EE123 Spring 2015
Discussion Section 11

Frank Ong

Plan

- Phase response
- System analysis

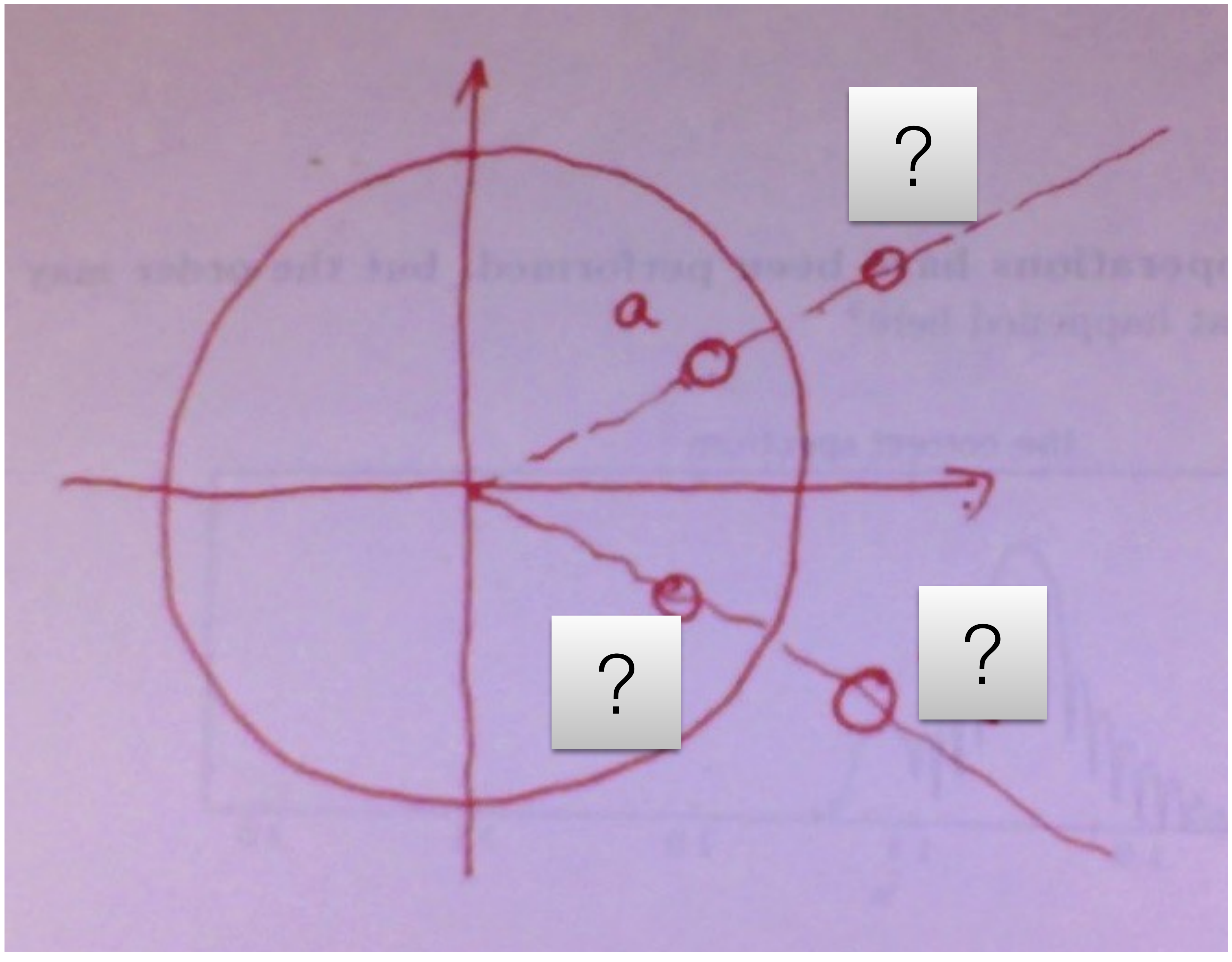
Why do we care?

Linear difference equations :

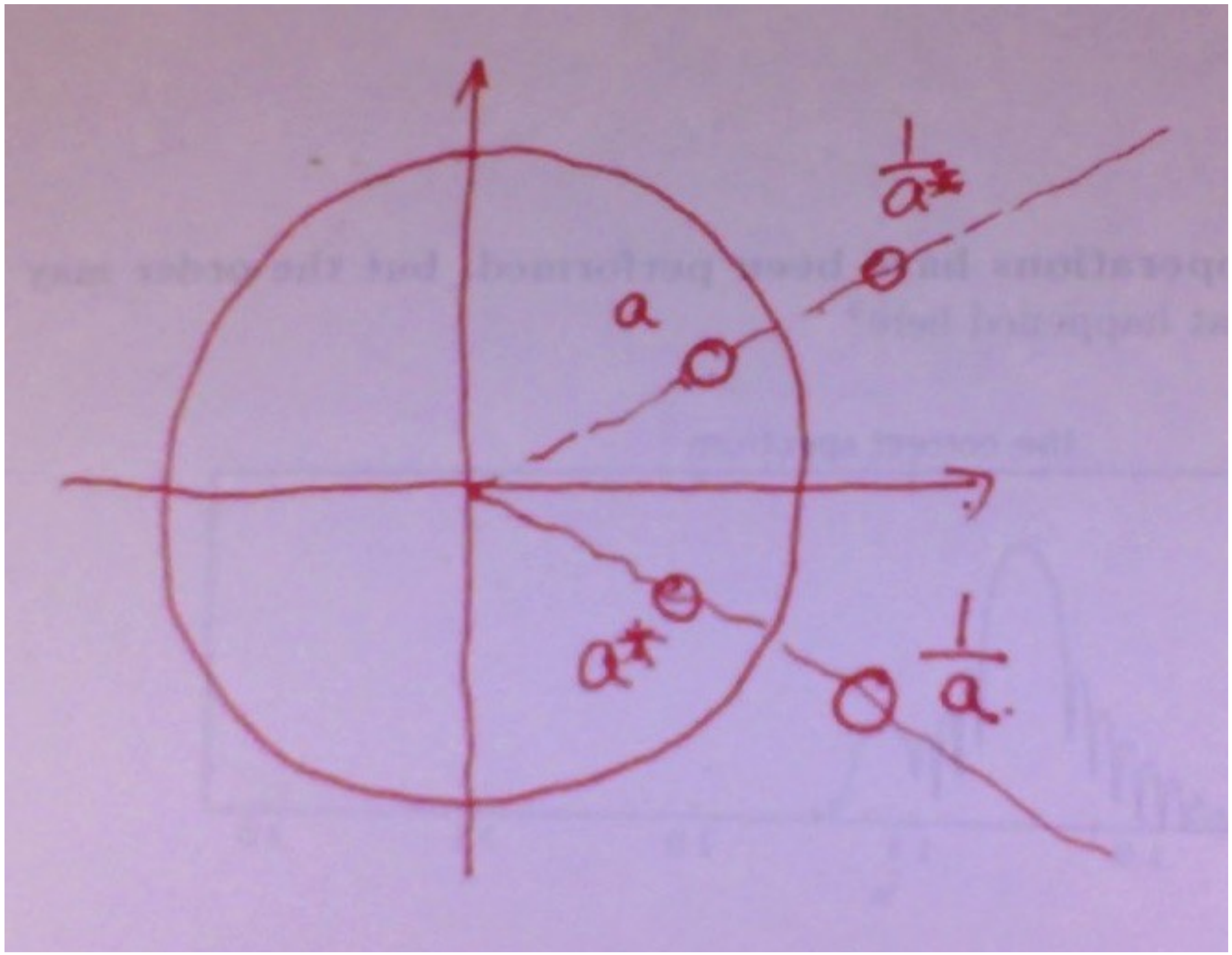
$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \ldots + a_N z^{-N}} = \frac{b_0}{a_0} \frac{\prod_{k=1}^{M}(1 - c_k z^{-1})}{\prod_{k=1}^{N}(1 - d_k z^{-1})}$$

