EE123 Spring 2015
Discussion Section 13

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FIR System design

1. Windows => TBW
2. Optimal filter design

Time Bandwidth Product

TBW is an approximation, but useful

$$T(BW) = (M+1)\omega/2\pi$$

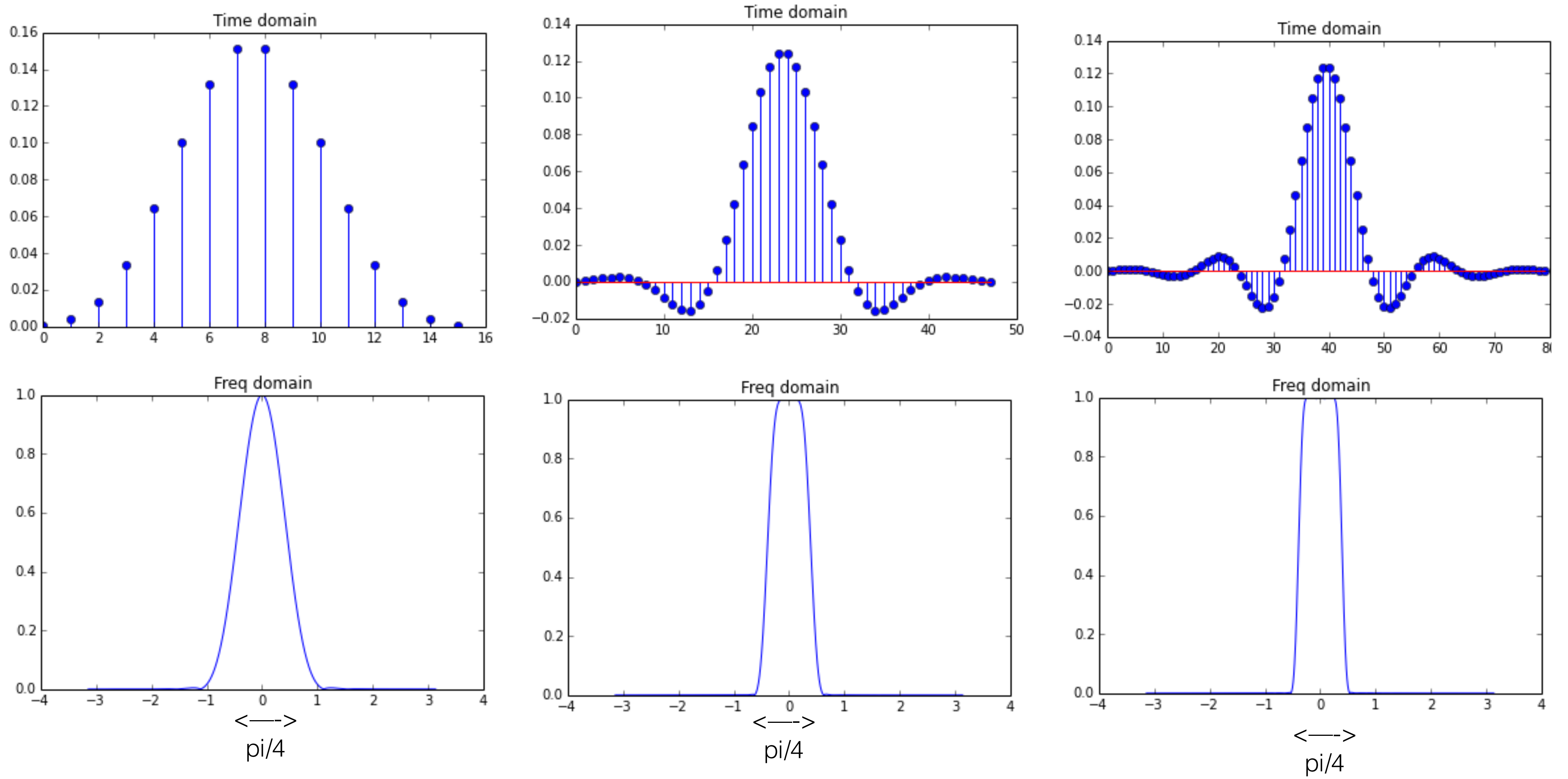
M+1 is the length of the filter in time domain

w is the bandwidth of the filter in freq domain (both sides)

