EE123 Spring 2015
Discussion Section 12

Giulia Fanti (slides by Frank Ong)

5. a) (10 points) Give a *minimum-phase* filter that has the same magnitude response as:

$$H(z) = \frac{(0.3 + z^{-1})(0.5 + z^{-1})}{(1 - 0.2z^{-1})(1 + 0.5z^{-1})}.$$

b) (10 points) Give two causal generalized linear phase FIR filters that satisfy the following constraints:

i) The group delay is equal to 1.

ii) $\frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) = 1.$

iii) $\frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega})|^2 = 2.$

Specify the type of each filter (I, II, III, or IV).

a) $H(z) = H_{min}(z) \cdot H_{ap}(z)$

$$= \underbrace{\frac{1+0.3z^{-1}}{1-0.2z^{-1}}}_{H_{min}(z)} \cdot \underbrace{\frac{(0.3+z^{-1})(0.5+z^{-1})}{(1+0.3z^{-1})(1+0.5z^{-1})}}_{H_{ap}(z)}$$

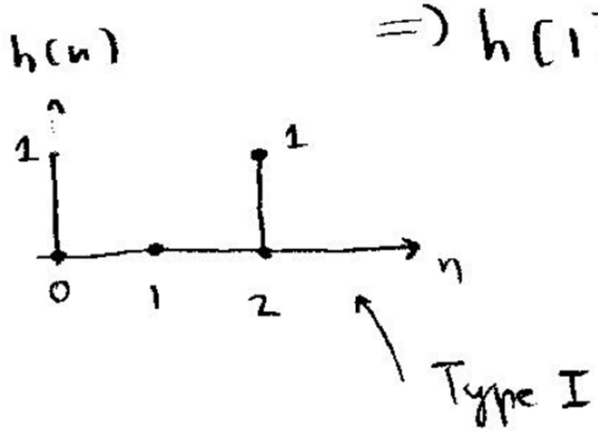
$$\Rightarrow \boxed{H_{min}(z) = \frac{1+0.3z^{-1}}{1-0.2z^{-1}}}$$

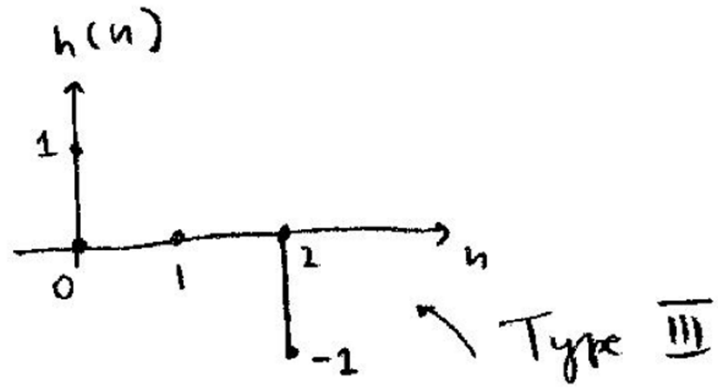
b) i) $\Rightarrow \frac{M}{2} = 1 \Rightarrow M = 2$ (Type I, Type III)

ii) $\Rightarrow h[0] = 1$

iii) $\Rightarrow h^2[0] + h^2[1] + h^2[2] = 2$, also $h^2[0] = h^2[2]$

$\Rightarrow h[1] = 0$





6. A FIR filter of order M is to be designed by windowing the impulse response of the ideal filter:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\pi/2} & 0 < \omega < \pi \\ e^{j\pi/2} & -\pi < \omega < 0. \end{cases}$$

a) (10 points) Determine the impulse response corresponding to:

$$H_d(e^{j\omega})e^{-j\omega M/2}.$$

b) (5 points) What types of filters (I, II, III, or IV) would result from windowing the impulse response determined in part (a)?

c) (5 points) Which window would you select if your aim was to minimize the mean-square error $\int_{-\pi}^{\pi} |H(e^{j\omega}) - H_d(e^{j\omega})|^2 d\omega$?

