EE123 Spring 2015
Discussion Section 6

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Outline

- Lab 2 short overview
- Wavelet transform short overview
- Wavelet problems

Lab 2

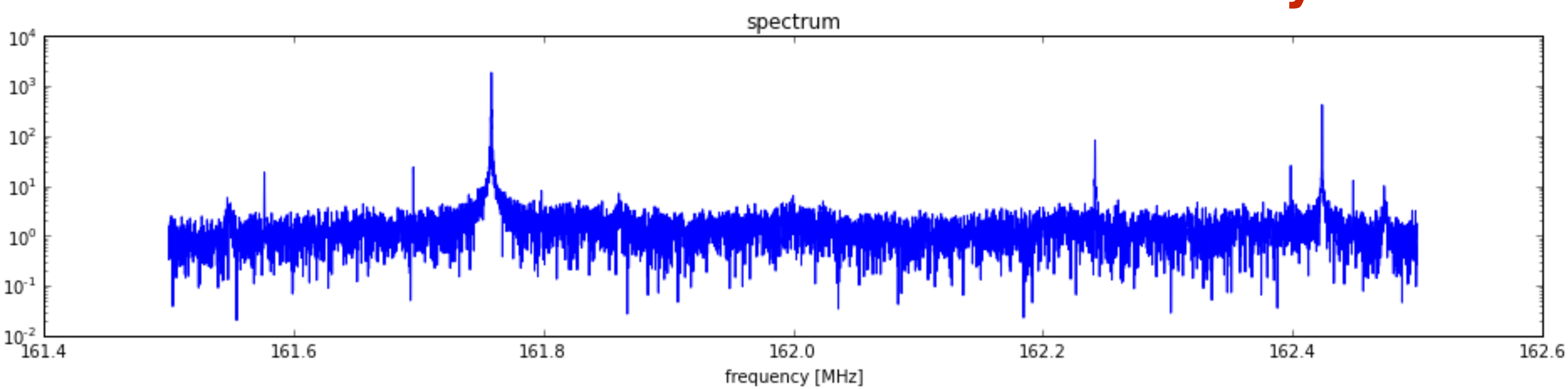
- Two parts:

1. **Install rtl-sdr**
2. Compute power spectrum for NOAA weather station
3. Preamble detection for ADS-B

Part I: How to get spectrum in practice?

1. First need to crop signal
    - Gibbs ringing
    - Spectral broadening
2. Window chunk
3. FFT

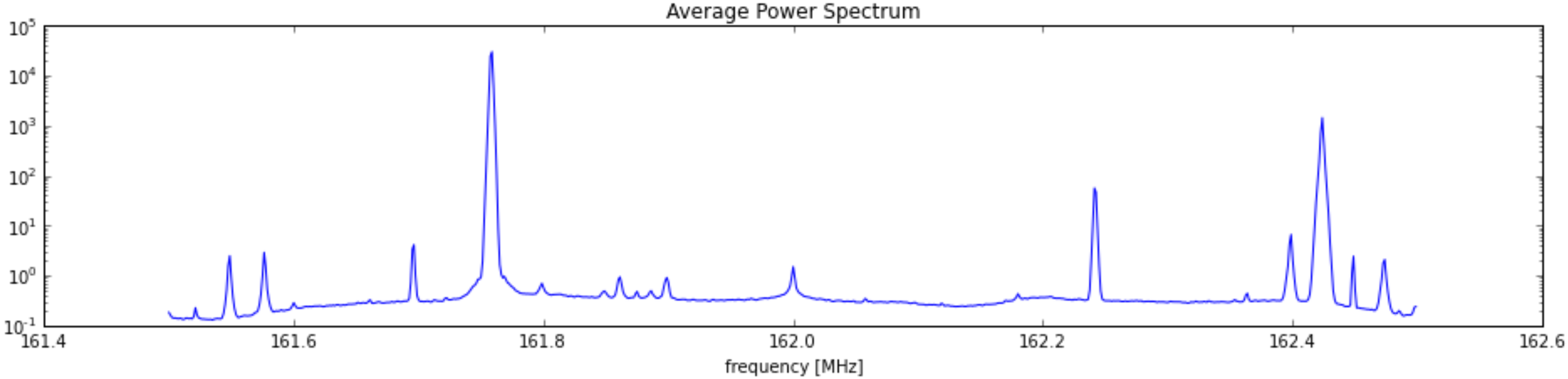
Noisy



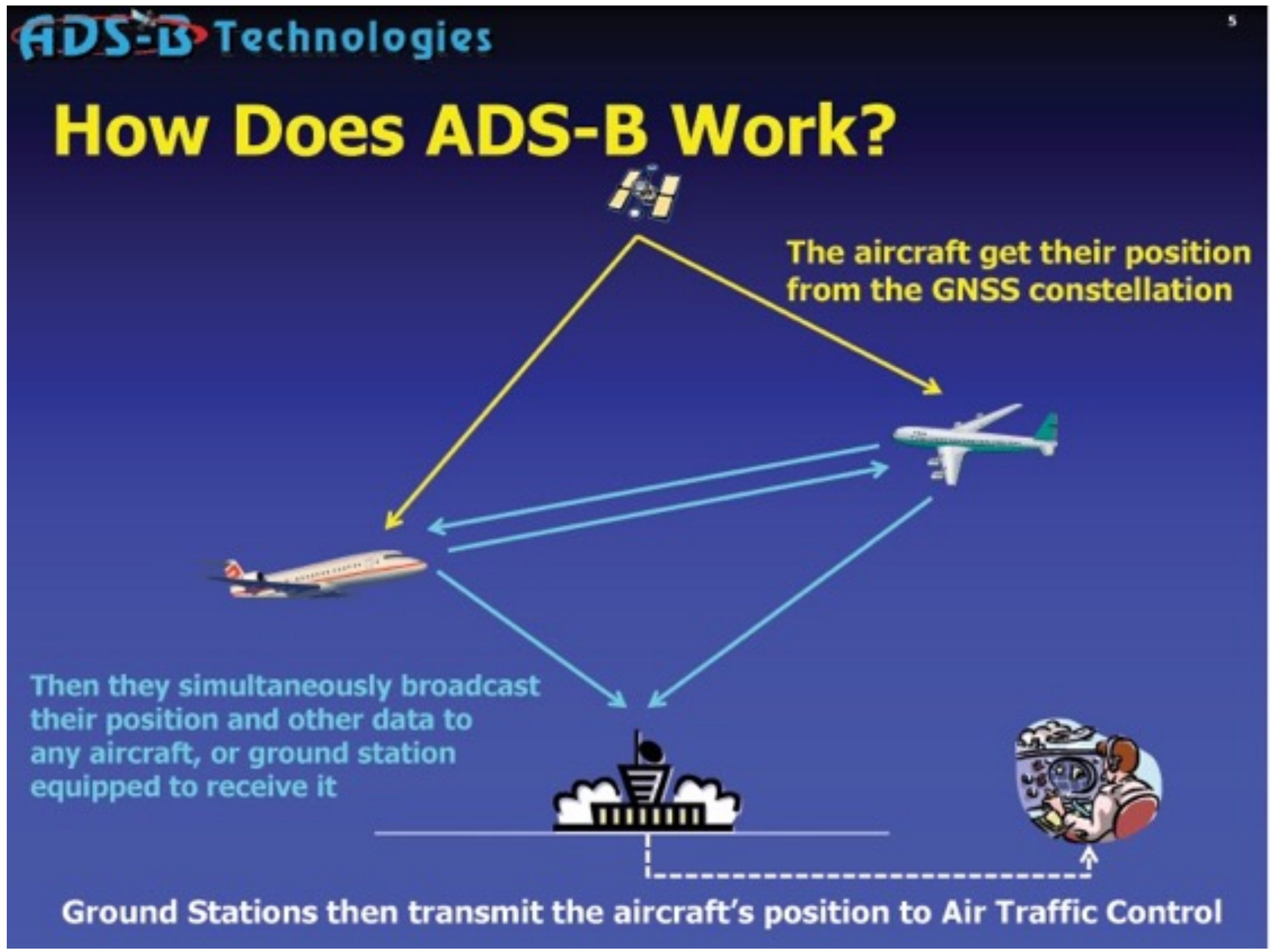
Part I: How to get spectrum in practice?