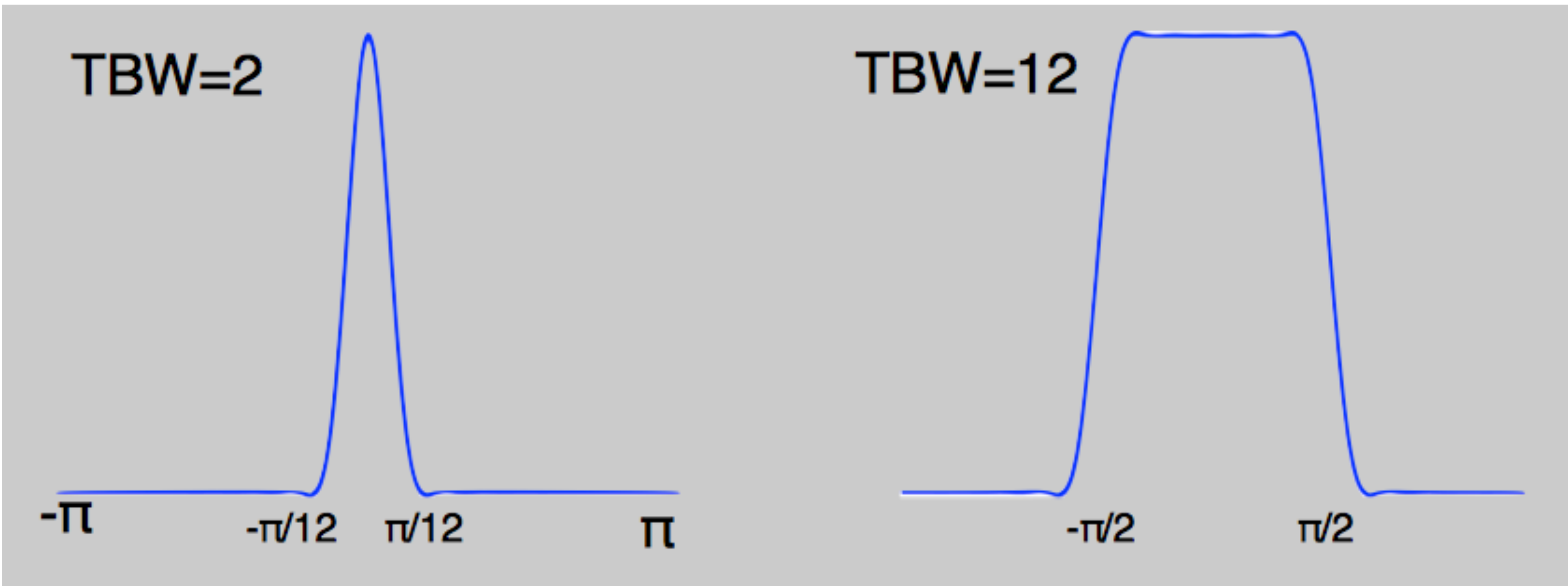
TBW = number of zero crossings in **time domain**

Example: Design a filter with BW = pi/4 (from -pi/8 to pi/8)