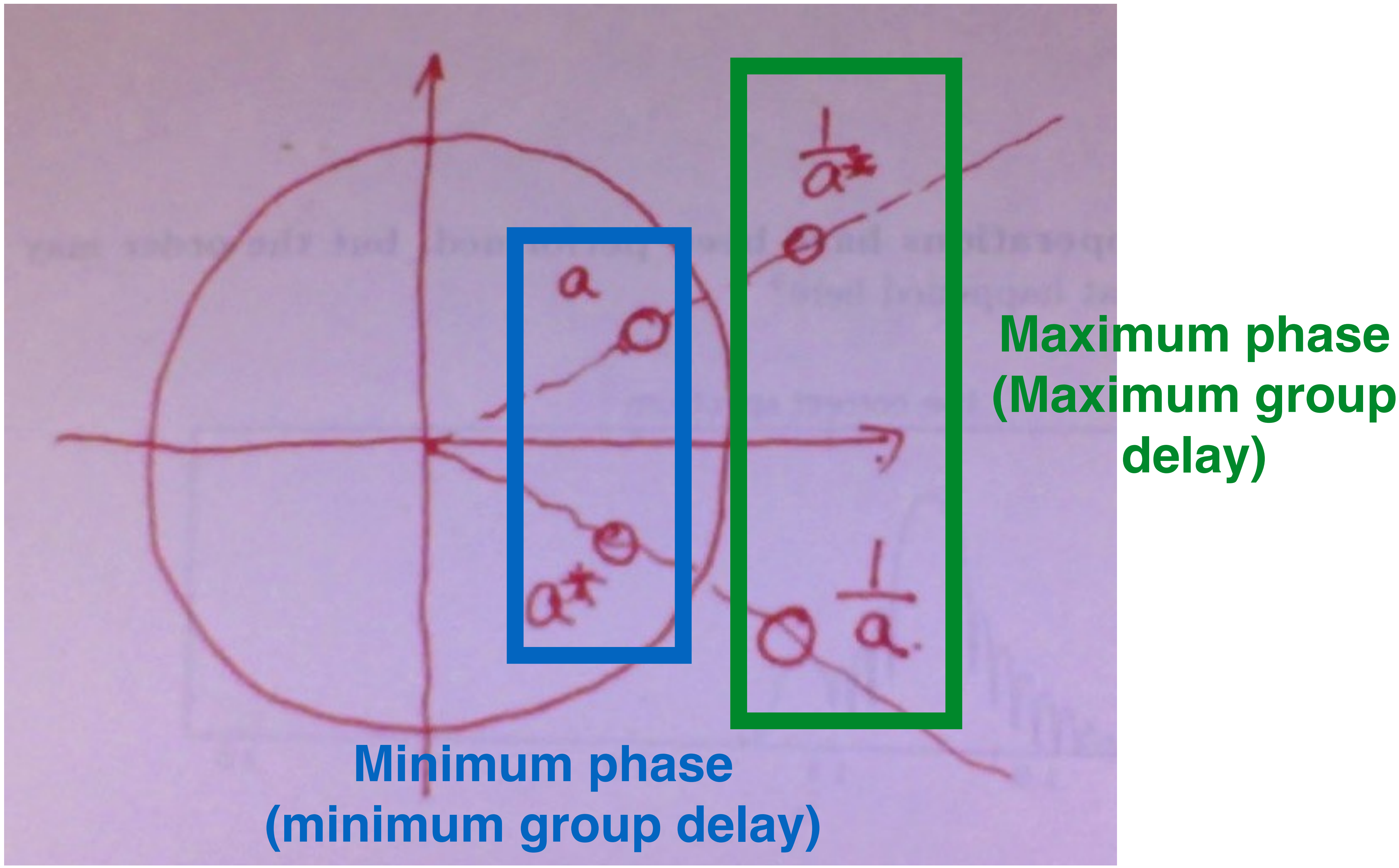
Flipping poles and zeros



Flipping poles and zeros



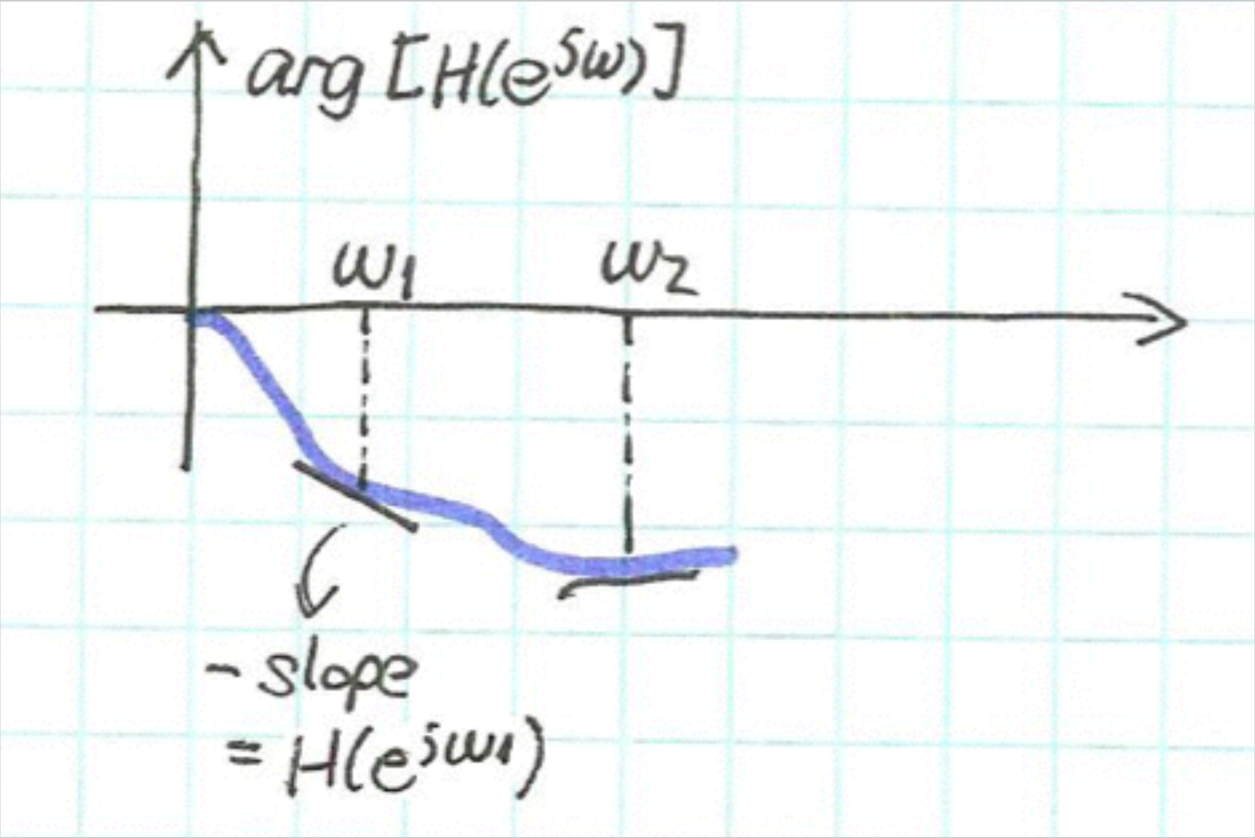
Flipping poles and zeros



Group delay

To characterize general phase response, look at the group delay:

$$\mathrm{grd}[H(e^{j\omega})] = -\frac{d}{d\omega}\{\arg[H(e^{j\omega})]\}$$



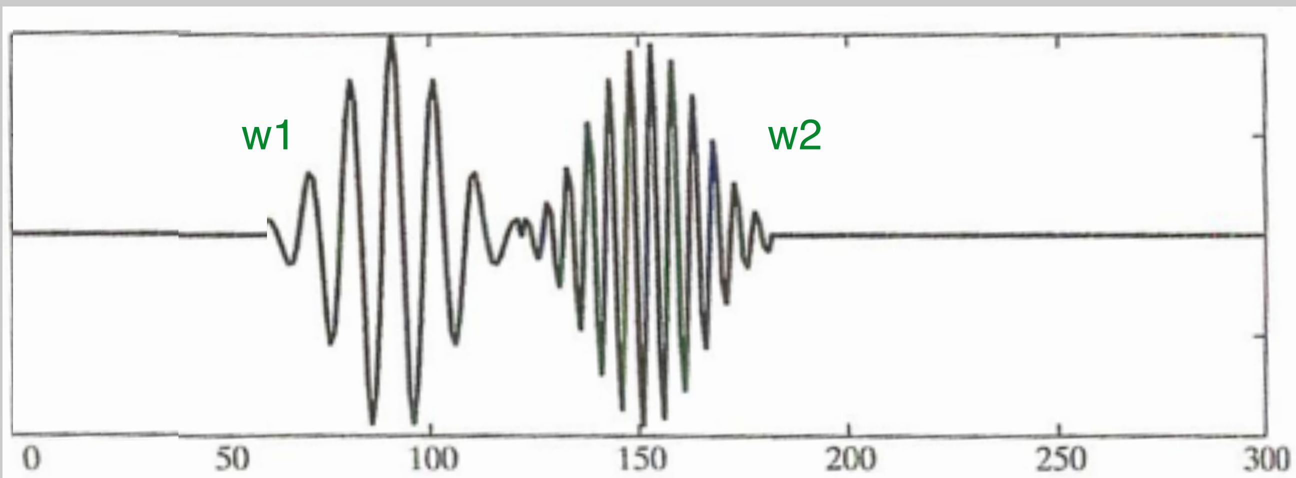
For narrowband signals, phase response looks like a linear phase

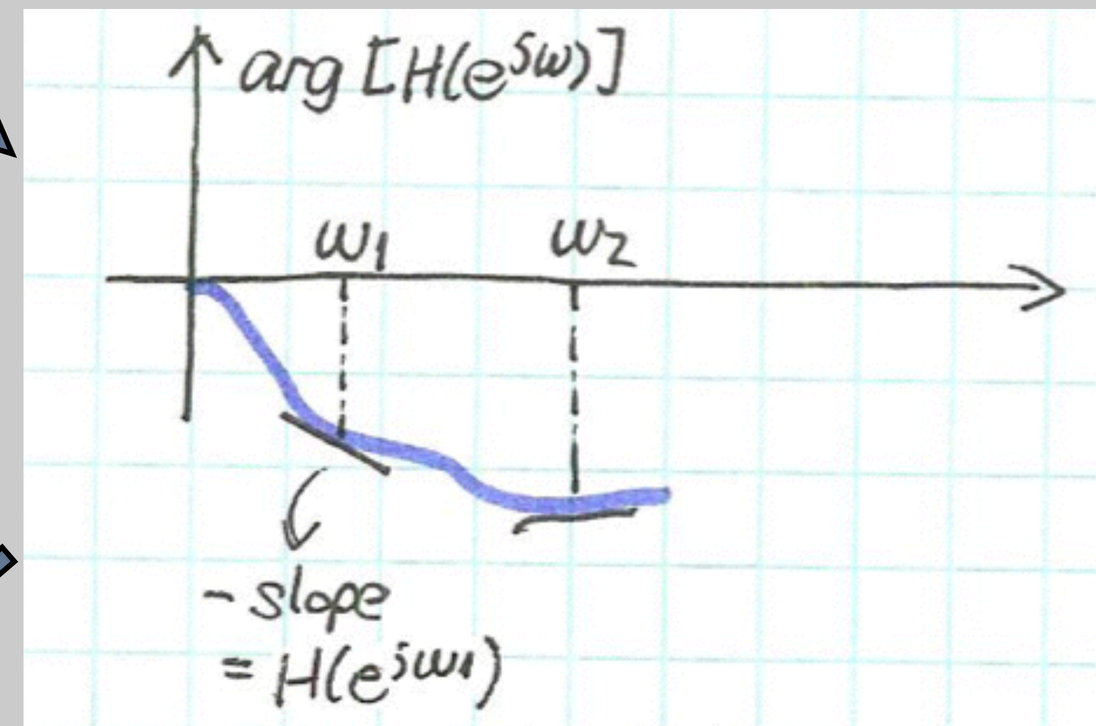
Can be negative!