1. First need to crop signal
    - Gibbs ringing
    - Spectral broadening
2. Window chunk
3. FFT

Average



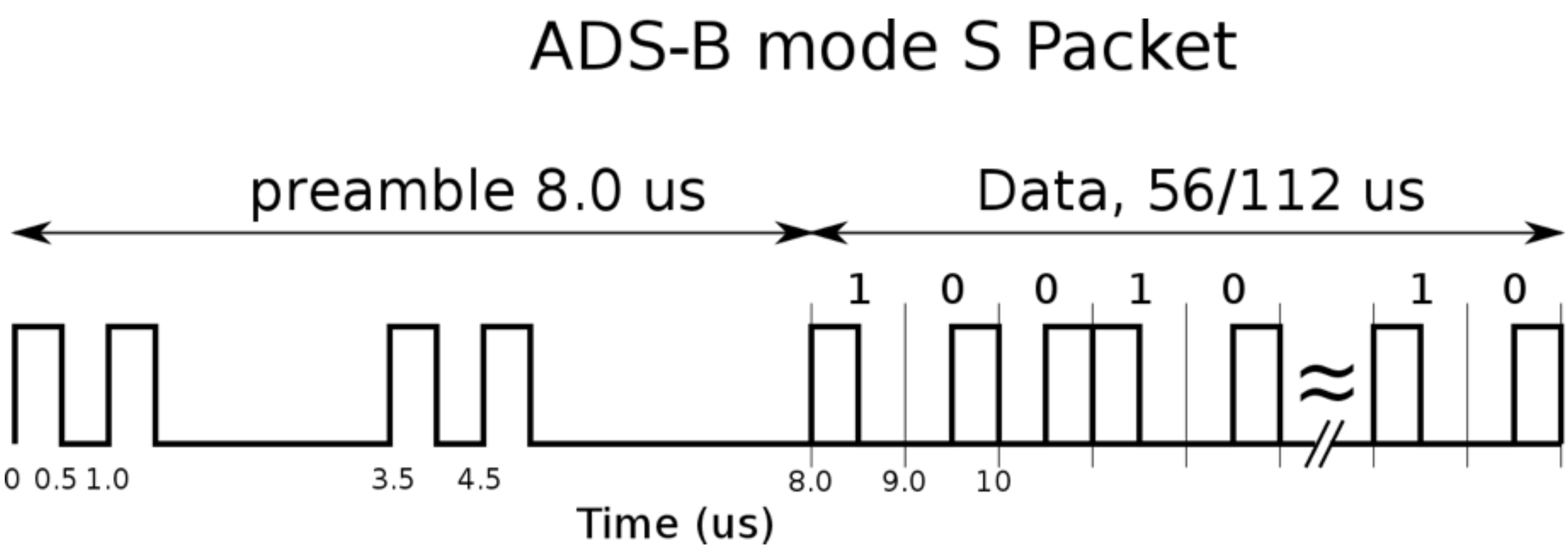
Part II: ADS-B



By 2020, all aircraft operating in the US airspaces (A, B, C) will be required to carry equipment that produces an ADS-B Out broadcast.

