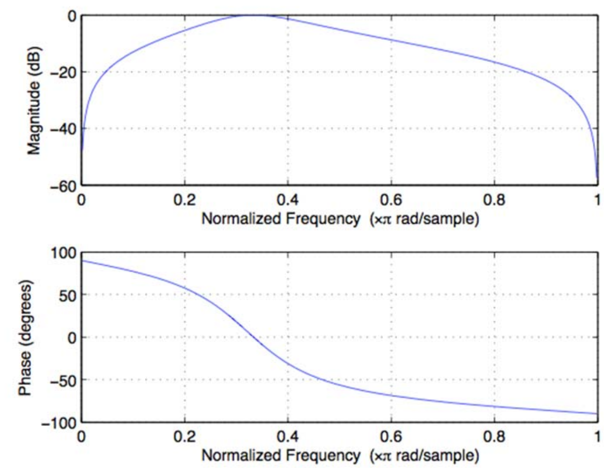
$$H_1(z) = 0.25\frac{1-z^{-2}}{1-0.75z^{-1}+0.5z^{-2}} \quad H_2(z) = 0.75\frac{1+z^{-2}}{1+0.75z^{-2}}$$

$$H_3(z) = \frac{\frac{4}{9}+\frac{2}{3}z^{-1}+z^{-2}}{1+\frac{2}{3}z^{-1}+\frac{4}{9}z^{-2}} \qquad H_4(z) = 0.2(1+z^{-1}+z^{-2}+z^{-3}+z^{-4}).$$

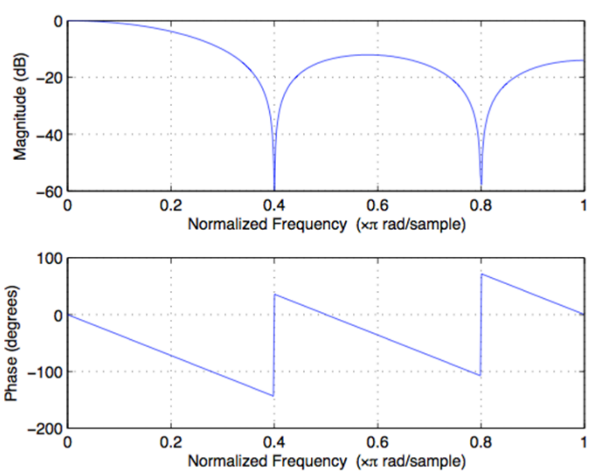
A



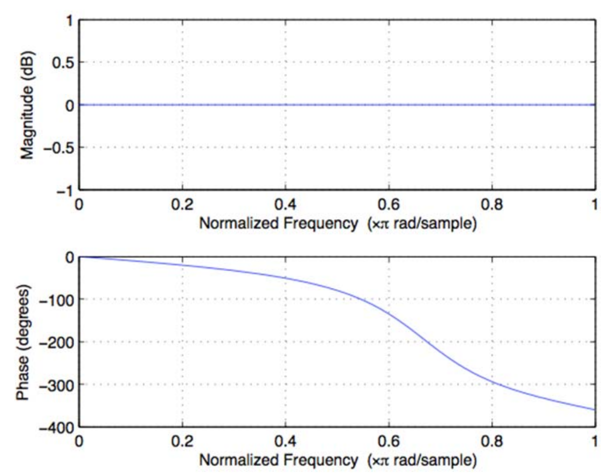
B



C



D



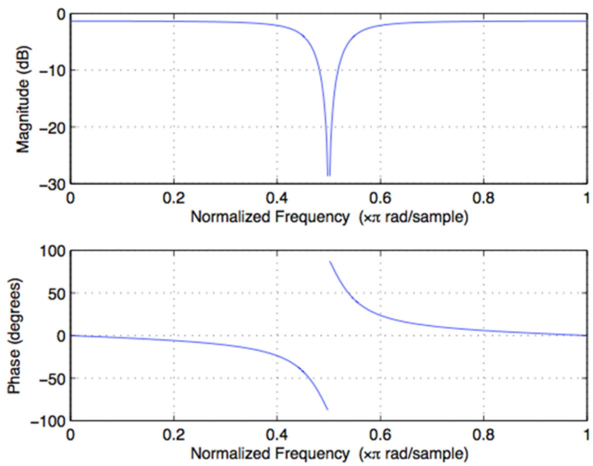
Solution 3

b) (10 points) Match the frequency responses below with the transfer functions:

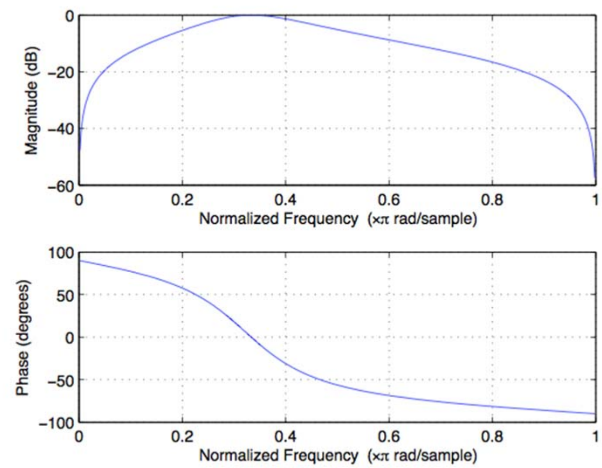
B $H_1(z) = 0.25\frac{1-z^{-2}}{1-0.75z^{-1}+0.5z^{-2}}$ $H_2(z) = 0.75\frac{1+z^{-2}}{1+0.75z^{-2}}$ A

D $H_3(z) = \frac{\frac{4}{9}+\frac{2}{3}z^{-1}+z^{-2}}{1+\frac{2}{3}z^{-1}+\frac{4}{9}z^{-2}}$ $H_4(z) = 0.2(1+z^{-1}+z^{-2}+z^{-3}+z^{-4})$. C

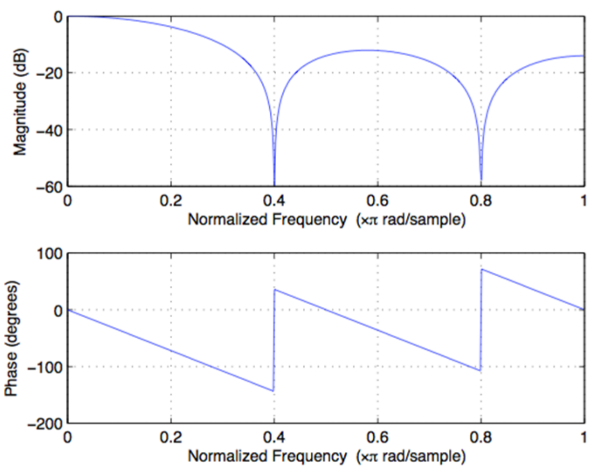
A



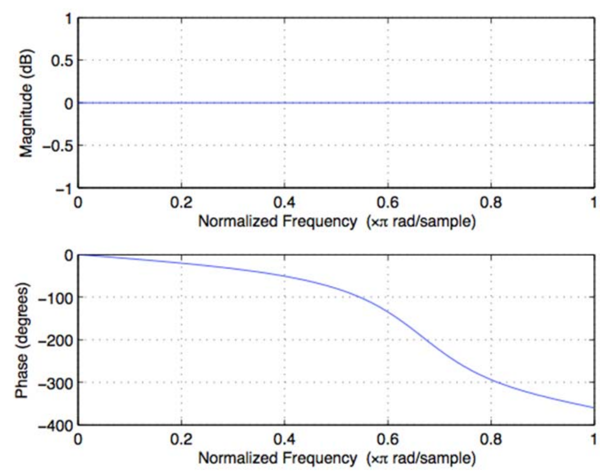
B



C



D



Problem 4

Consider the problem of reconstructing the signal from only its Fourier magnitude $|H(e^{jw})|^2$

a How does the pole-zero plot of $|H(e^{jw})|^2$ look like compared to $H(e^{jw})$?
Hint: the z-transform of $|H(e^{jw})|^2$ is $H(z)H^*(1/z)$

b If the system $h[n]$ is causal and stable, can you uniquely recover $h[n]$ from $|H(e^{jw})|^2$