Group delay

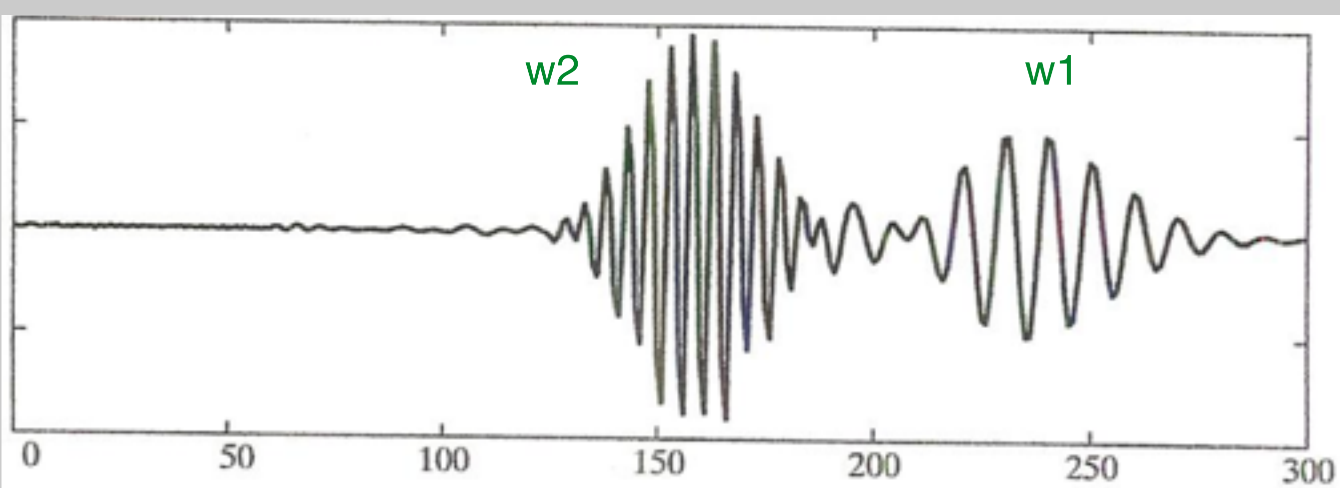
$$\text{grd}[H(e^{j\omega})] = -\frac{d}{d\omega}\{\arg[H(e^{j\omega})]\}$$

Input





Output



For narrowband signals, phase response looks like a linear phase

Faster-than-light effects and negative group delays in optics and electronics, and their applications

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Abstract

Recent manifestations of apparently faster-than-light effects confirmed our predictions that the group velocity in transparent optical media can exceed c. Special relativity is not violated by these phenomena. Moreover, in the electronic domain, the causality principle does not forbid negative group delays of analytic signals in electronic circuits, in which the peak of an output pulse leaves the exit port of a circuit *before* the peak of the input pulse enters the input port. Fur-