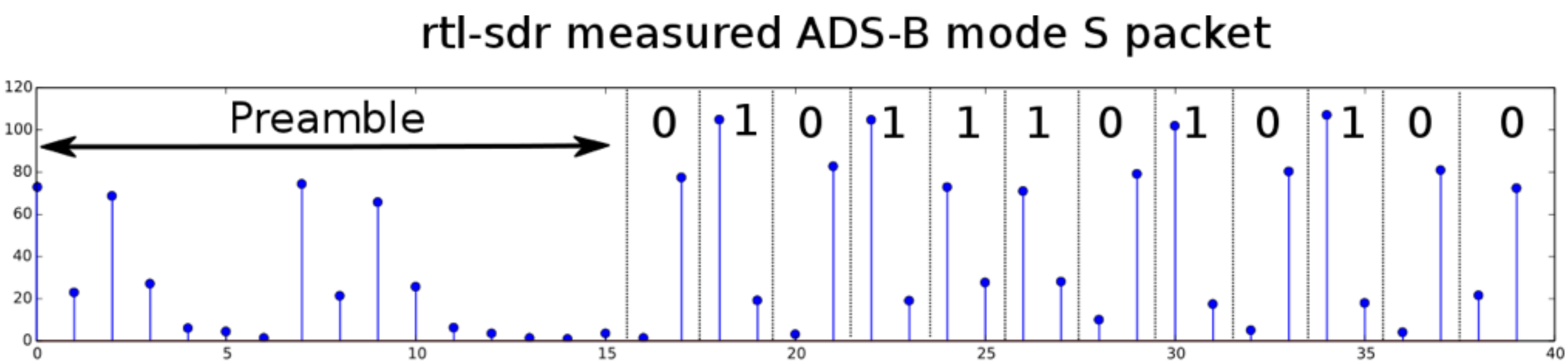
Part II: ADS-B Detection

- Use preamble for detection
  - Data does not contain pattern like preamble

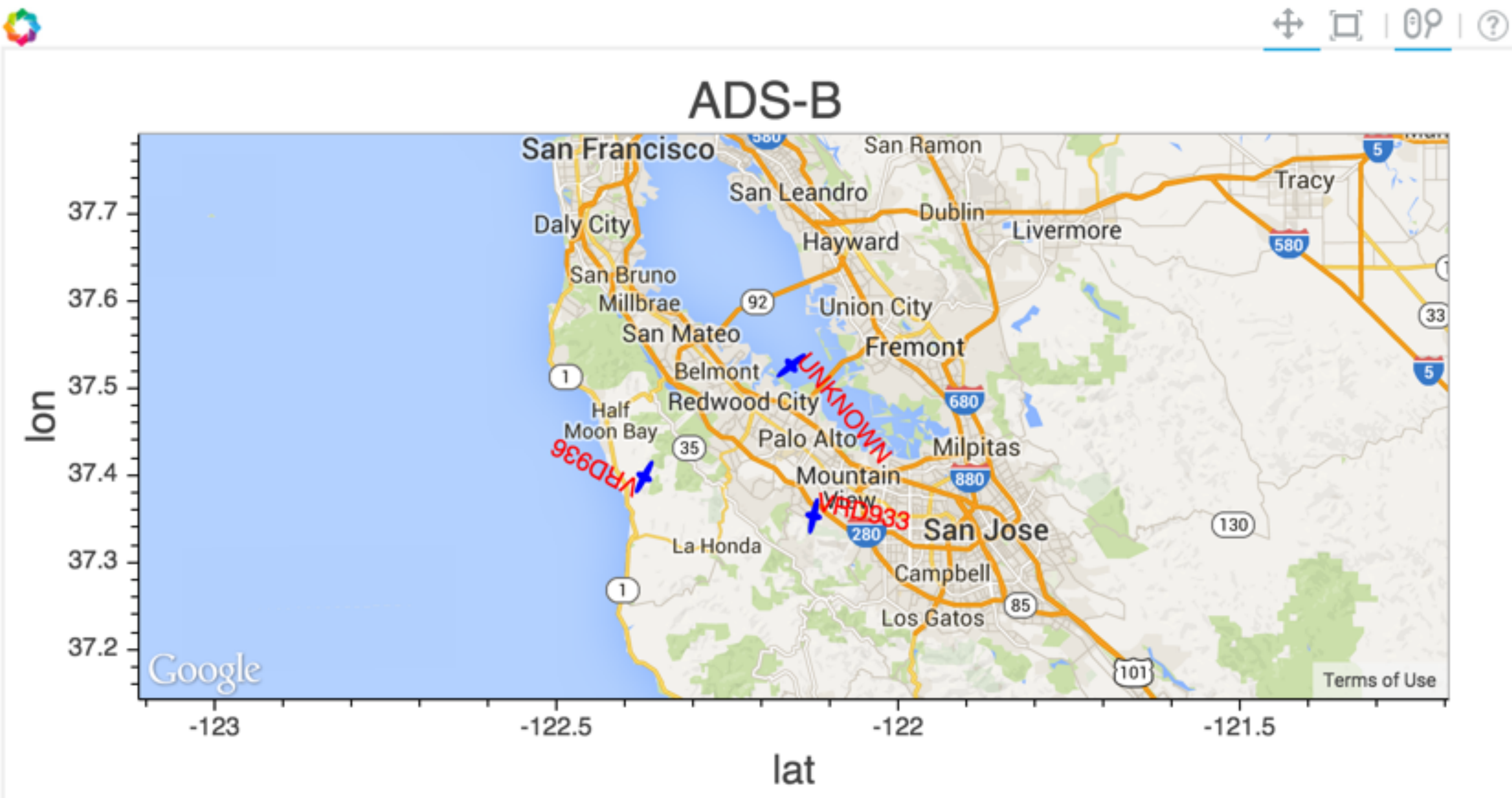


Part II: ADS-B Detection

1. Matched filtering
2. Logic



Real-Time ADS-B

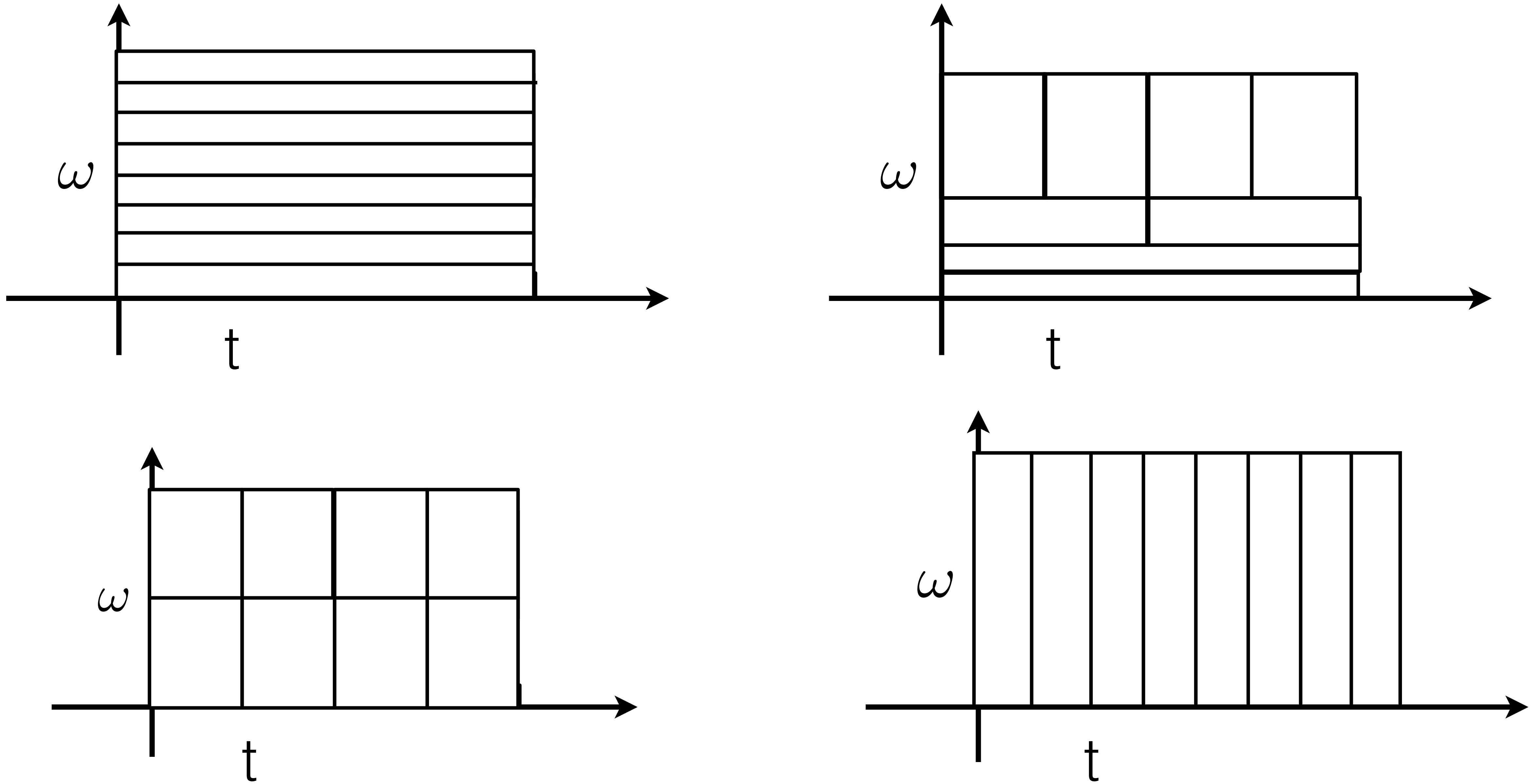



```
Found 1 plane
Found 2 planes
Found 3 planes
Found 5 planes
```

Motivation for wavelet transform

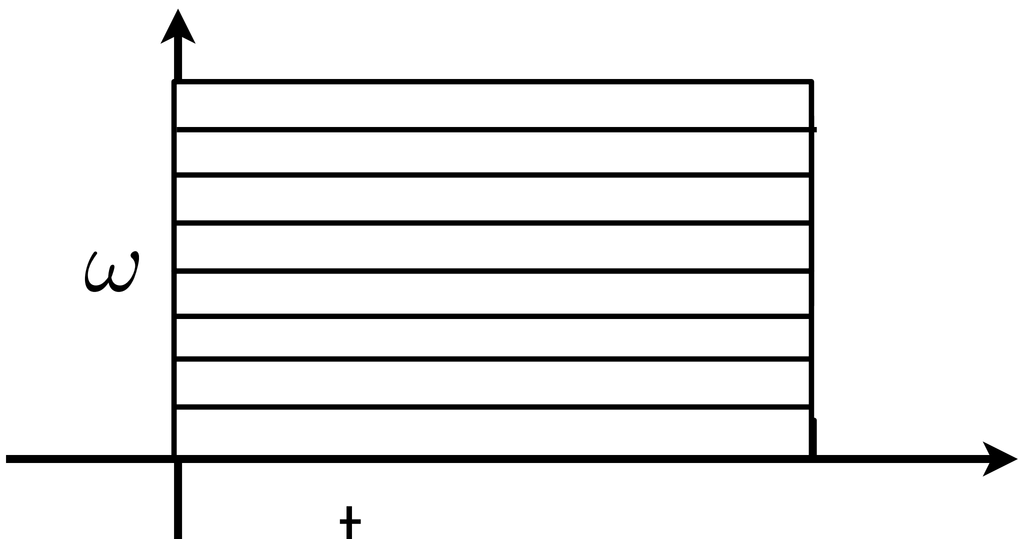
- **Fourier Transform**
    - does not show temporal information
- **Short Time Fourier Transform**
    - Same tradeoff between temporal and spectral resolution across all frequency (Same Heisenberg boxes)
- **Wavelet Transform**
    - high temporal resolution for high frequency
    - high spectral resolution for low frequency

Which frequency tiling is which?

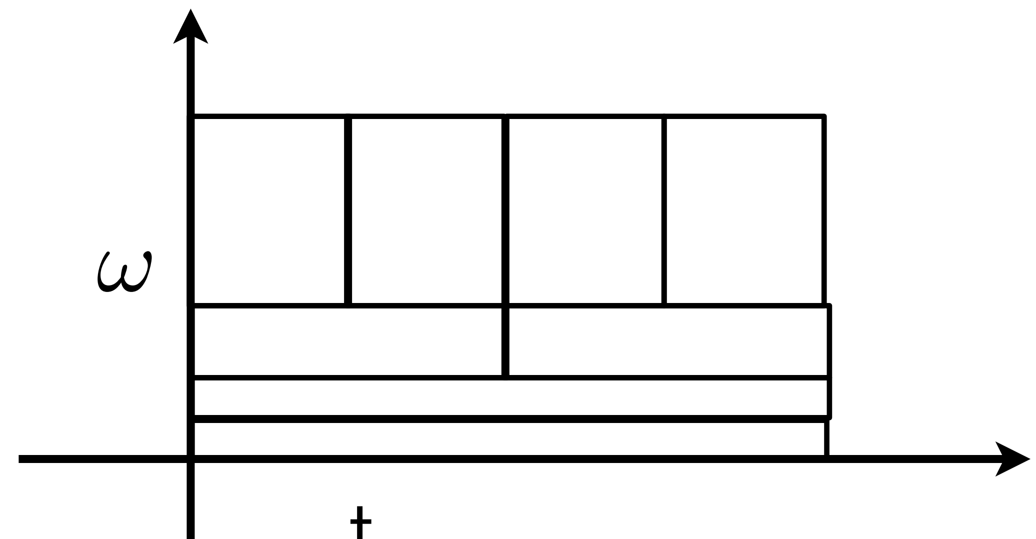


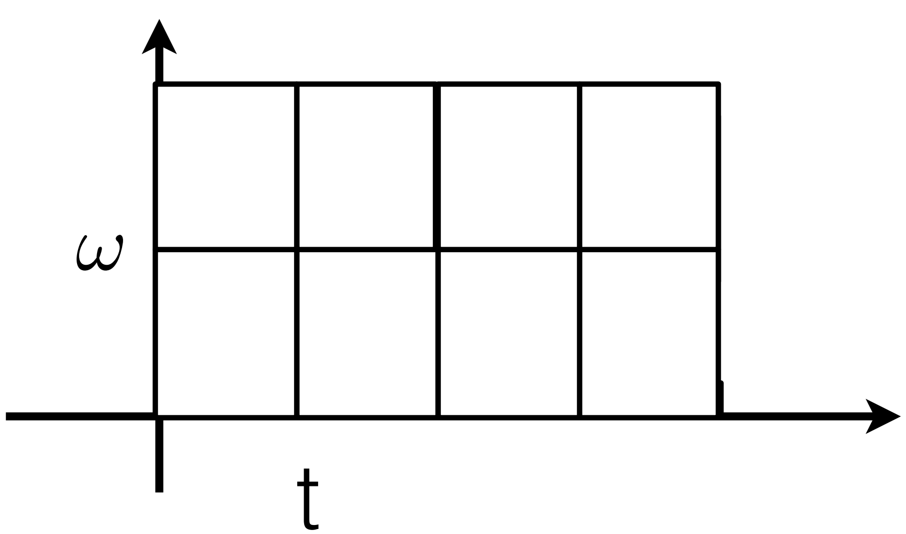
Which frequency tiling is which?