TBW = 2 | TBW = 6 | TBW = 10



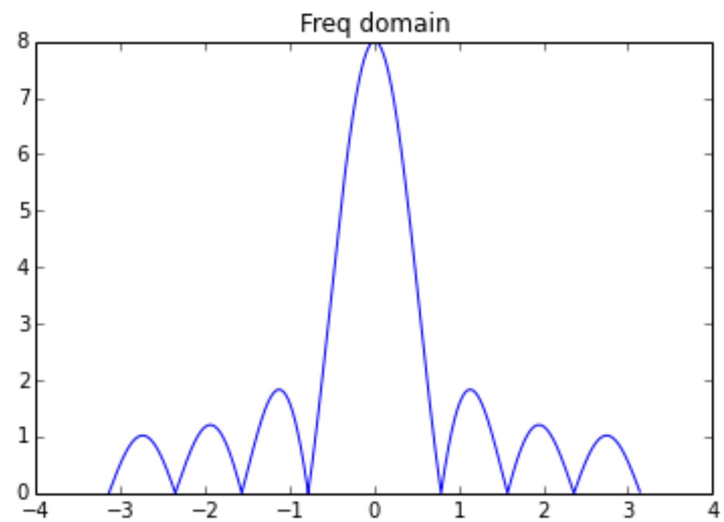
Fix M = 23, increase BW



Same length in time domain => same transition

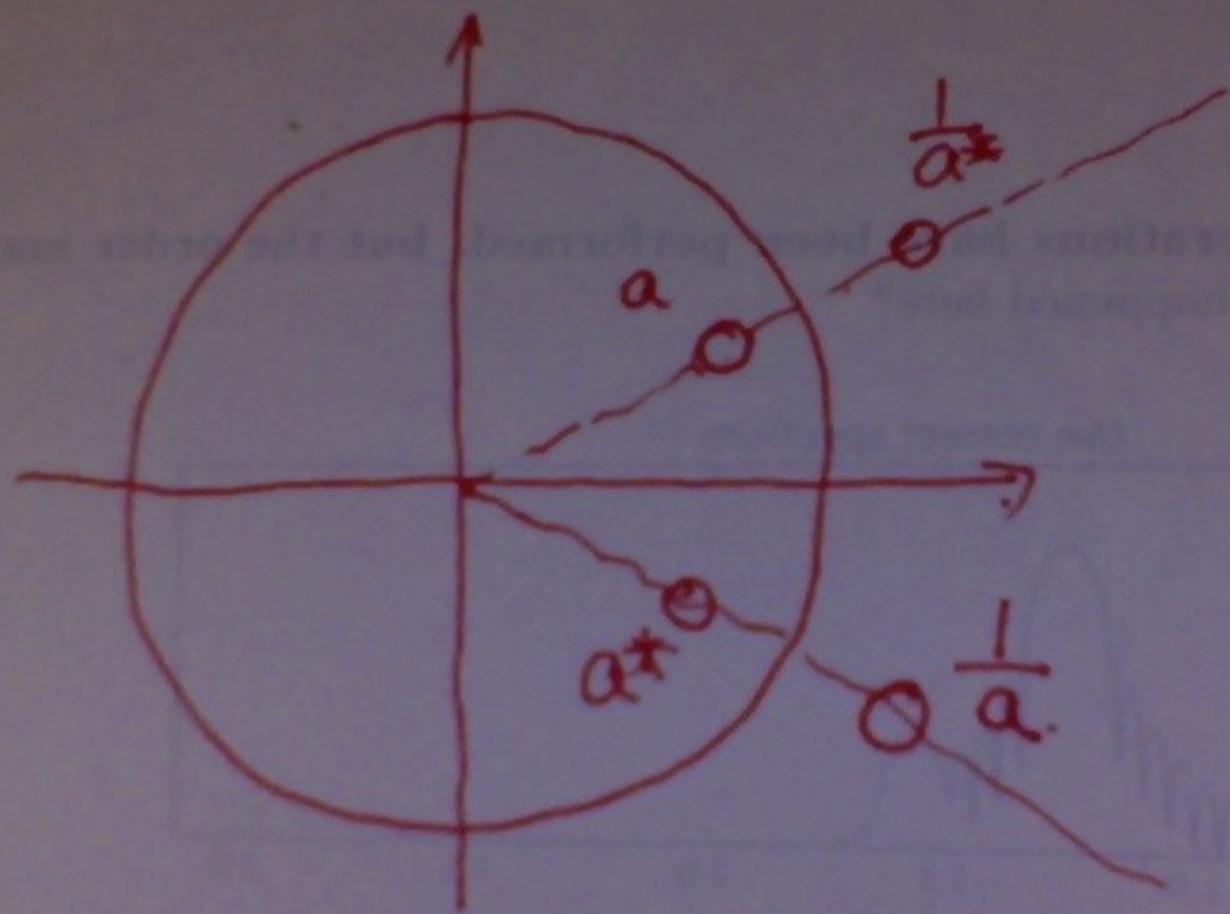
Special case

TBW = 1: Time domain box, Freq domain sinc



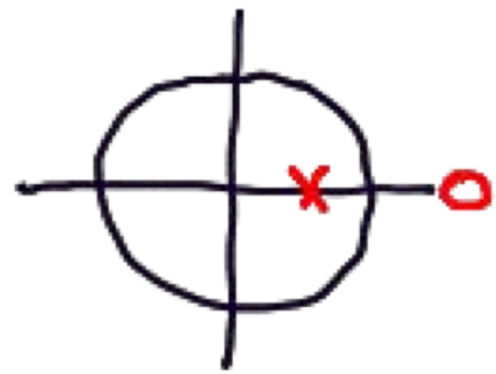
System design

- All-pass
- Minimum phase
- Linear Phase


1/a*
a
a*
1/a

All pass system

$$H_{ap}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}$$



$$|H_{ap}(e^{j\omega})| = 1$$

Stable/causal

$$\text{grd}[H_{ap}(e^{j\omega})] \geq 0,$$

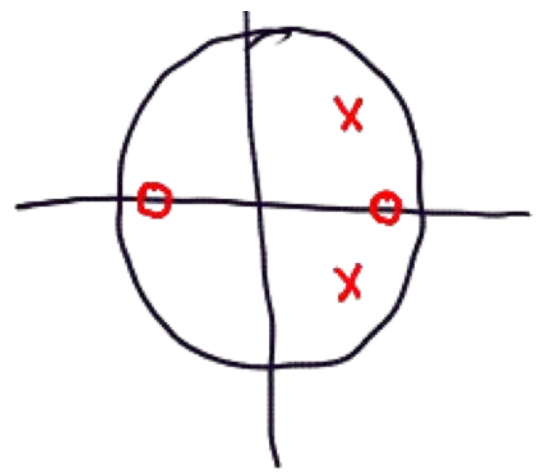
$$\arg[H_{ap}(e^{j\omega})] \leq 0 \qquad \text{for } 0 \leq \omega < \pi.$$

Minimum phase system

The Minimum Group-Delay Property

$$\text{grd}[H(e^{j\omega})] = \text{grd}[H_{\min}(e^{j\omega})] + \text{grd}[H_{ap}(e^{j\omega})].$$