All pass system

$$H_{\mathrm{ap}}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}$$

$$|H_{\mathrm{ap}}(e^{j\omega})| = 1$$

All pass system

$$H_{ap}(z) = \frac{z^{-1} - a^*}{1 - a z^{-1}}$$

$$|H_{ap}(e^{j\omega})| = 1$$

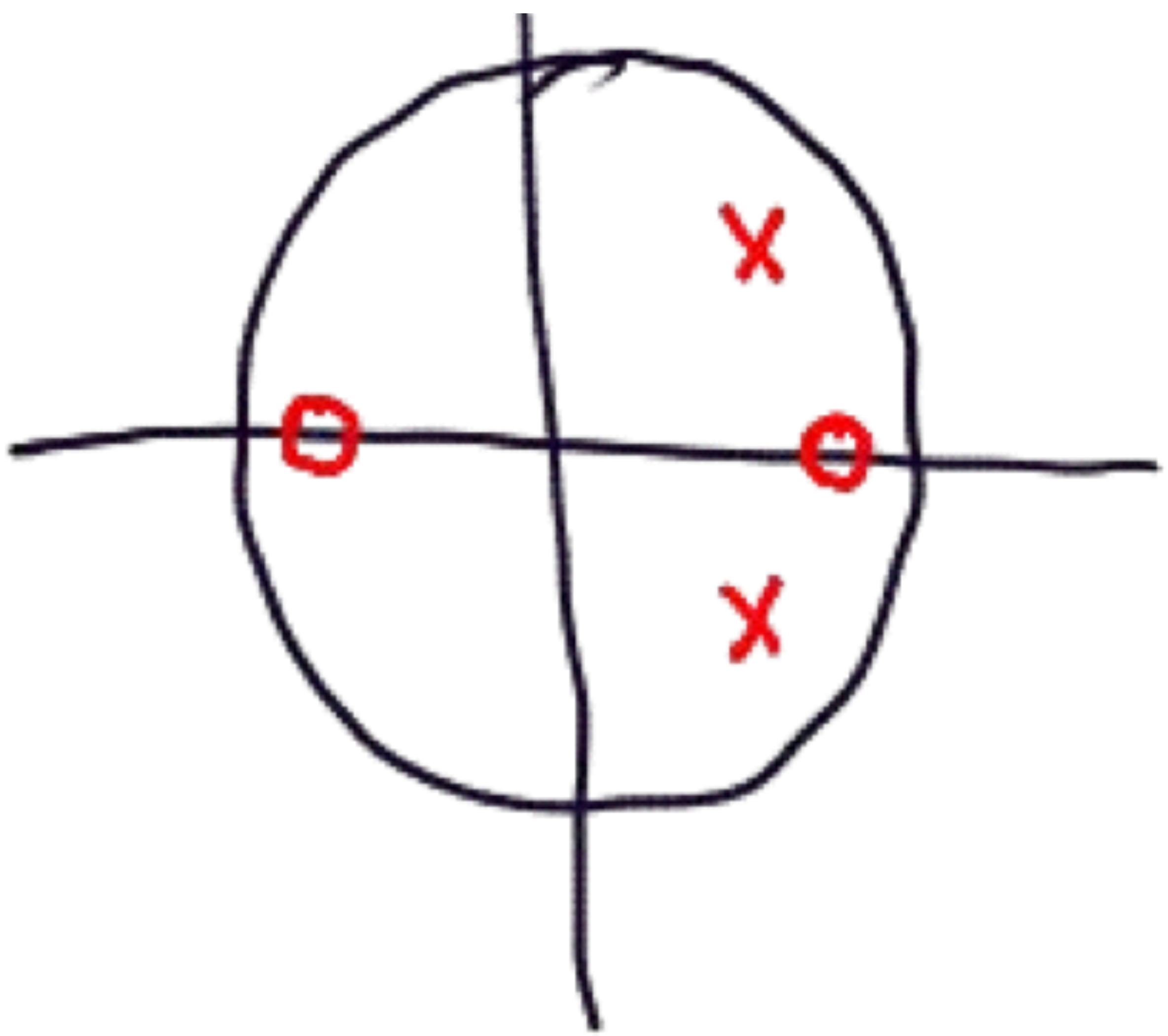
Stable/causal

$$\text{grd}[H_{ap}(e^{j\omega})] \geq 0,$$

$$\arg[H_{ap}(e^{j\omega})] \leq 0 \qquad \text{for } 0 \leq \omega < \pi.$$

Minimum phase system

- Poles and zeros inside the unit circle



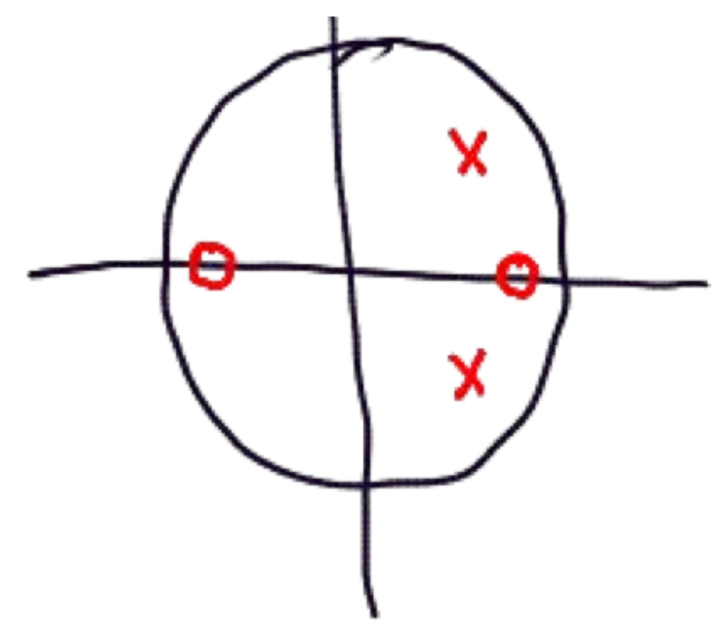
Minimum phase system

The Minimum Group-Delay Property

$$\text{grd}[H(e^{j\omega})] = \text{grd}[H_{\text{min}}(e^{j\omega})] + \text{grd}[H_{\text{ap}}(e^{j\omega})].$$

The Minimum Phase-Lag Property

$$\arg[H(e^{j\omega})] = \arg[H_{\text{min}}(e^{j\omega})] + \arg[H_{\text{ap}}(e^{j\omega})].$$



The Minimum Energy-Delay Property

$\forall h$ such that $|H(e^{j\omega})| = |H_{\text{min}}(e^{j\omega})|$.

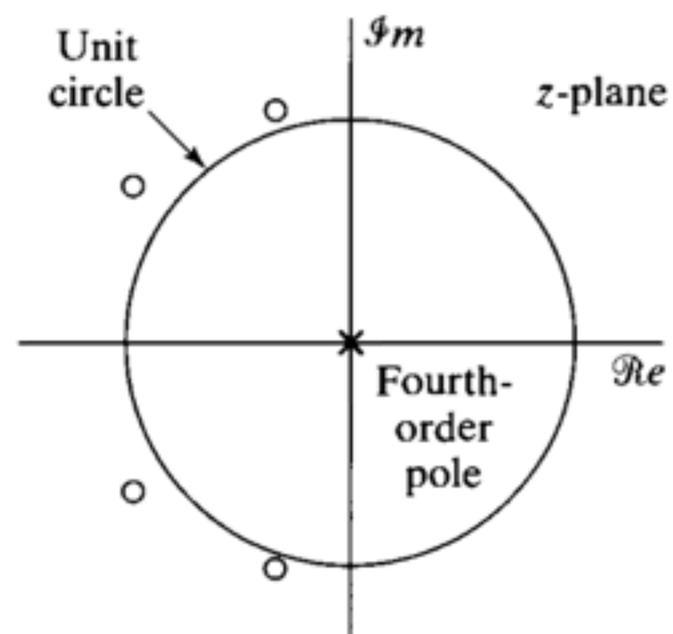
$$\sum_{m=0}^{n} |h[m]|^2 \leq \sum_{m=0}^{n} |h_{\text{min}}[m]|^2$$

All pass/minimum phase decomposition

$$H(z) = H_{min}(z) \cdot H_{ap}(z)$$

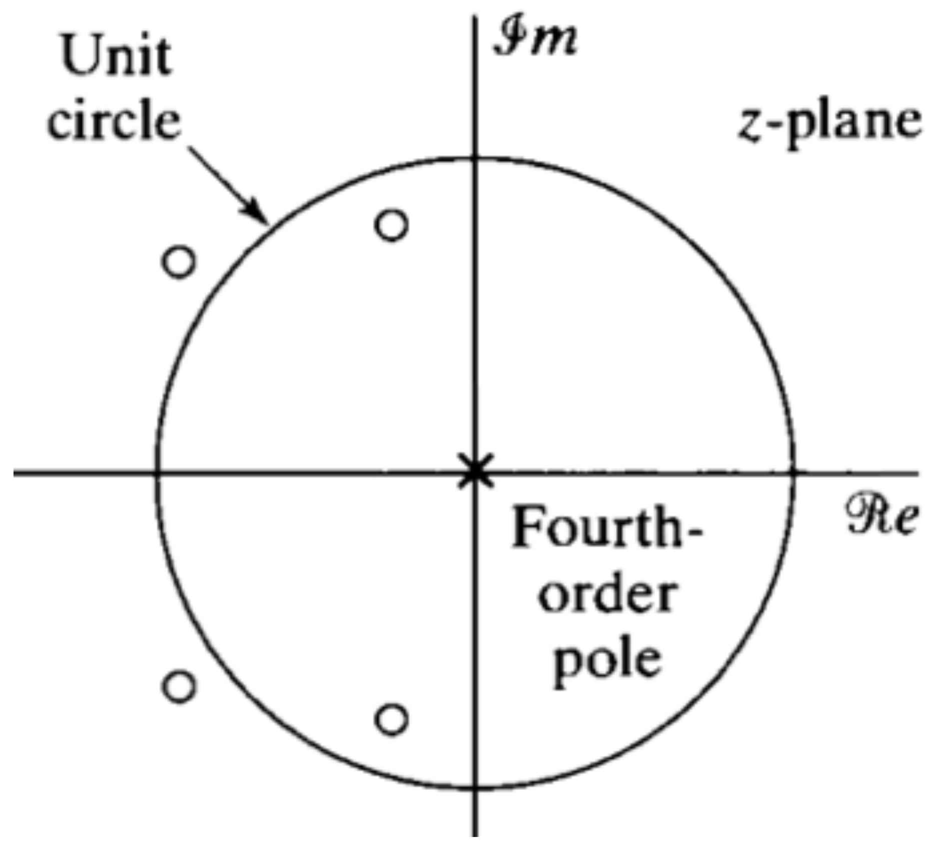
1. Pick zeros outside the unit circle. Flip them inside as poles. Put them together as H_{ap}

2. Construct H_{min} taking in the zeros and poles inside the unit circle and compensating the poles created by H_{ap}



Problem 1

- Do minimum phase/ all pass decomposition:



1. Pick zeros outside the unit circle. Flip them inside as poles. Put them together as H_{ap}
2. Construct H_{min} taking in the zeros and poles inside the unit circle and compensating the poles created by H_{ap}