- Fourier Transform

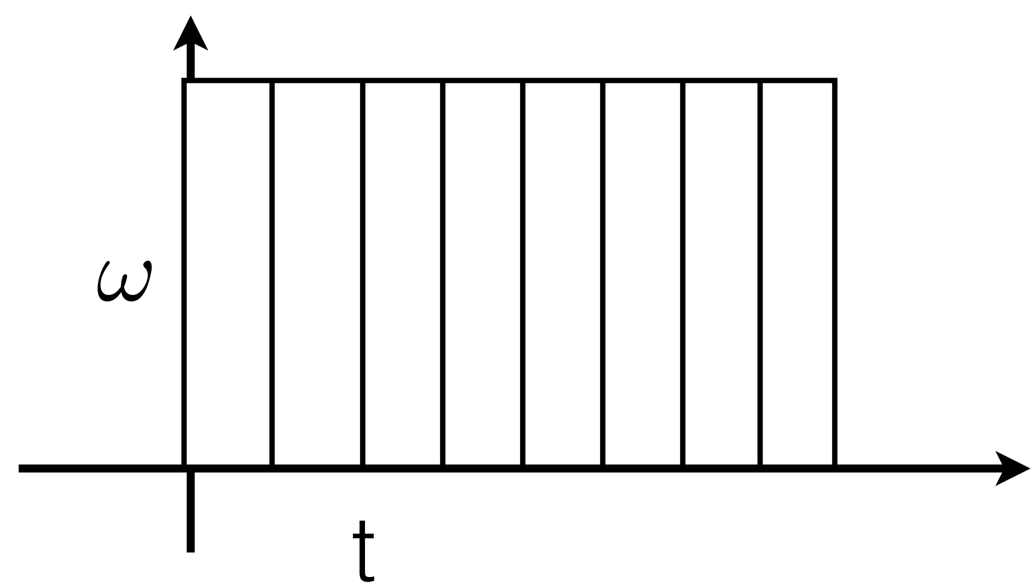


- Wavelet





- Short Time Fourier Transform

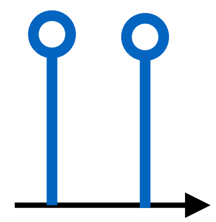


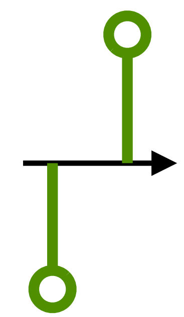
- Time Domain

How to realize wavelet time-frequency tiles?

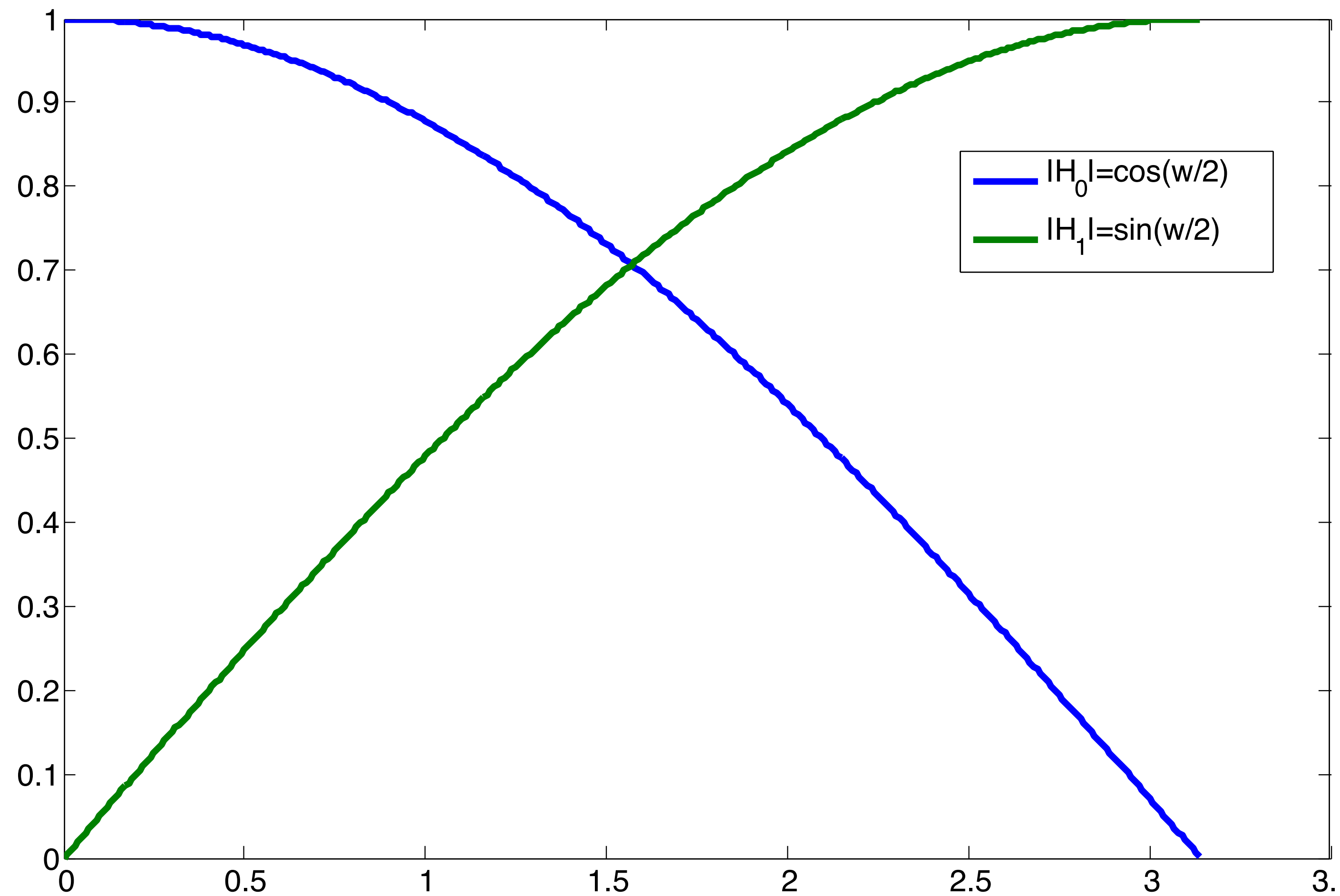
Use filters!

Average



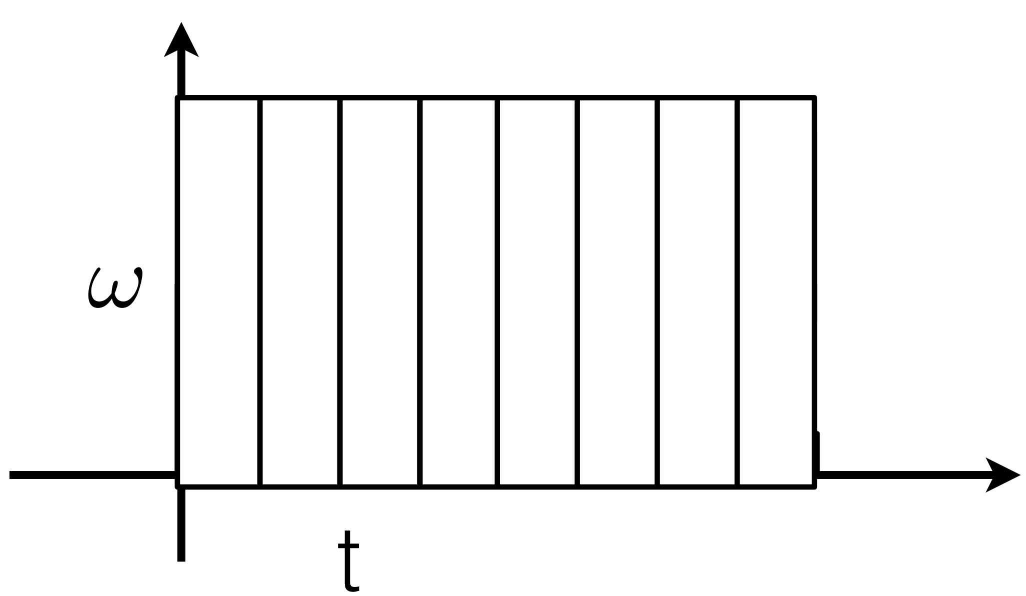


Difference

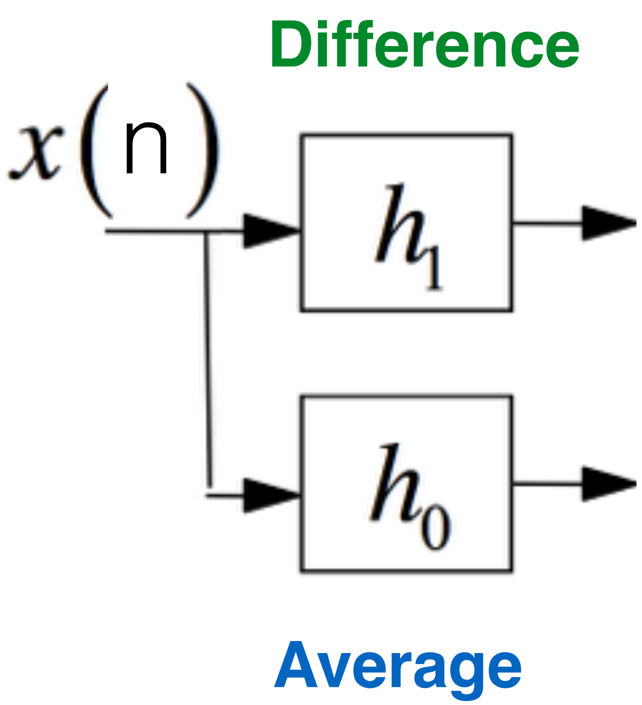


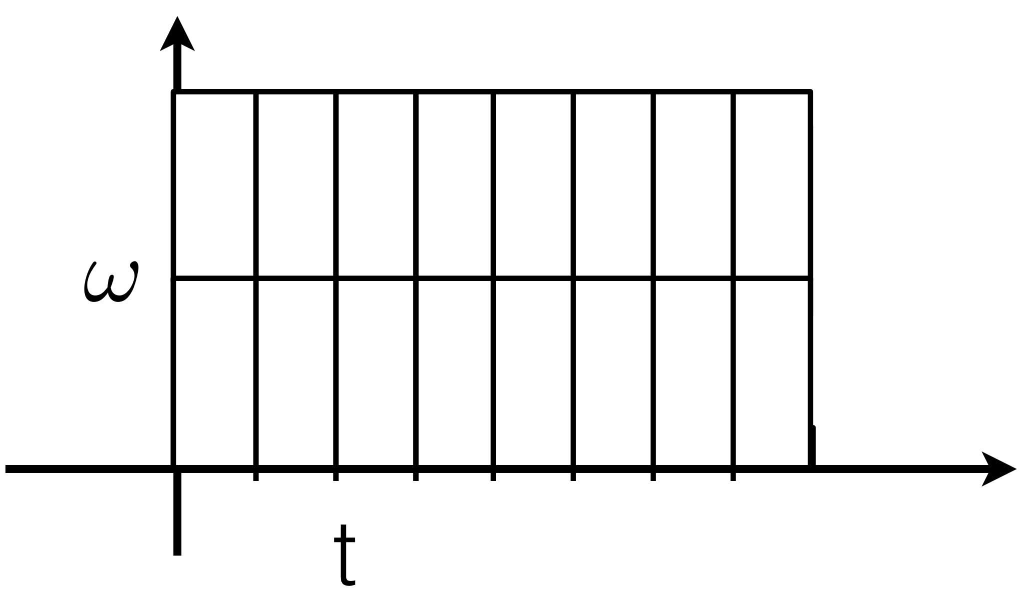
How to realize wavelet time-frequency tiles?