a) $H_s(e^{jw}) = H_d(e^{jw}) \cdot e^{-jwM/2}$

$$h_s[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_s(e^{jw})\, e^{jwn}\, dw = \frac{1}{2\pi}\left[e^{j\cdot\frac{\pi}{2}} \int_{-\pi}^{0} e^{jw(n-\frac{M}{2})}\, dw \right.$$

$$\left. + e^{-j\frac{\pi}{2}} \int_{0}^{\pi} e^{jw(n-\frac{M}{2})}\, dw \right]$$

$$= \frac{1}{2\pi} \cdot \left[j \cdot \frac{1 - e^{-j\pi(n-\frac{M}{2})}}{j(n-\frac{M}{2})} - j \cdot \frac{e^{j\pi(n-\frac{M}{2})} - 1}{j(n-\frac{M}{2})} \right]$$

$$= \frac{1}{2\pi} \cdot \frac{2 - 2\cos\left(\pi(n-\frac{M}{2})\right)}{\pi(n-\frac{M}{2})} = \frac{1 - \cos\left(\pi(n-\frac{M}{2})\right)}{\pi(n-\frac{M}{2})}$$

b) $h[n] = h_s[n] \cdot w[n]$, where $w[n]$ is symmetric window of length $M+1$: $w[n] = \begin{cases} \neq 0 & , n=0,\dots,M \\ = 0 & , \text{otherwise} \end{cases}$

and $w[n] = w[M-n]$

$$h[M-n] = \frac{1-\cos\left(\pi\left(\frac{M}{2}-n\right)\right)}{\pi\left(\frac{M}{2}-n\right)} \cdot w[M-n]$$

$$= -\frac{1-\cos\left(\pi\left(n-\frac{M}{2}\right)\right)}{\pi\left(n-\frac{M}{2}\right)} w[n] = -h[n] \Rightarrow \text{Types } \underline{\text{III}} \text{ or } \underline{\text{IV}}$$

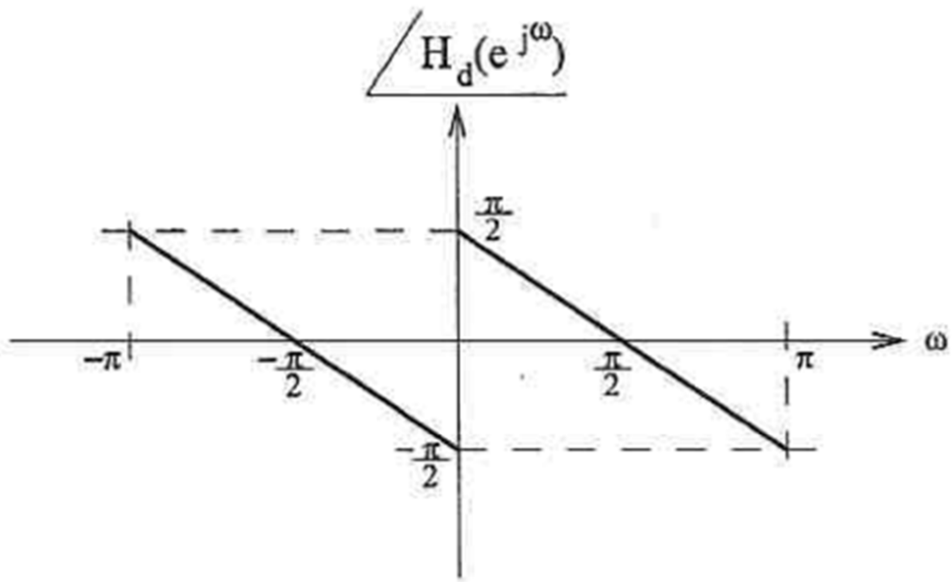
c) $h[n] = w[n] \cdot h_d[n]$

Since minimizing $\int_{-\pi}^{\pi} |H(e^{j\omega}) - H_d(e^{j\omega})|^2 d\omega$

is the same as minimizing $\sum_n |h[n] - h_d[n]|^2$

then rectangular window will minimize the mean-square error.

Given the phase characteristics of a generalized linear phase FIR filter $H_d(e^{j\omega})$ shown below, answer the following questions. Include brief explanations to get credit.



(a) (6 pts) Is this a symmetric (i.e., Type-I or Type-II) or an anti-symmetric (i.e., Type-III or Type-IV) filter? Why?

(b) (8 pts) Can the filter length be determined from the given information? If yes, what is the length? If not, why not?

(c) (6 pts) The filter magnitude response has a DC gain of 1. True or False? Why?

(a) (6 pts) Is this a symmetric (i.e., Type-I or Type-II) or an anti-symmetric (i.e., Type-III or Type-IV) filter? Why?

$$\text{slope} = \frac{-\pi/2}{\pi/2} = -1 \rightarrow \alpha = 1 \rightarrow M = 2 \text{ (even)}$$

$$\beta = \pi/2 \text{ and } M \text{ even} \rightarrow \boxed{\text{Type} - \text{III (anti-symmetric) filter}}$$

(b) (8 pts) Can the filter length be determined from the given information? If yes, what is the length? If not, why not?