Problem 2

5. Given the filters:

$$H_1(z) = 1 + 16z^{-4} \quad H_2(z) = \frac{-1 + 8z^{-3}}{8 - z^{-3}} \quad H_3(z) = \frac{1 + 0.81z^{-2}}{1 - 0.25z^{-2}},$$

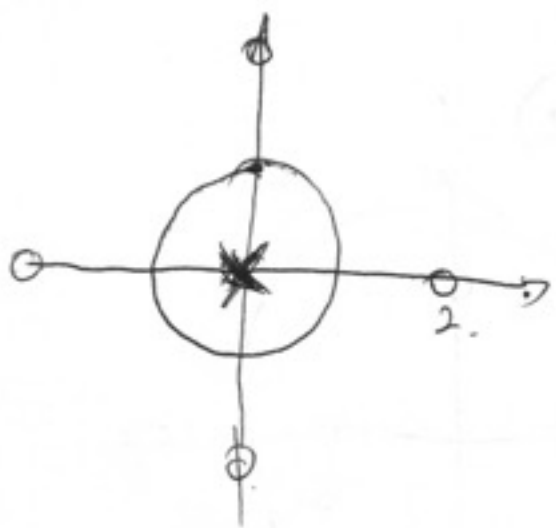
a) (10 points) indicate which of the following properties apply: stable, IIR, FIR, minimum phase, all-pass.

b) (5 points) For the non-minimum phase filter(s) above, give another filter that is minimum phase, but that has the same magnitude response.

along with the all pass/minimum phase decomposition

Solution 2

$H_1(z) = 1 + 16z^{-4}$

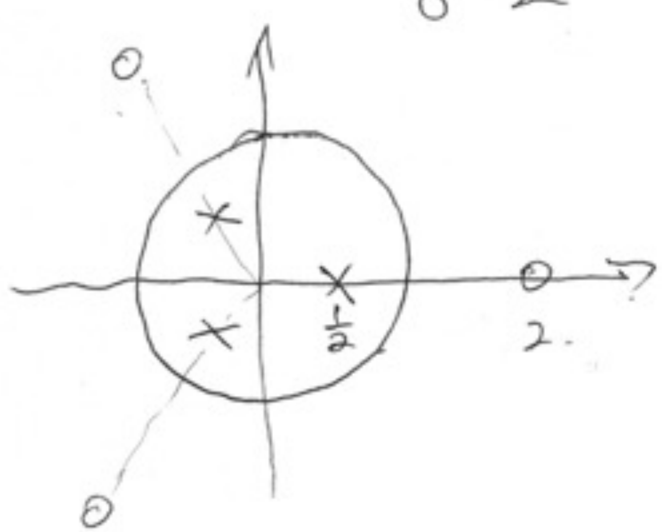


All zeros ⇒ **Stable**

⇒ **FIR**

⇒ **Maximum Phase**

$H_2(z) = \dfrac{-1 + 8z^{-3}}{8 - z^{-3}}$



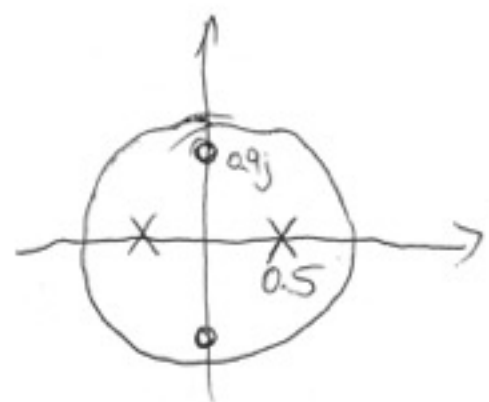
Stable

IIR

All Pass

Solution 2

$$H_3(z) = \frac{1+0.81z^{-2}}{1-0.25z^{-2}}$$

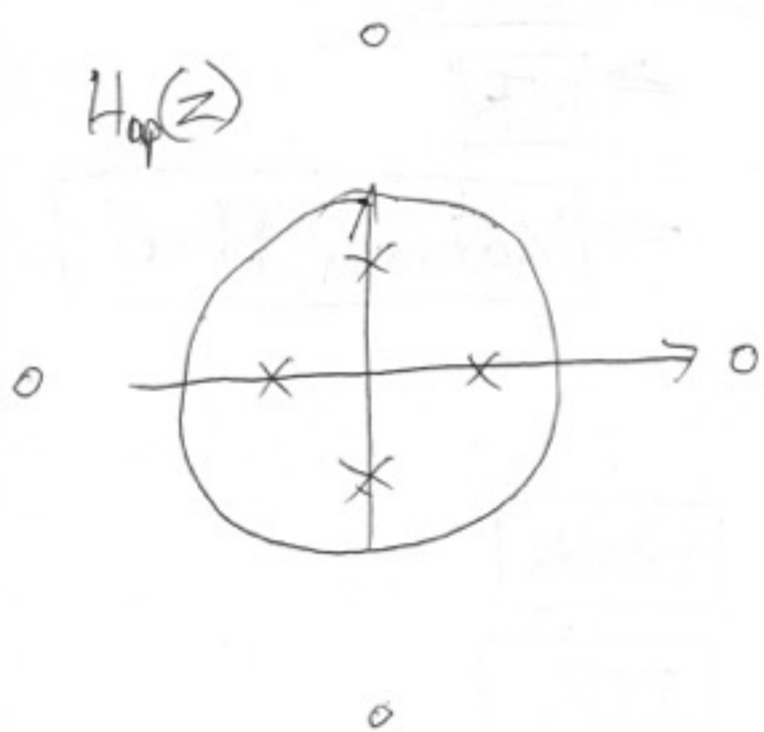


Stable

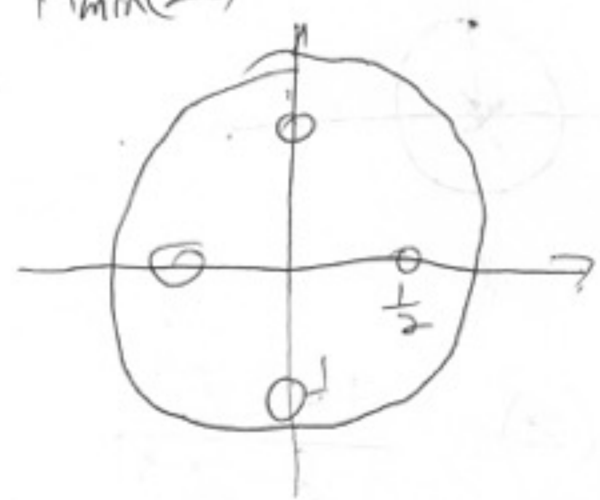
IIR

Minimum Phase

$H_{ap}(z)$



$H_{min}(z)$



Problem 3

5.48. Figure P5.48-1 shows the pole–zero plots for three different causal LTI systems with real impulse responses. Indicate which of the following properties apply to each of the systems pictured: stable, IIR, FIR, minimum phase, all-pass, generalized linear phase, positive group delay at all ω.