$x(n)$

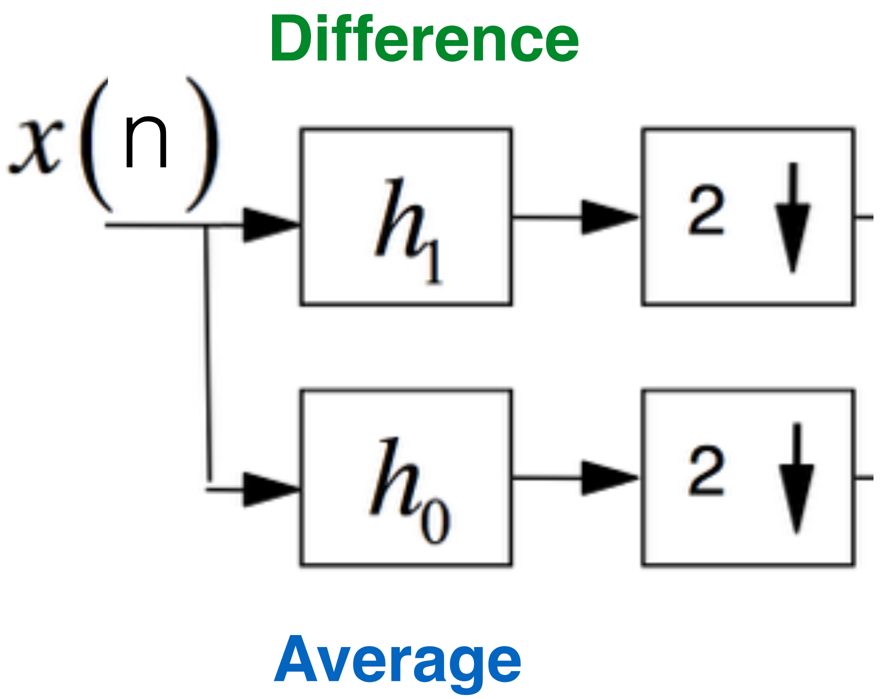


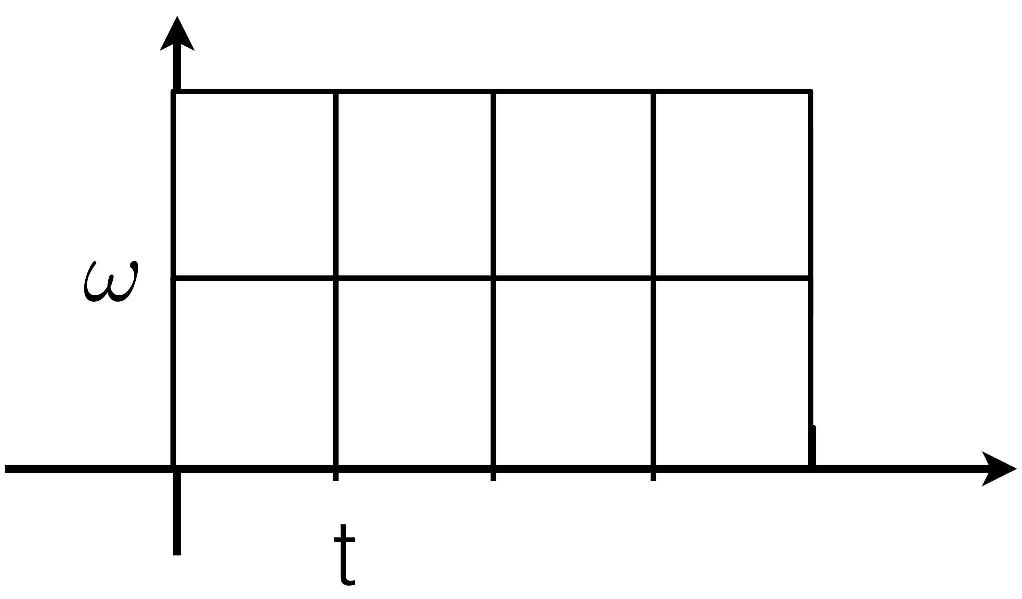
How to realize wavelet time-frequency tiles?



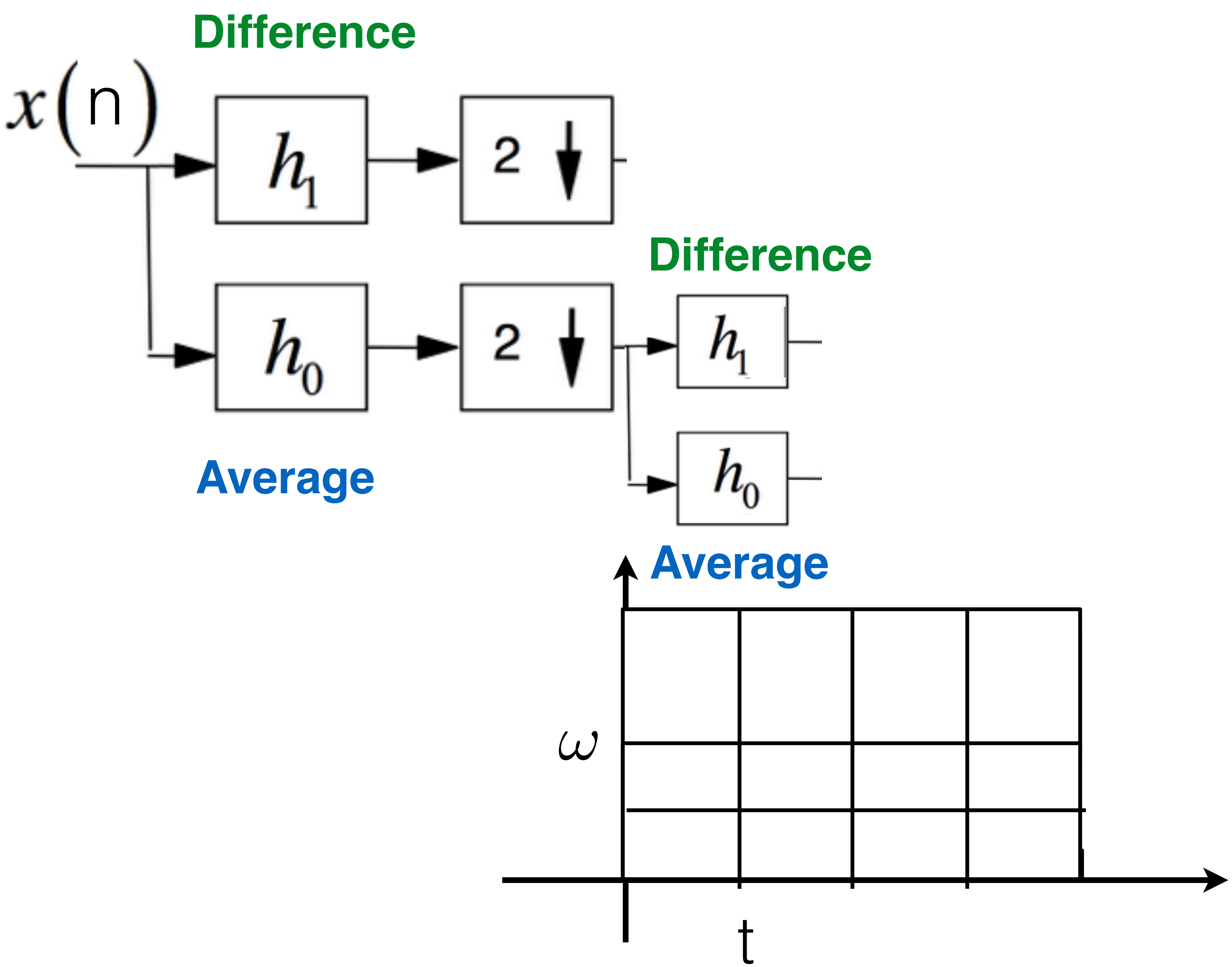


How to realize wavelet time-frequency tiles?