(c) (6 pts) The filter magnitude response has a DC gain of 1. True or False? Why?

Problem 4

5.32. Suppose that a causal LTI system has an impulse response of length 6 as shown in Figure P5.32, where c is a real-valued constant (positive or negative).

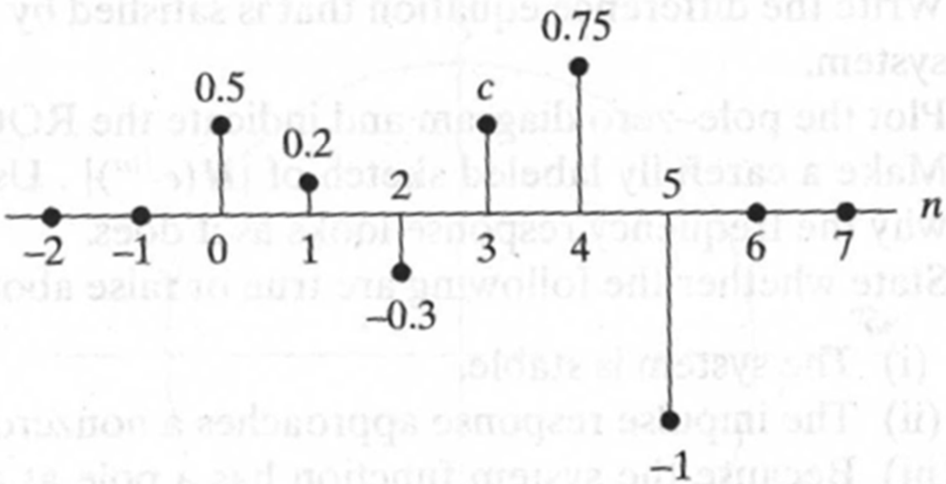


Figure P5.32

Which of the following statements is true:

(a) This system must be minimum phase.
(b) This system cannot be minimum phase.
(c) This system may or may not be minimum phase, depending on the value of c.

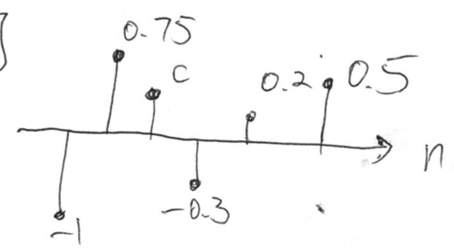
Justify your answer.

Solution 4

Flip in time domain corresponds to flipping the zeros/poles along the unit circle.

- If $h[n]$ is minimum phase then $h[-n+N-1]$ is maximum phase

- $h[-n+N-1]$



has better energy compactness

Specifically, $|h[5]| = 1 > |h[0]| = 0.5$.

$\Rightarrow h[-n+N-1]$ is not maximum phase

$\Rightarrow h[n]$ not minimum phase.

Problem 5

5.32. The Fourier transform of a stable linear time-invariant system is purely real and is shown in Figure P5.32-1. Determine whether this system has a stable inverse system.

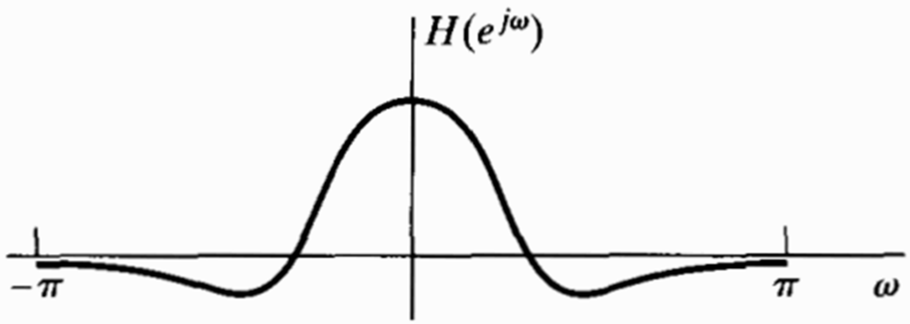


Figure P5.32-1

Solution 5

5.32. Since $H(e^{j\omega})$ has a zero on the unit circle, its inverse system will have a pole on the unit circle and thus is not stable.