EE123 Discussion Section 2

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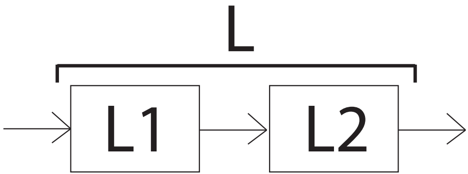
General

Office hours: Thursdays 5-6pm Cory 212
Questions about homework1 / lab0?

Question 1

Let L_1 and L_2 be two separate systems and L be the cascaded system:



Determine whether each of following statements is true or false:

- If L_1 is LTI and L_2 is not LTI, then L cannot be LTI
- If L_1 is not LTI and L_2 is not LTI, then L cannot be LTI

Solution 1

Determine whether each of following statements is true or false:

- If L_1 is LTI and L_2 is not LTI, then L cannot be LTI
  False
  Consider the system $L_1 = 0$. $L = 0$
- If L_1 is not LTI and L_2 is not LTI, then L cannot be LTI
  False
  Consider the systems $L_1\{x\} = x^3$ and $L_2\{x\} = x^{1/3}$. $L = I$

Question 2 (From old exam)

A discrete-time system H produces a corresponding output signal y that is the symmetric part of the input. That is,

$$y[n] = \frac{x[n] + x[-n]}{2}$$

Select the strongest true assertion from the list below.

- The system must be an LTI system.
- The system could be an LTI system, but does not have to be.
- The system cannot be an LTI system.

Solution 2 (From old exam)

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Select the strongest true assertion from the list below.

- The system must be an LTI system.
- The system could be an LTI system, but does not have to be.
- **The system cannot be an LTI system.**

Solution 2 cont'd (From old exam)

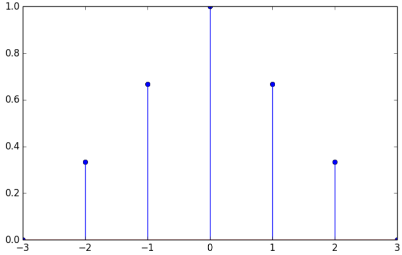
Not Time Invariant, counterexample:

- Consider $x_1[n] = \delta[n]$, then $y_1[n] = \delta[n]$.
- But for $x_2[n] = \delta[n-1]$, $y_2[n] = \frac{\delta[n-1]+\delta[n+1]}{2} \neq y_1[n-1]$.
- Hence, H is not an TI

However, the system is linear

Question 3

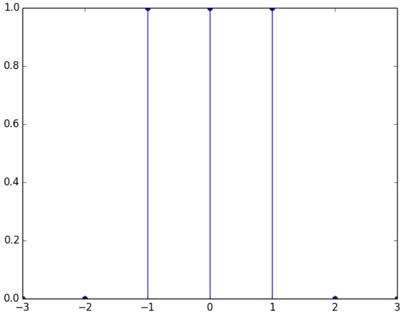
What is the Discrete-Time Fourier transform $X(e^{j\omega})$ of the below signal $x[n]$:



(Hint: $x[n]$ is a convolution of two signals)

Solution 3

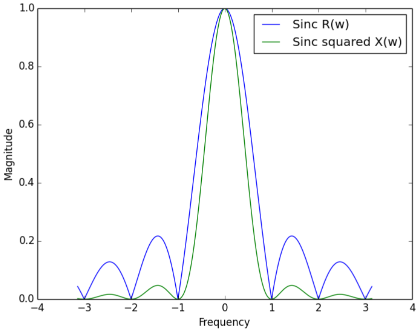
$x[n] = r[n] * r[n]$, where $r[n]$ is the periodic sinc



Hence using the convolution property,
$X(e^{j\omega}) = R(e^{j\omega})^2 = (\frac{\sin[\omega(3/2)]}{\sin(\omega/2)})^2$

Solution 3

Normalized magnitude response for $X(e^{j\omega})$ and $R(e^{j\omega})$

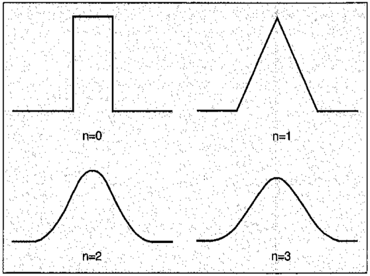


Solution 3

These signals are known as splines. In the continuous domain, the generalizations are called B-splines, which converges to a Gaussian in the limit (Unser 1999). They are often useful in interpolation.

$$\beta^0(x) = \begin{cases} 1, & -\frac{1}{2} < x < \frac{1}{2} \\ \frac{1}{2}, & |x| = \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$\beta^n(x) = \underbrace{\beta^0 * \beta^0 * \cdots * \beta^0(x)}_{(n+1)\text{ times}}.$$