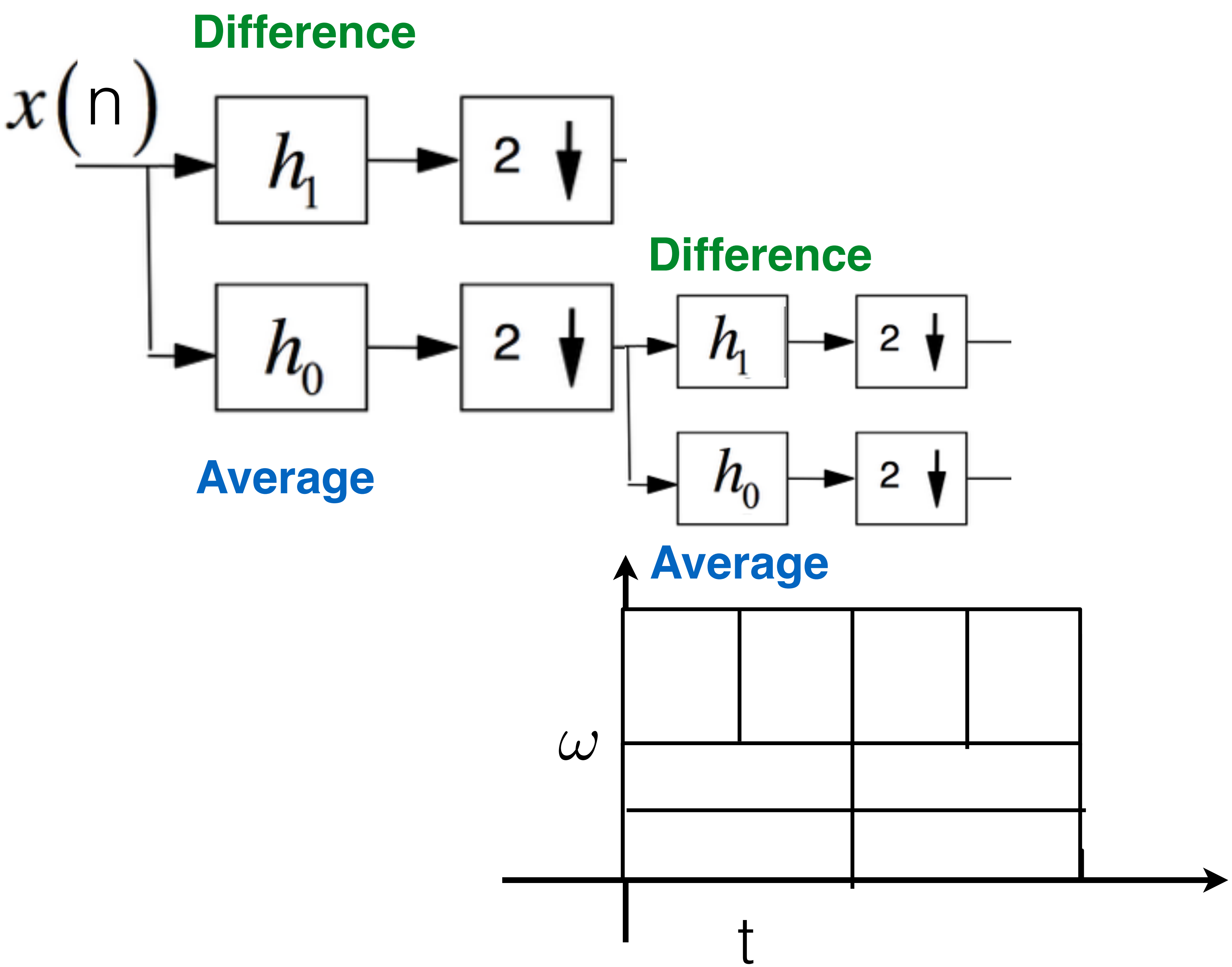




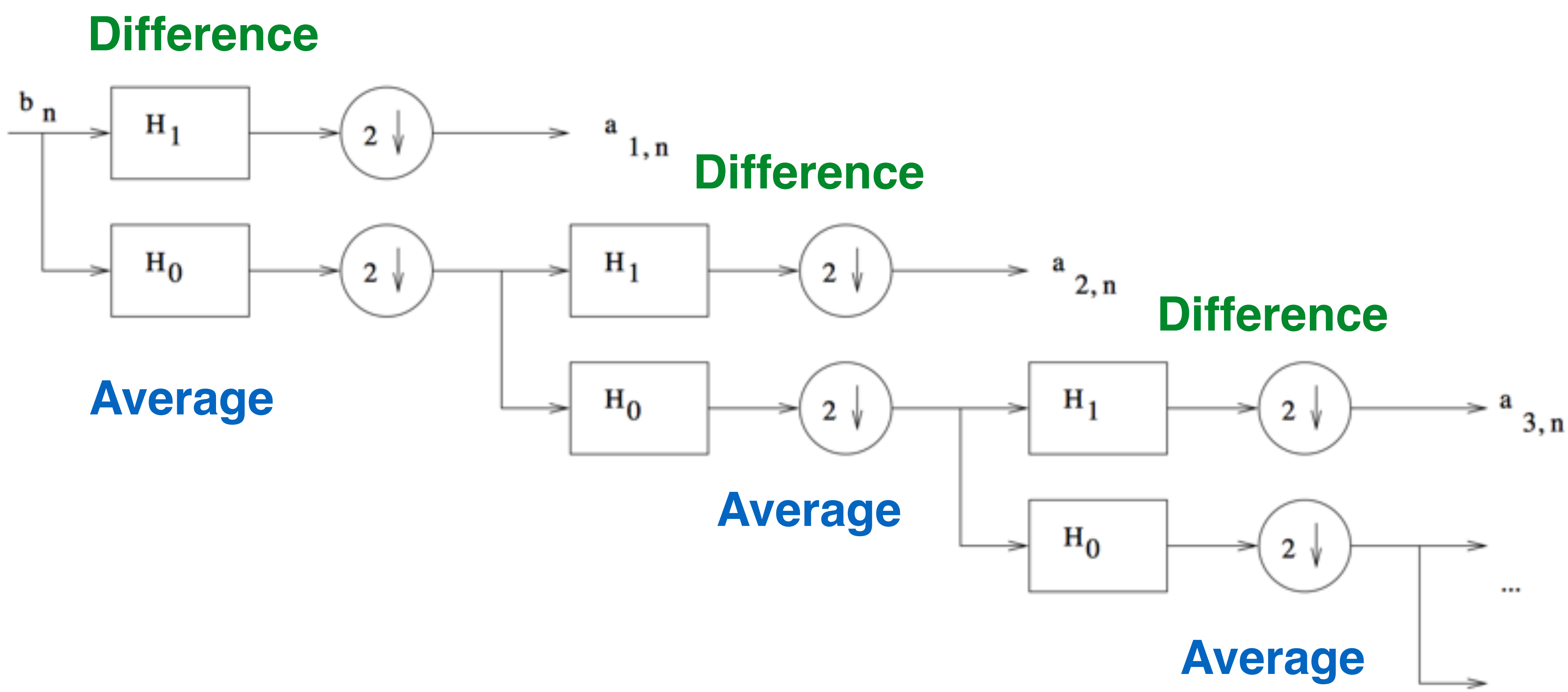
How to realize wavelet time-frequency tiles?



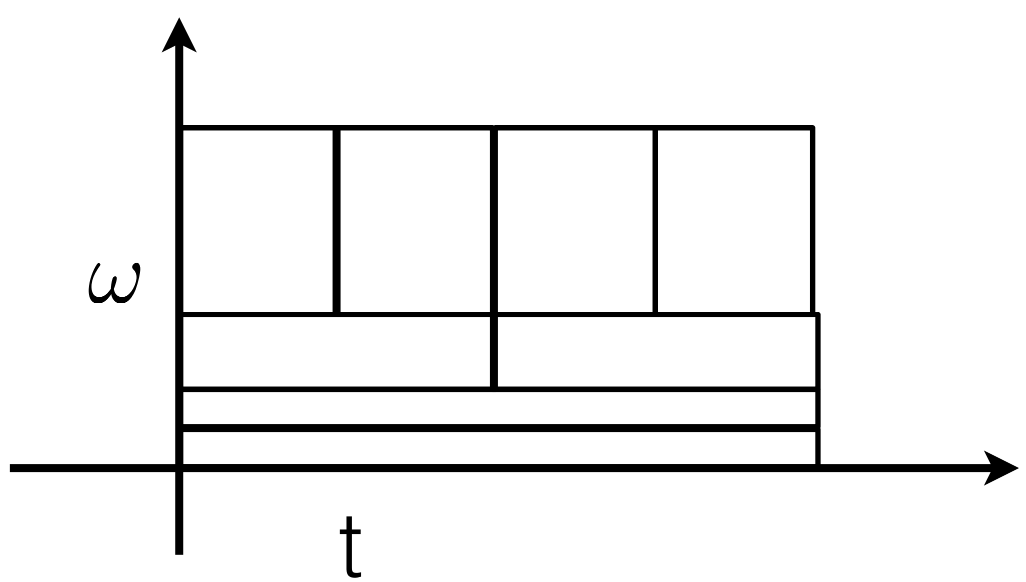
How to realize wavelet time-frequency tiles?