Flipping zero/pole outside unit circle increases group delay



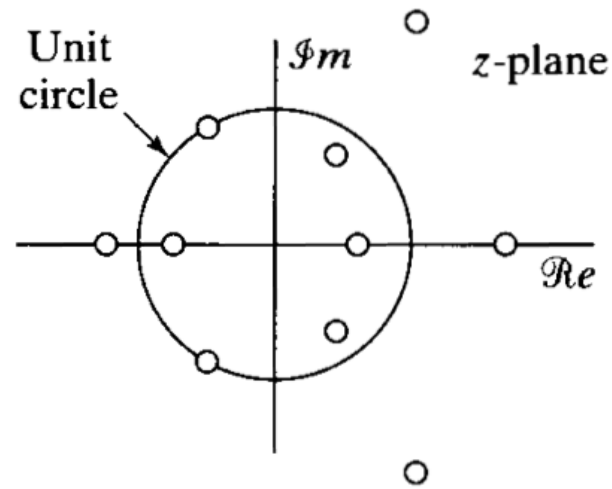
The Minimum Energy-Delay Property

$$\forall h \text{ such that } |H(e^{j\omega})| = |H_{\min}(e^{j\omega})|.$$

$$\sum_{m=0}^{n} |h[m]|^2 \leq \sum_{m=0}^{n} |h_{\min}[m]|^2$$

Linear Phase

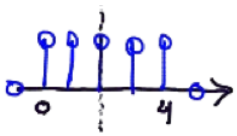
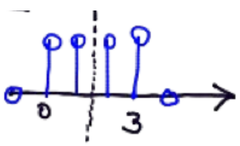
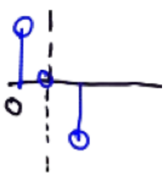
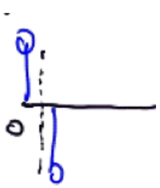
length $(M+1)$



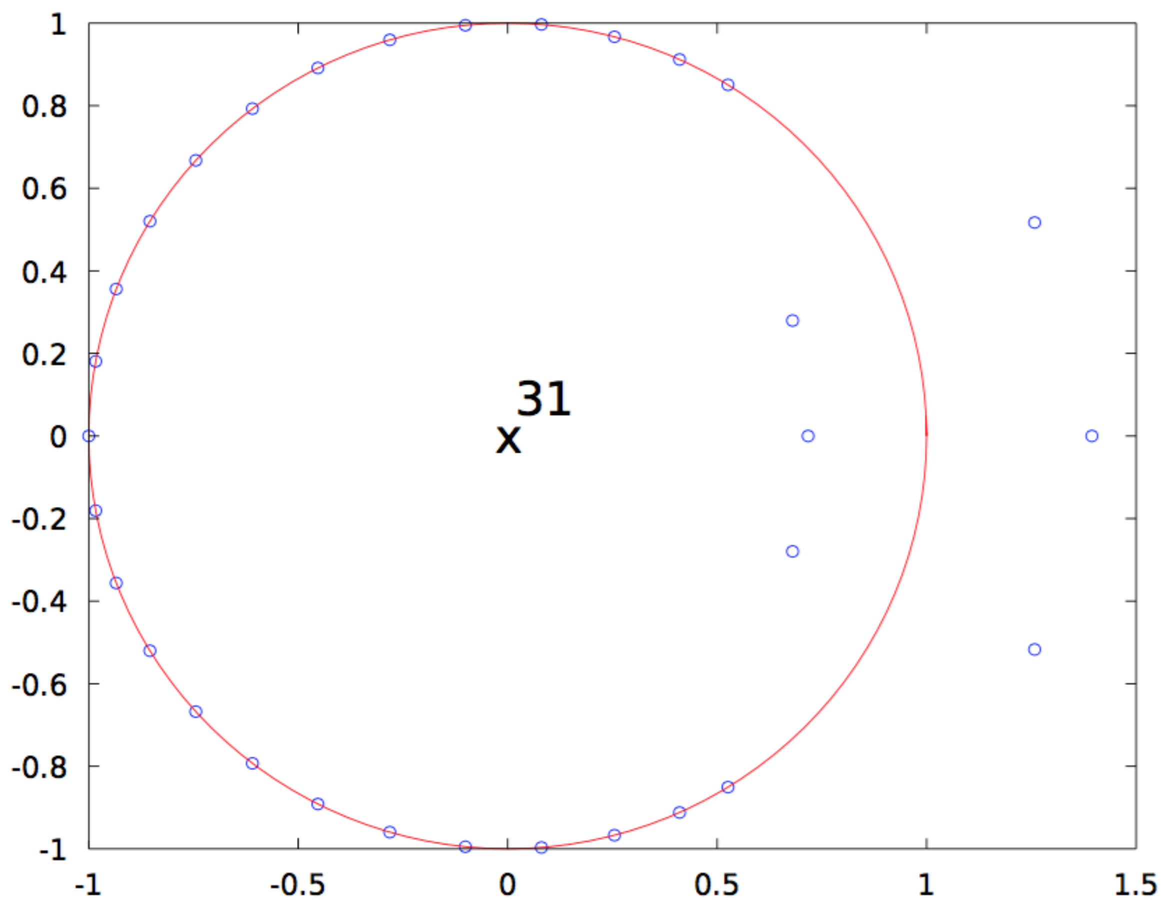
$$H(e^{j\omega}) = A_e(e^{j\omega})e^{-j\omega M/2},$$