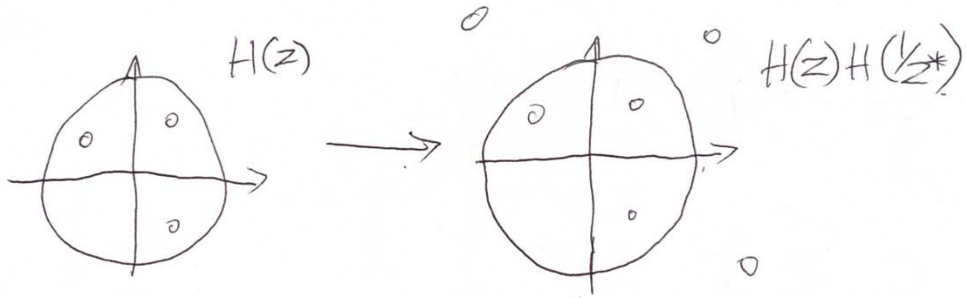
c Assume that $h[n]$ is causal and stable and that, in addition, you know that the system function has the form

$$H(z) = \frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

for some finite a_k. Can you uniquely recover $h[n]$ from $|H(e^{jw})|^2$

Solution 4

a)



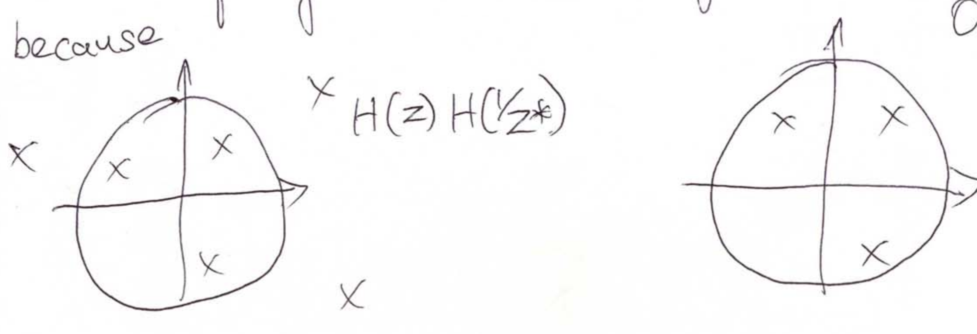
b) h[n] causal and stable does not help

The above example is a counterexample.

c) h[n] causal and stable + all pole system

can uniquely recover the signal

because



Problem 5

Consider the linear time-invariant discrete-time system represented by the rational system function

$$H(z) = \frac{1}{1 - \alpha z^{-N}}$$

a) How many poles are there in this system?
b) Write the corresponding difference equation and sketch the system (i.e., adders, multipliers, and delays).
c) If the sampling rate is 16 kHz, $N = 16000$, $\alpha = .5$, and the input is a short speech segment of someone saying "ba", what would the output sound like (assuming all the necessary machinery like antialiasing filters, A/D, D/A, etc.)?
d) For $N=4$, α slightly less than 1, sketch the frequency response.

Solution 5

a) $$H(z) = \frac{1}{1 - \alpha z^{-N}}$$

Poles: $z_k = \alpha \cdot e^{j\frac{2\pi}{N}k}$ $\quad k = 0, \cdots N-1$

N poles

b) $$y[n] - \alpha \cdot y[n-N] = x[n]$$

Solution 5

c) Assuming "ba" lasts less than 1 second.

that means it ~~lasts~~ takes less than

16000 samples time. Since $y[n] = x[n] + \alpha \cdot y[n-16000]$

$\Rightarrow$ ba ba ba ba ba ba . . . –

d)