Multiscale Representation



- Iterate on the average subband



Haar decomposition example (ignoring scaling)

- x[n] = [5, 5, 5, 5, 6, 6, 7, 7]
  - xd1 = [0, 0, 0, 0]
  - xa1 = [10, 10, 12, 14]
    - xd2 = [0, 2]
    - xa2 = [20, 26]
      - xd3 = [-6]
      - xa3 = [46]

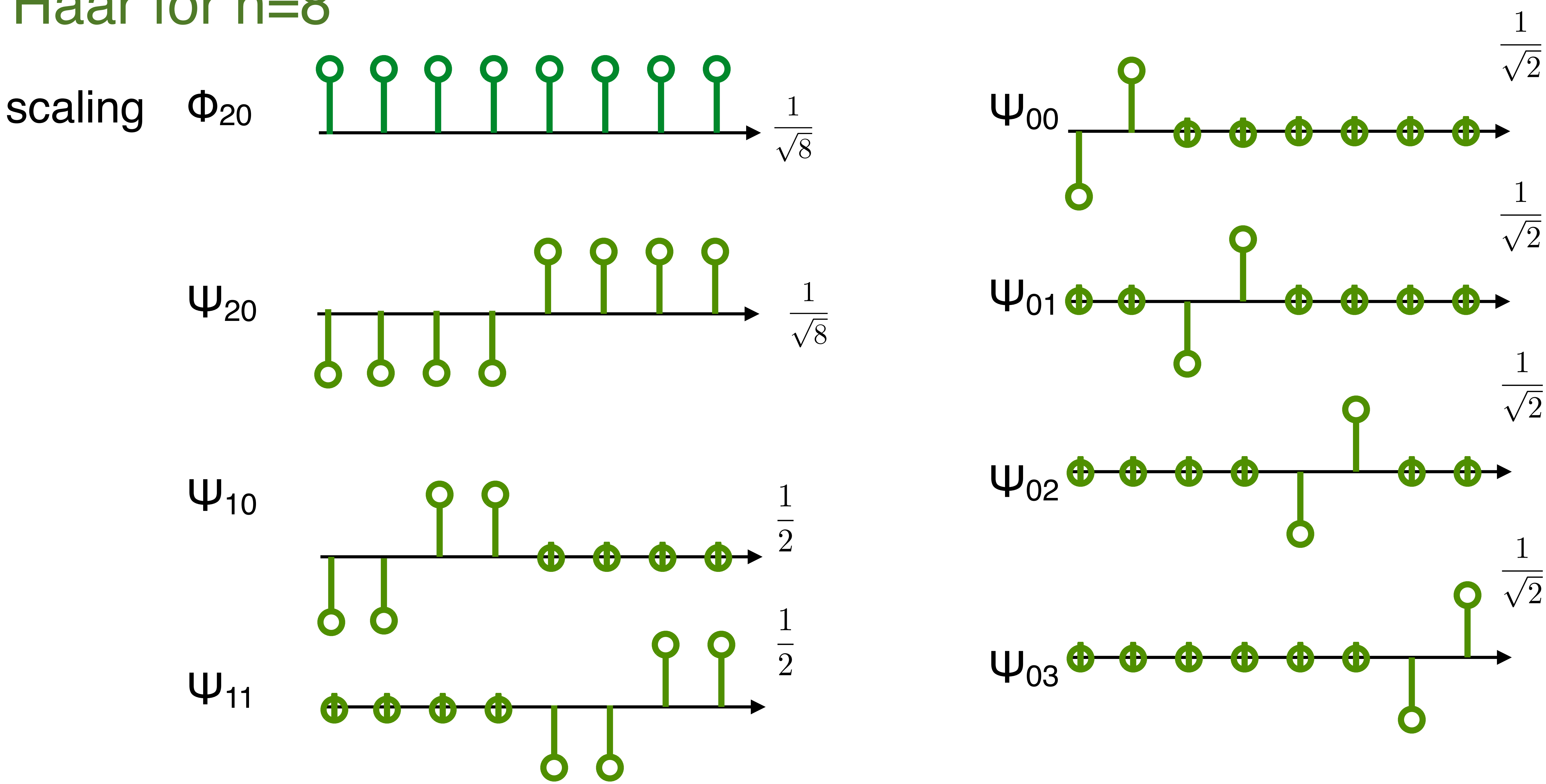
Haar decomposition example (ignoring scaling)

- x[n] = [5, 5, 5, 5, 6, 6, 7, 7]
  - **xd1 = [0, 0, 0, 0]**
  - xa1 = [10, 10, 12, 14]
    - **xd2 = [0, 2]**
    - xa2 = [20, 26]
      - **xd3 = [6]**
      - **xa3 = [46]**