Can be decomposed into
minimum phase and maximum phase

Linear Phase

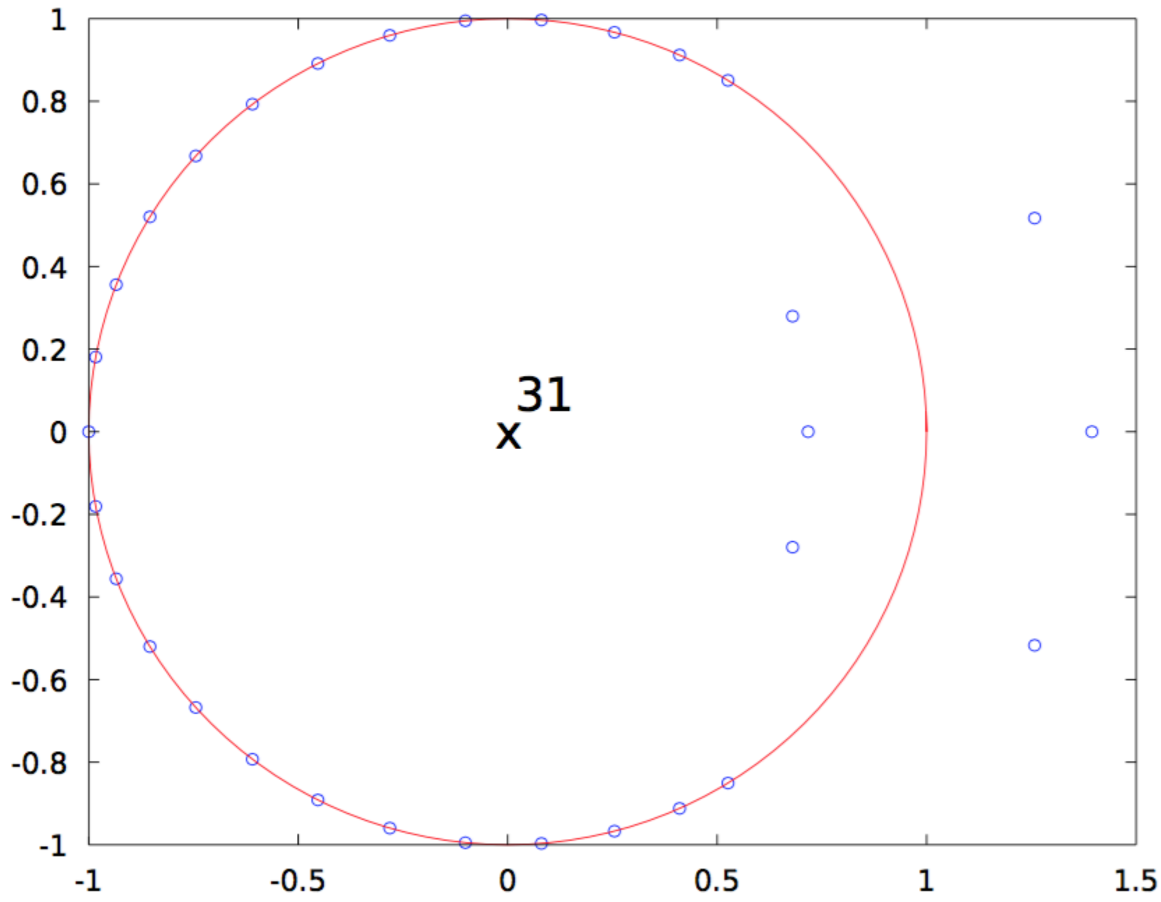
Impulse Response		M = (length-1)	Symmetry	Zero at w=0	Zero at w=pi
	**Type I**	even	Symmetric	No	No
	**Type II**	odd	Symmetric	No	Yes
	**Type III**	even	Anti-symmetric	Yes	Yes
	**Type IV**	odd	Anti-symmetric	Yes	No

Consider the following zero-pole diagram of a M=31 (the original problem had a mistake of $M = 32$), TBW=8 filter