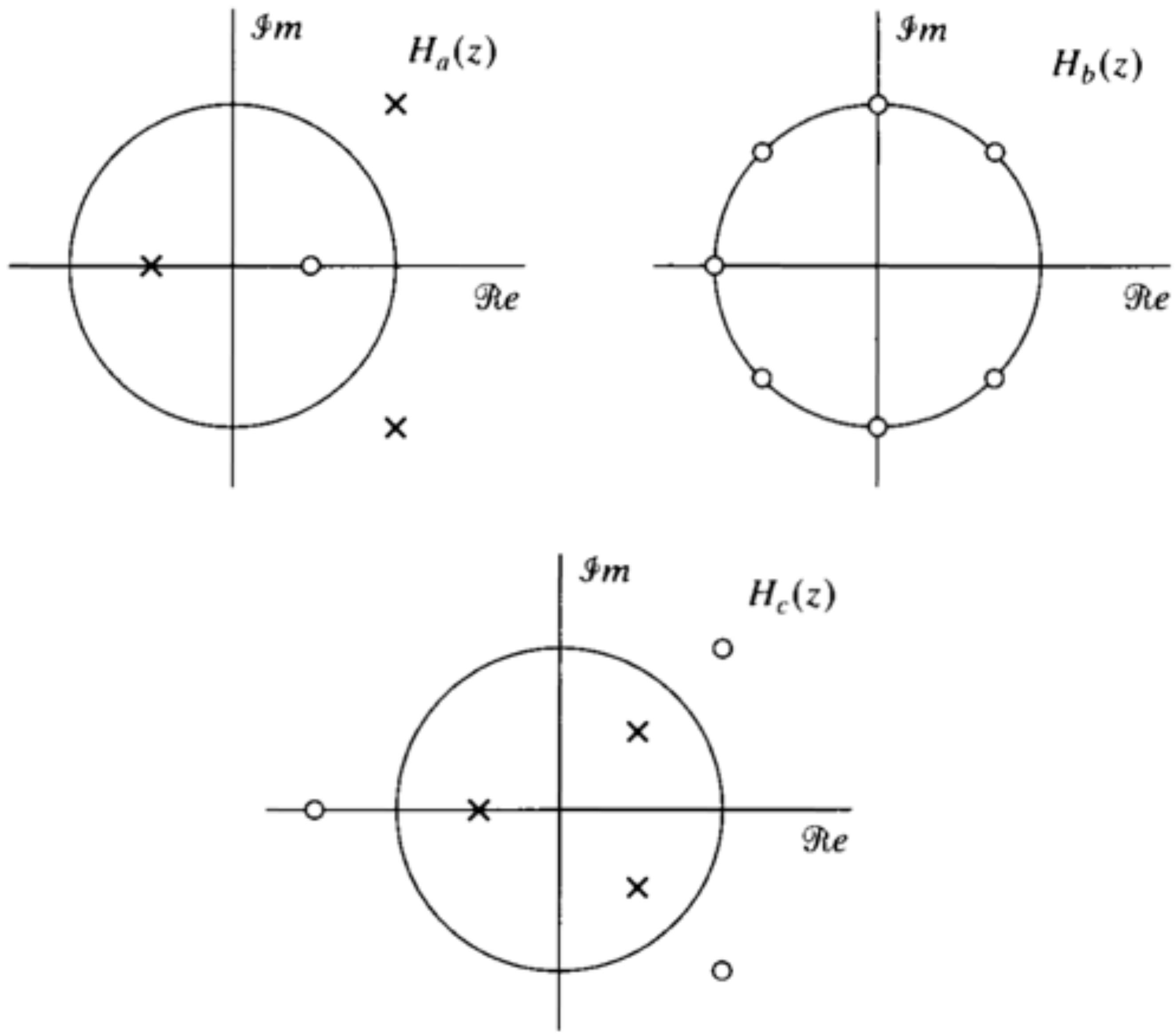
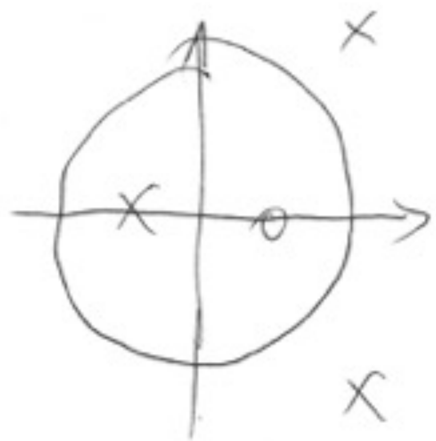


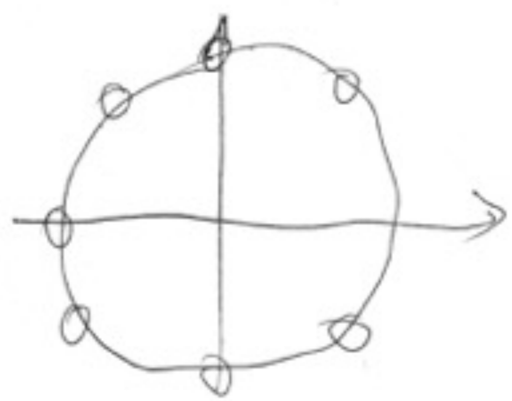
Figure P5.48-1



Not stable.

IIR.

Can have non-positive group delay

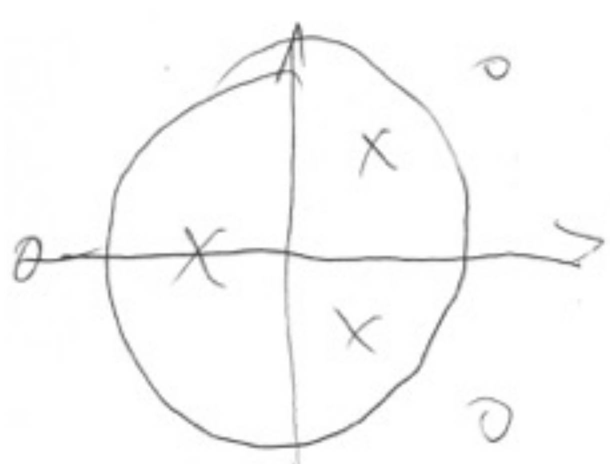


Stable

FIR

linear phase

Causal $\Rightarrow$ Positive group delay.



Stable.

All pass

Nonlinear phase

Positive group delay.

(by properties of all pass)

Problem 4

5.32. Suppose that a causal LTI system has an impulse response of length 6 as shown in Figure P5.32, where c is a real-valued constant (positive or negative).