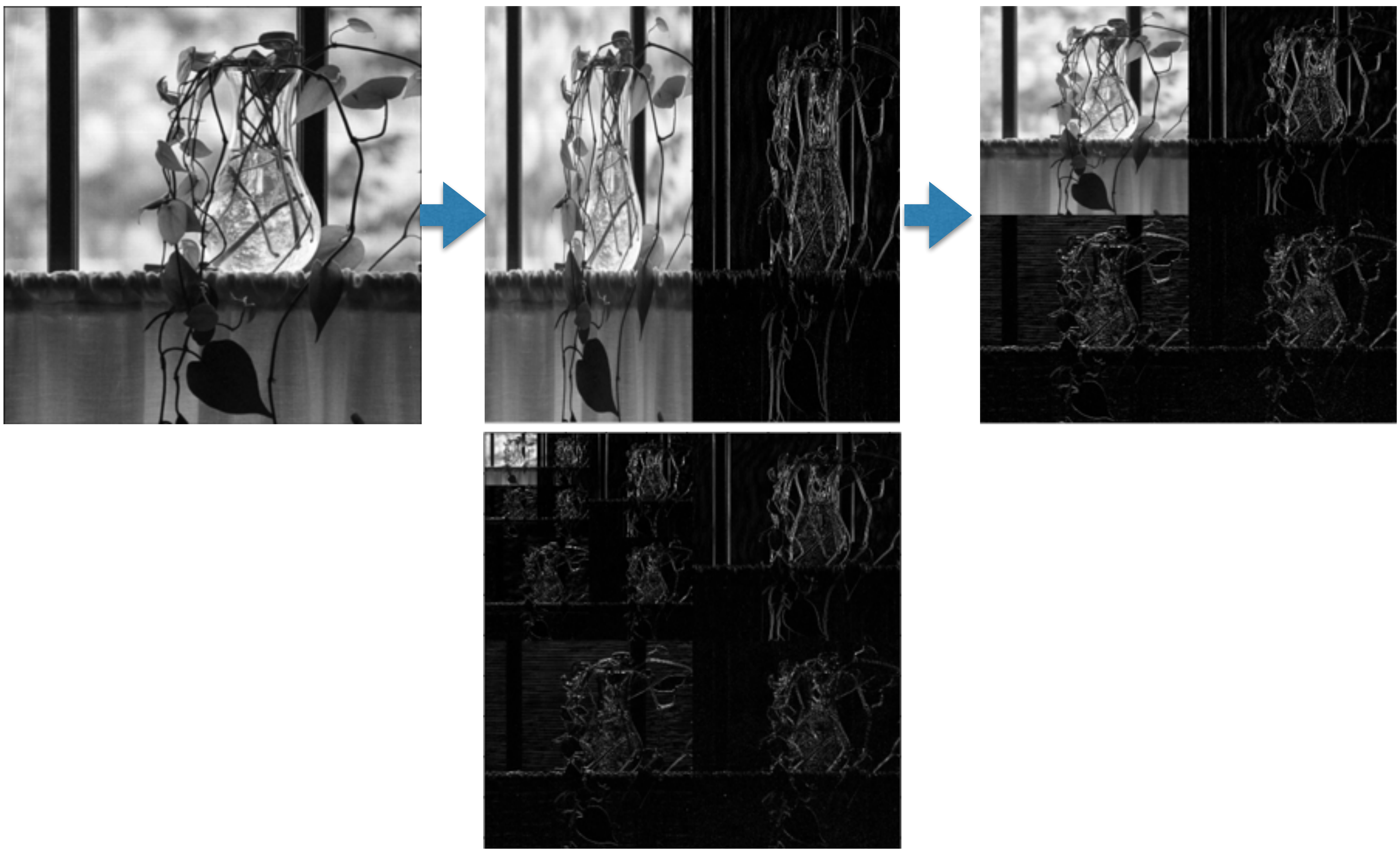
Save

Haar Wavelet
Basis Functions

Haar for n=8



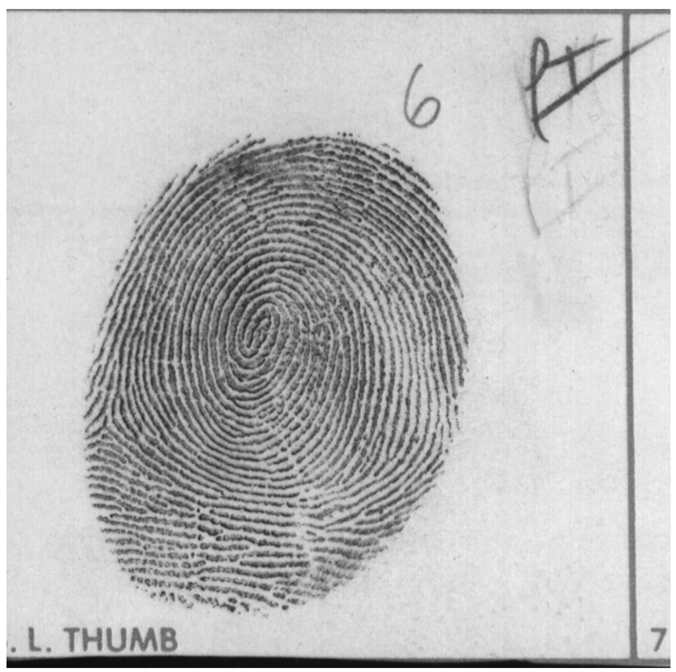
2D Wavelet Transform

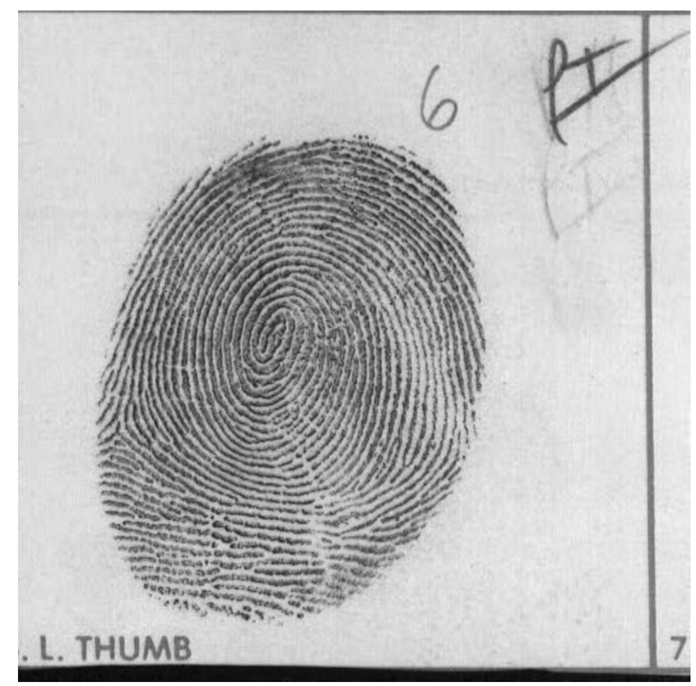


Wavelet application: Fingerprint compression

The FBI needs to digitize its ~200 million fingerprint records
=> need compression

Original



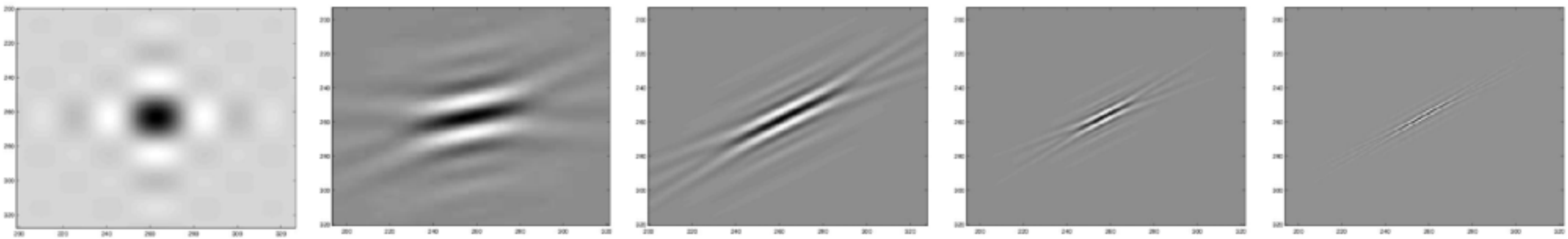


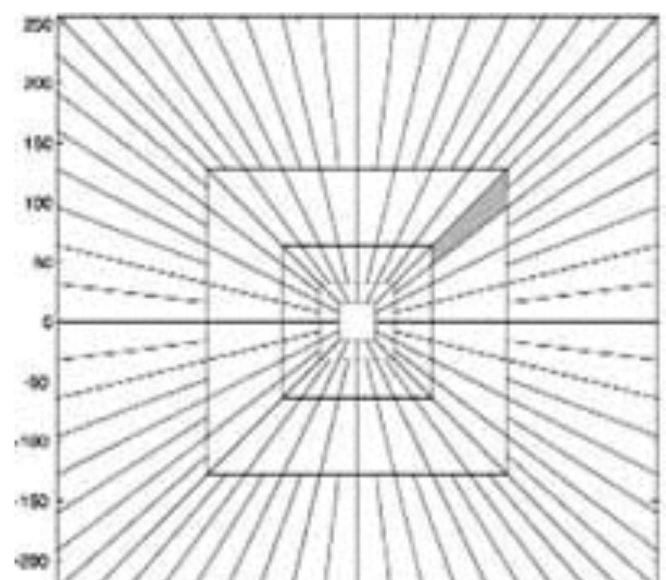
Wavelet

Wavelet

JPEG

Going beyond wavelet: Curvelet





[Activelet] [AMlet] [Aniset*] [Armlet] [Bandlet] [Barlet] [Bathlet] [Beamlet] [Binlet] [Bumplet*] [Brushlet] [Camplet] [Caplet] [Chirplet] [*Chordlet] [Circlet] [Coiflet] [Contourlet] [Cooklet] [Coslet*] [Craplet] [Cubelet*] [CURElet] [Curvelet] [Daublet] [Directionlet] [Dreamlet*] [Edgelet] [ERBlet] [FAMlet*] [FLaglet*] [Flatlet] [Formlet] [Fourierlet*] [Framelet] [Fresnelet] [Gaborlet] [Gabor shearlet*] [GAMlet] [Gausslet] [Graphlet] [Grouplet] [Haarlet] [Haardlet] [Heatlet] [Hutlet] [Hyperbolet] [Icalet (Icalette)] [Interpolet] [Lesslet (cf. Morelet)] [Loglet] [Marrlet*] [MIMOlet] [Monowavelet*] [Morelet] [Morphlet] [Multiselectivelet] [Multiwavelet] [Needlet] [Noiselet] [Ondelette/wavelet] [Ondulette] [Prewavelet*] [Phaselet] [Planelet] [Platelet] [Purelet] [Quadlet/q-Quadlet*] [QVlet] [Radonlet] [RAMlet] [Randlet] [Ranklet] [Ridgelet] [Riezlet*] [Ripplet (original, type-I and II)] [Scalet] [S2let*] [Seamlet] [Seislet] [Shadelet*] [Shapelet] [Shearlet] [Sinclet] [Singlet] [Sinlet*] [Slantlet] [Smoothlet] [Snakelet*] [SOHOlet] [Sparselet] [Speclet*] [Spikelet] [Splinelet] [Starlet*] [Steerlet] [Stokeslet*] [Subwavelet (Sub-wavelet)] [Superwavelet] [SURE-let (SURElet)] [Surfacelet] [Surflet] [Symlet/Symmlet] [S2let*] [Tetrolet] [Treelet] [Vaguelette] [Walet*] [Wavelet-Vaguelette] [Wavelet] [Warblet] [Warplet] [Wedgelet] [Xlet/X-let]

Problem 1

- Compute the Haar wavelet transform of the following signal (ignore scaling):
  - $x[n] = [1, 1, 2, 10, 4, 4, 1, 1]$

Solution 1

- x[n] = [1, 1, 2, 10, 4, 4, 1, 1]
- **xd1 = [0, 8, 0, 0]**
- xa1 = [2, 12, 8, 2]
    - **xd2 =[10, -6]**
    - xa2 = [14, 10]
        - **xd1 = [-4]**
        - **xa1 = [24]**

Problem 2

- x[n] = ???
- **xd1 = [0, 0, 0, 0]**
- xa1 = ???
    - **xd2 =[10, -5]**
    - xa2 = ???
        - **xd1 = [-4]**
        - **xa1 = [24]**

Reconstruct x[n] from its Haar wavelet coefficients

Solution 2

- x[n] = [1, 1, 6, 6, 3.75, 3.75, 1.25, 1.25]
- **xd1 = [0, 0, 0, 0]**
- xa1 = [2, 12, 7.5, 2.5]
    - **xd2 =[10, -5]**
    - xa2 = [14, 10]
        - **xd1 = [-4]**
        - **xa1 = [24]**

Problem 3

Problem 2. Consider the uncertainty relation $\Delta_w^2 \cdot \Delta_t^2 \geq \pi/2$, where $\Delta_t^2 = \int_{-\infty}^{+\infty} t^2 \left|f(t)\right|^2 dt$ and $\Delta_\omega^2 = \int_{-\infty}^{+\infty} \omega^2 \left|F(\omega)\right|^2 d\omega$

Can you give the time-bandwidth product of a rectangular pulse, $p(t) = 1,\ -1/2 \leq t \leq 1/2$, 0 else?

Hint: Approximate the sinc(x) function with 1/x

Solution 3

- The frequency deviation for a rect function blows up to infinity, so the time-frequency product is infinity

- Interestingly, the time-frequency product for a triangular pulse is finite because we have a sinc^2 instead. The product has the value:

$$\Delta_t^2 \Delta_\omega^2 = \left(\frac{1}{10}\right)(6\pi) = 0.6\pi \geq \frac{\pi}{2}$$