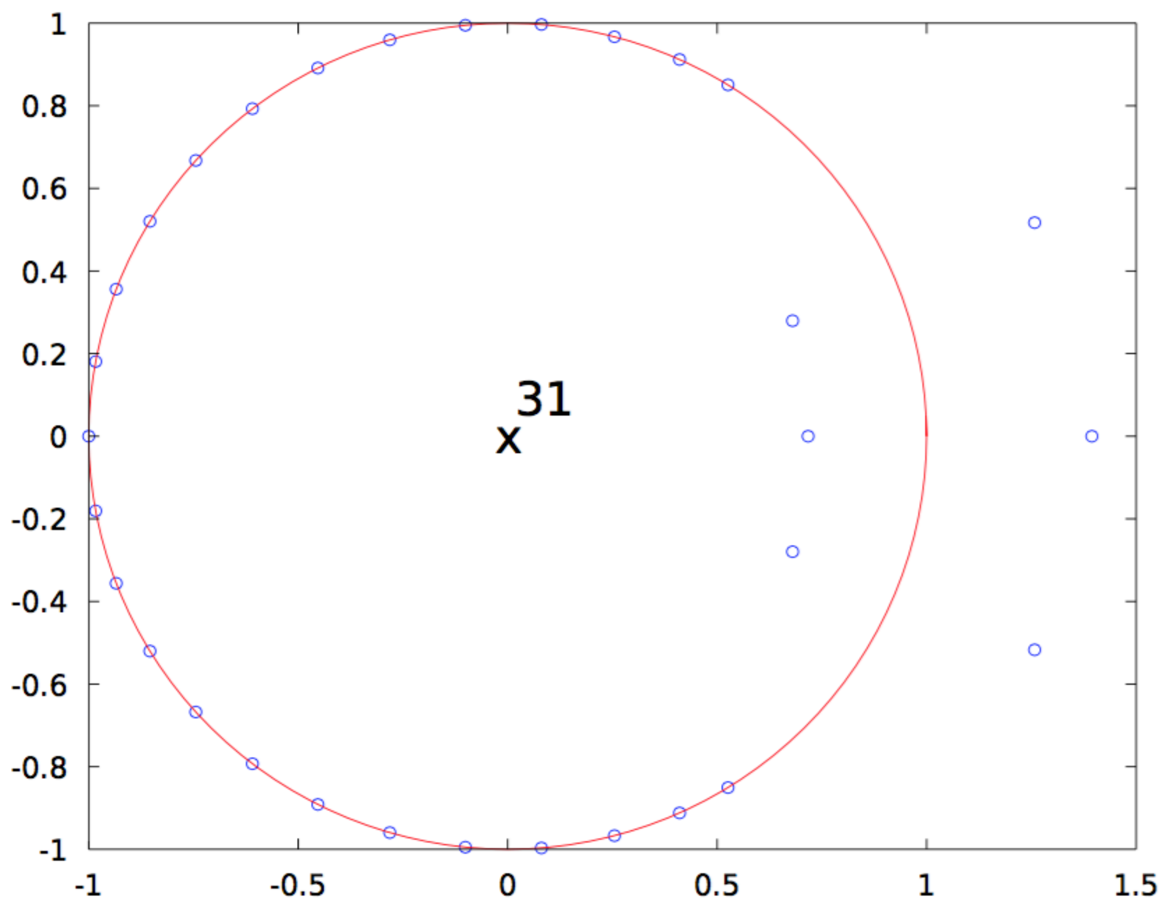
a. Qualitatively draw its impulse response

Consider the following zero-pole diagram of a M=31 (the original problem had a mistake of $M = 32$), TBW=8 filter



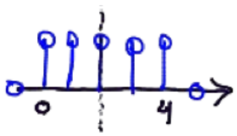
b. Given a sampling rate of 48000Hz. What's the approximate cutoff frequency of the filter?

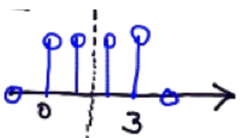
Consider the following zero-pole diagram of a M=31 (the original problem had a mistake of $M = 32$), TBW=8 filter

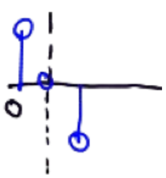


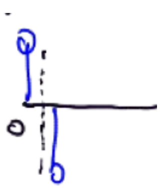
c. How would the zero-pole diagram change if you design a causal M=63 (originally M=64), TBW=8 filter? Qualitatively draw the zero-pole diagram and its impulse response. Emphasize the differences to part (a)

Linear Phase









	M = (length-1)	Symmetry	Zero at w=0	Zero at w=pi
Type I	even	Symmetric	No	No
Type II	odd	Symmetric	No	Yes
Type III	even	Anti-symmetric	Yes	Yes
Type IV	odd	Anti-symmetric	Yes	No

You would like to design a linear-phase "Rho" filter for tomographic reconstruction using least-squares design. The filter should approximate the frequency response

$$H_d(e^{j\omega}) = A(e^{j\omega})e^{-j\alpha\omega + j\beta},$$

where $A(e^{j\omega}) = |w|$.

a. This is a high-pass filter. Can you design a type III or IV GLP "Rho" filter? Explain.

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where $A(e^{j\omega}) = |w|$.

b. You go and design a type-II $M = 7$ filter. Qualitatively draw the frequency response of your designed filter $\tilde{H}(e^{j\omega})$. Explain.