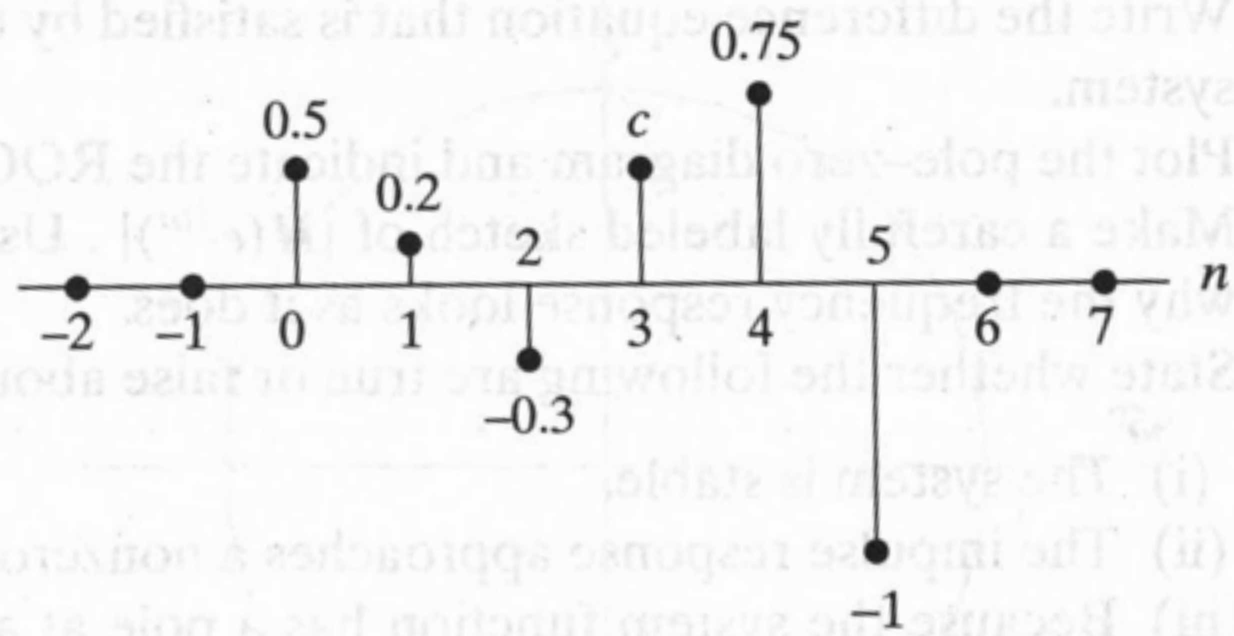


Figure P5.32

Which of the following statements is true:

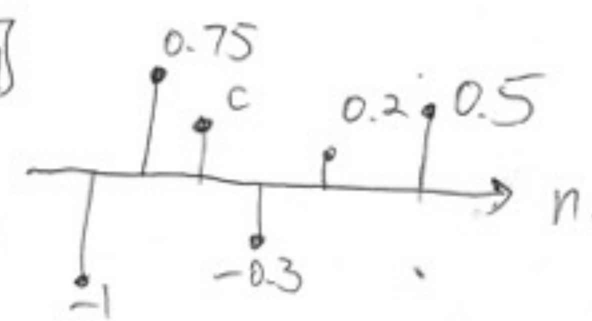
(a) This system must be minimum phase.
(b) This system cannot be minimum phase.
(c) This system may or may not be minimum phase, depending on the value of c.

Justify your answer.

Solution 4

Flip in time domain corresponds to flipping the zeros/poles along the unit circle.

- If $h[n]$ is minimum phase then $h[-n+N-1]$ is maximum phase
- $h[-n+N-1]$



has better energy compactness

Specifically, $|h[5]| = 1 > |h[0]| = 0.5$

$\Rightarrow h[-n+N-1]$ is not maximum phase

$\Rightarrow h[n]$ not minimum phase.

Problem 5

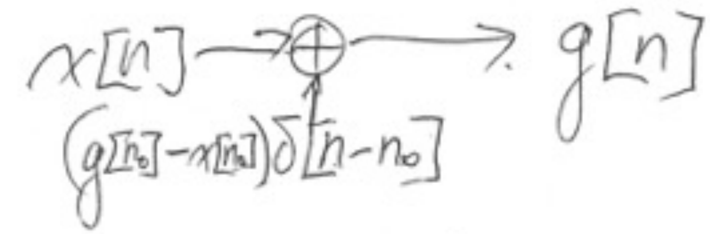
5.80. Consider a real-valued sequence $x[n]$ for which $X(e^{j\omega}) = 0$ for $\frac{\pi}{4} \le |\omega| \le \pi$. One sequence value of $x[n]$ may have been corrupted, and we would like to recover it approximately or exactly. With $g[n]$ denoting the corrupted signal,

$$g[n] = x[n] \qquad \text{for } n \neq n_0,$$

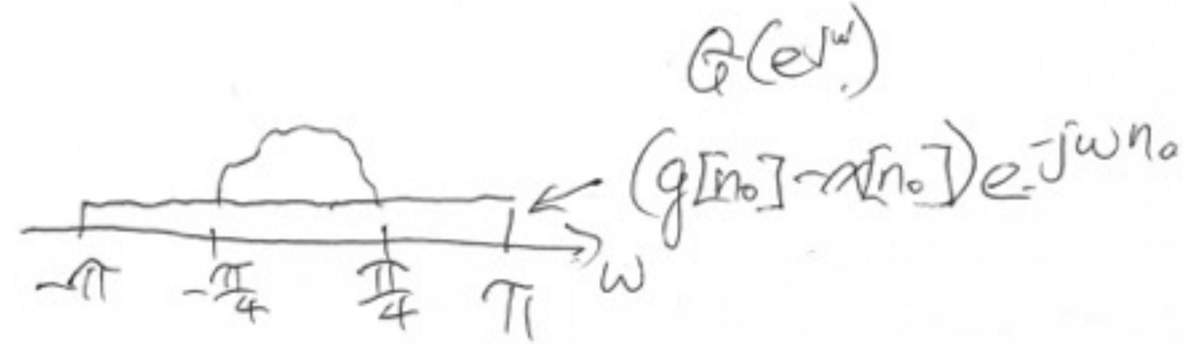
and $g[n_0]$ is real but not related to $x[n_0]$. specify a practical algorithm for recovering $x[n]$ from $g[n]$ exactly or approximately.

- 

- Model $g[n]$ as



ie. $g[n] = x[n] + (g[n_0] - x[n_0])\delta[n-n_0]$

- 

- To recover $x[n_0]$,
pick a point between $\frac{\pi}{4}$ to π
say $\frac{\pi}{2}$.

- Pass a narrowband signal $e^{j\frac{\pi}{2}n}$ to $g[n]$. Then the output is $G(e^{j\frac{\pi}{2}})e^{j\frac{\pi}{2}n}$.
$|G(e^{j\frac{\pi}{2}})| = |x[n_0]|$. Delay = group delay $= n_0$ (But with some phase wraps)