▲ *1. The centered B-splines of degree 0 to 3.*

Question 4 (From old exam)

Consider a system that produces the output:

$$y[n] = \cos(\frac{\pi}{4}n)$$

in response to the input signal x,

$$x[n] = e^{j\pi n/4}$$

Select the strongest true assertion from the list below:

- The system must be LTI
- The system could be LTI, but does not have to be
- The system cannot be LTI

Solution 4 (From old exam)

Consider a system that produces the output:

$$y[n] = \cos(\frac{\pi}{4}n)$$

in response to the input signal x,

$$x[n] = e^{j\pi n/4}$$

Select the strongest true assertion from the list below:

- The system must be LTI
- The system could be LTI, but does not have to be
- **The system cannot be LTI**

Because the output contains a pure tone $e^{-j\pi n/4}$ that is not present in the input.

Question 5a (From old exam)

For each of the following systems, determine if the system is linear, causal, shift-invariant and BIBO stable.
Indicate **Y** for Yes, **N** for No or **X** for cannot be determined

- $y[n] = \cos(\sqrt{|n|})x[n]$
  - Linear?
  - Causal?
  - Shift-Invariant?
  - Stable?

Solution 5a (From old exam)

For each of the following systems, determine if the system is linear, causal, shift-invariant and BIBO stable.
Indicate **Y** for Yes, **N** for No or **X** for cannot be determined

- $y[n] = \cos(\sqrt{|n|})x[n]$
  - Linear? **Y**
  - Causal? **Y**
  - Shift-Invariant? **N**
  - Stable? **Y**

Question 5b (From old exam)

For each of the following systems, determine if the system is linear, causal, shift-invariant and BIBO stable.
Indicate **Y** for Yes, **N** for No or **X** for cannot be determined

- The response of the system to an input of $\delta[n-1]$ is $(0.5)^n u[n]$
  - Linear?
  - Causal?
  - Shift-Invariant?
  - Stable?

Solution 5b (From old exam)

- The response of the system to an input of $\delta[n-1]$ is $(0.5)^n u[n]$
  - Linear? **X**
  - Causal? **X**
  - Shift-Invariant? **X**
  - Stable? **X**

  - Consider the system with impulse response $h[n] = (0.5)^{n+1} u[n+1]$
    $\Rightarrow$ linear, not causal, shift-invariant and stable
  - Consider the system that outputs $(0.5)^n u[n]$ regardless of input
    $\Rightarrow$ not linear, causal, shift-invariant and stable
  - Consider the system that outputs $(0.5)^n u[n]$ when the input is $\delta[n-1]$ and ∞ otherwise
    $\Rightarrow$ not linear, not causal, not shift-invariant and not stable

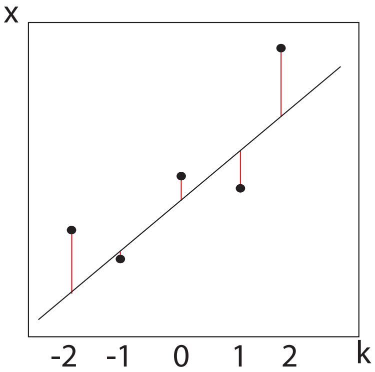
Homework Problem 7

In problem 7, you are asked to do a local quadratic polynomial regression for each n. The problem can be formulated as a **least squares problem**, in which you want to fit your parameters to many data.

Homework Problem 7

Example: Linear regression. We observe 5 data points $x[k]$ from $k = -2, -1, \ldots, 1, 2$.

We want to fit a line $x = mk + b$ by minimizing the squared distance between the line and the data points



Homework Problem 7

For $k = 2$, the squared distance is given by $(b + 2m - x[2])^2$.
If you write the equations out and put them in a matrix form, you get:

$$\min_{b,m} \| K \begin{pmatrix} b \\ m \end{pmatrix} - \begin{pmatrix} x[-2] \\ x[-1] \\ x[0] \\ x[1] \\ x[2] \end{pmatrix} \|_2^2$$

Homework Problem 7

To solve for b and m, take the "derivative" with respect to b and m and set it to zero:

$$K^T\left(K\begin{pmatrix} b \\ m \end{pmatrix} - \begin{pmatrix} x[-2] \\ x[-1] \\ x[0] \\ x[1] \\ x[2] \end{pmatrix}\right) = 0$$

$$\Rightarrow \begin{pmatrix} b \\ m \end{pmatrix} = (K^T K)^{-1} K^T \begin{pmatrix} x[-2] \\ x[-1] \\ x[0] \\ x[1] \\ x[2] \end{pmatrix}$$

Homework Problem 7

This is how you find inverses in python:

```
K = np.matrix( [ ... ] )
K_inv = np.linalg.inv( K.transpose() * K ) * K.transpose()
```