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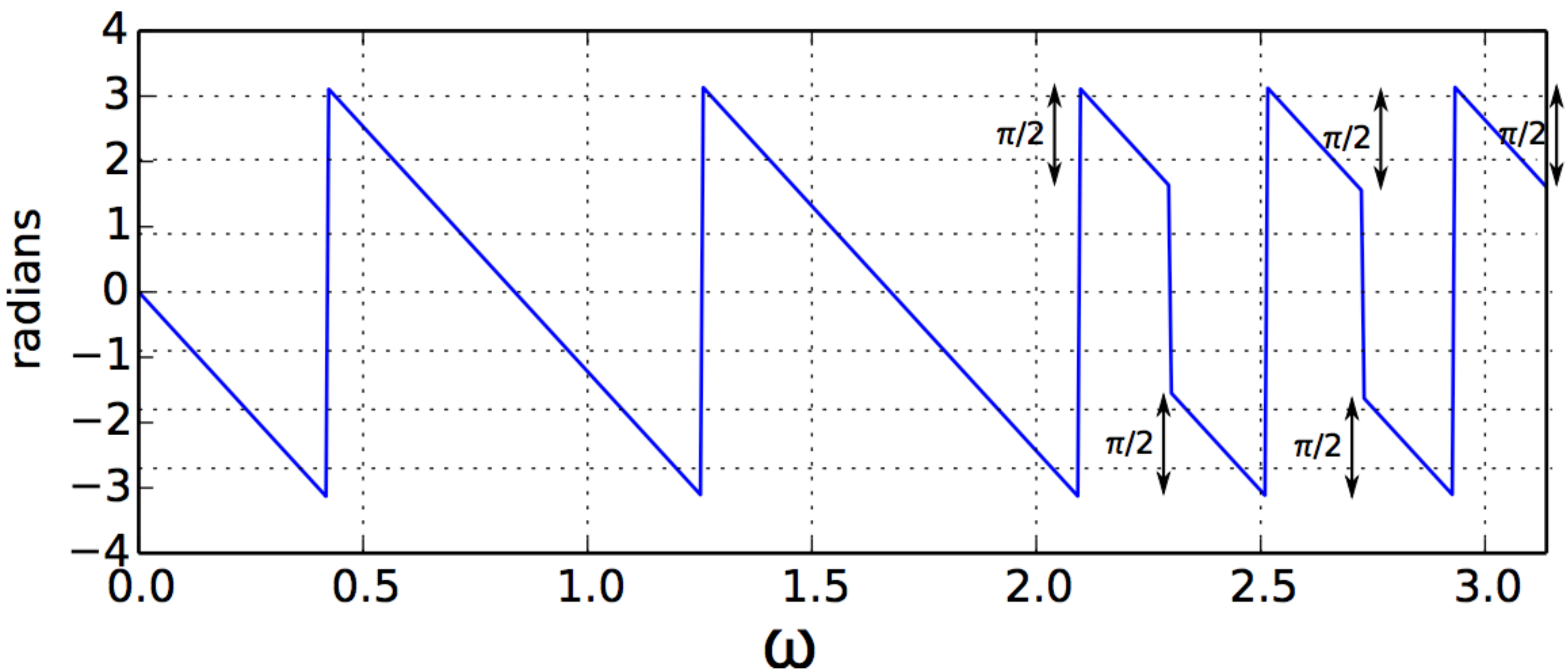
where $A(e^{j\omega}) = |w|$.

b. You go and design a type-II $M = 7$ filter. Qualitatively draw the frequency response of your designed filter $\tilde{H}(e^{j\omega})$. Explain.

c. Is there a way to design a filter with reduced approximation error $\int_{-\pi}^{\pi} |\tilde{H}(e^{j\omega}) - H_d(e^{j\omega})|^2$ compared to the filter in part b without increasing the filter order? If so, explain what you would do. If not, explain why not.

You are given a phase response of a causal, GLP low-pass FIR filter.



a What is the minimum number of zeros that the filter can have that are located on the unit circle? Explain

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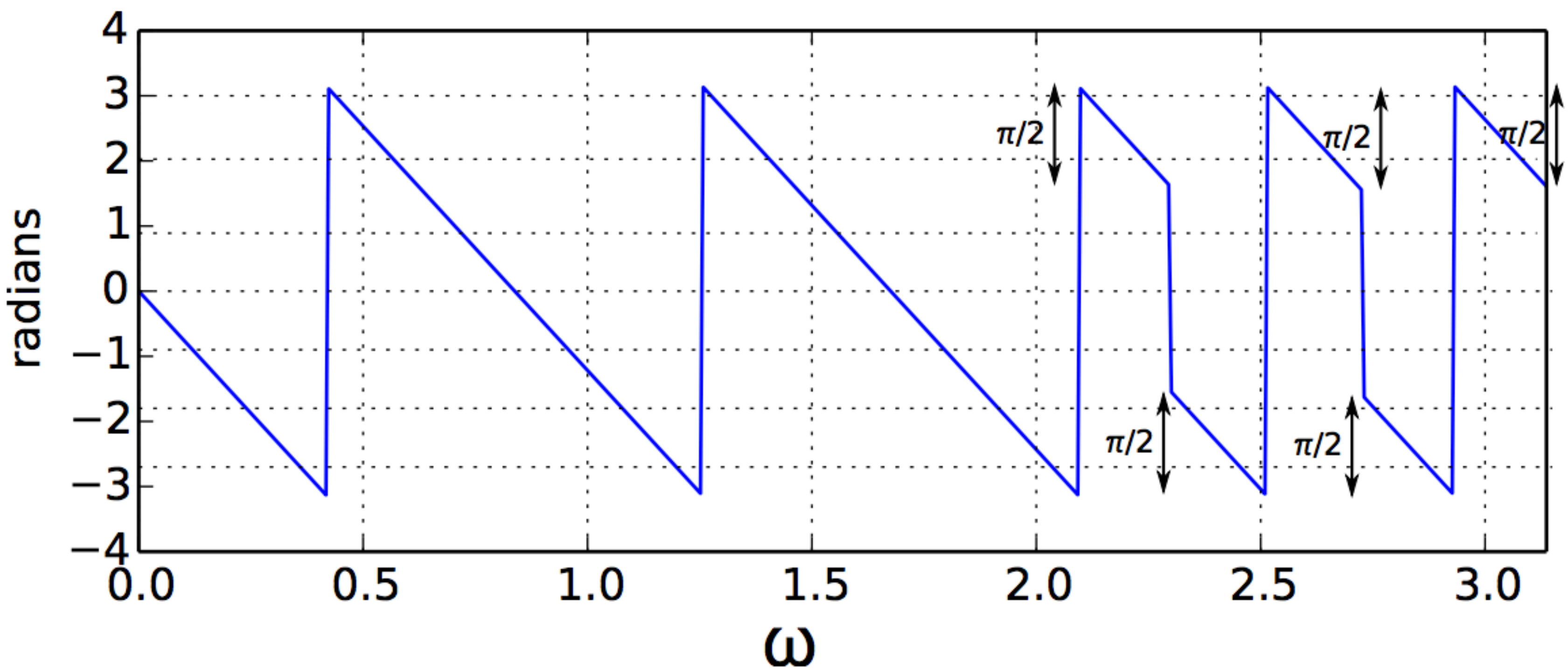
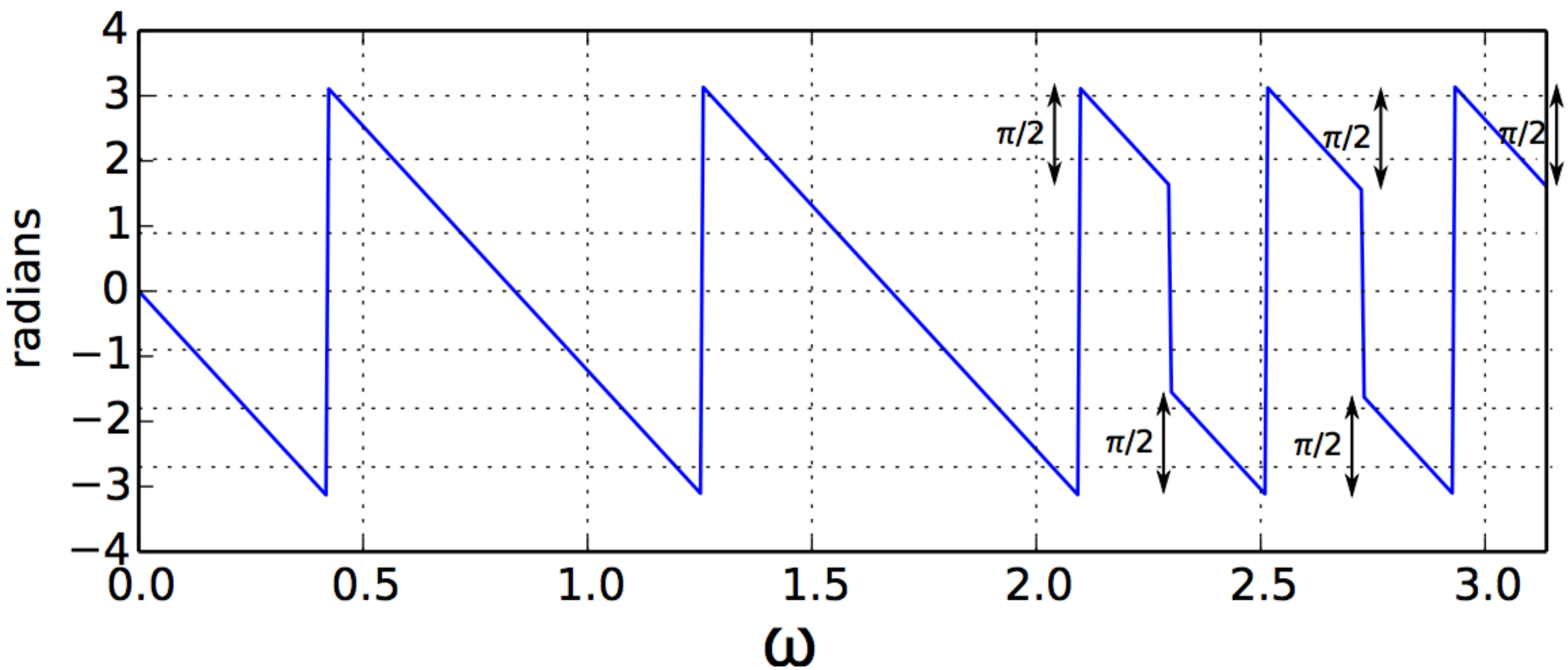


b. What type is the filter? Type I, II, III, IV

You are given a phase response of a causal, GLP low-pass FIR filter.



c. What is the filter delay?