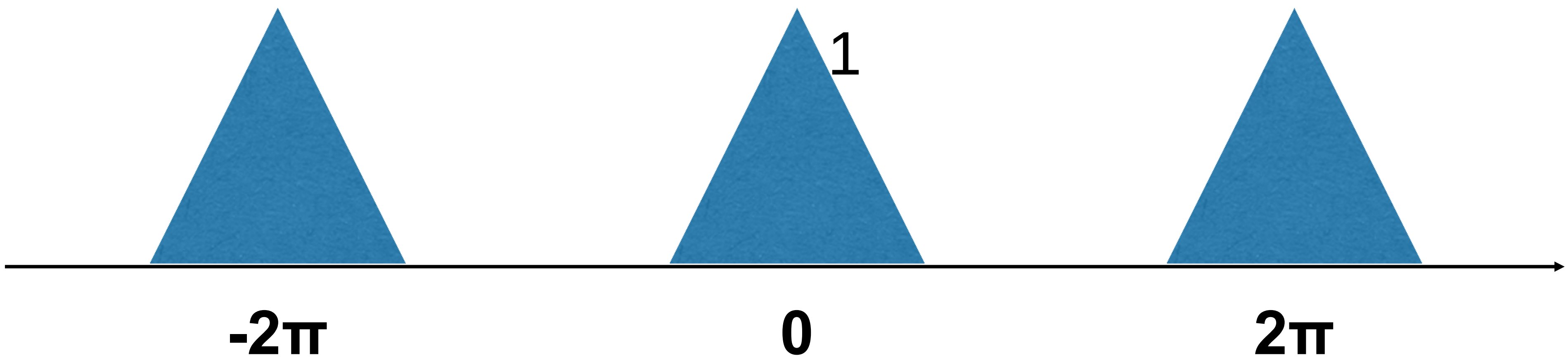
EE123 Spring 2015
Discussion Section 7

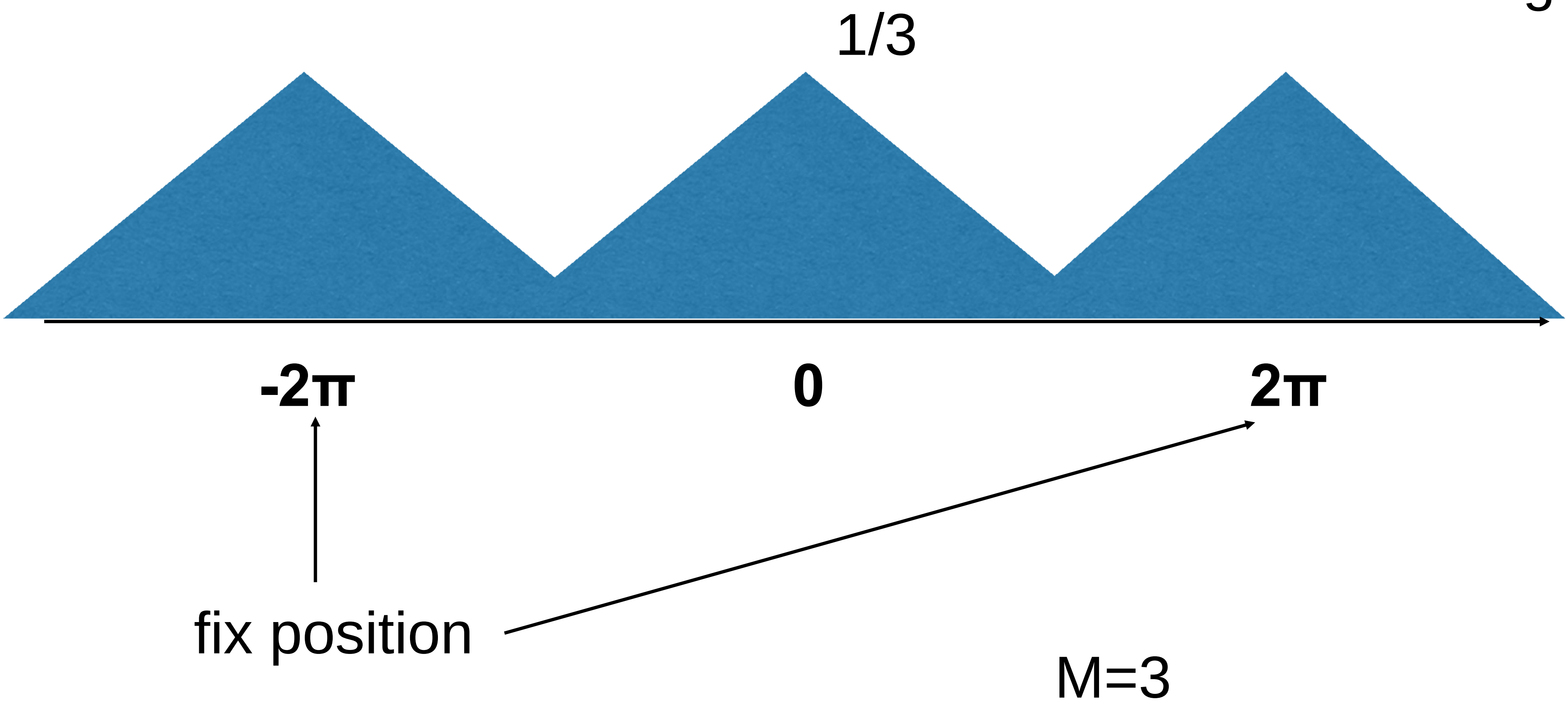
Giulia Fanti (based on slides by Frank Ong)

Effect of discrete time downsampling in spectral domain

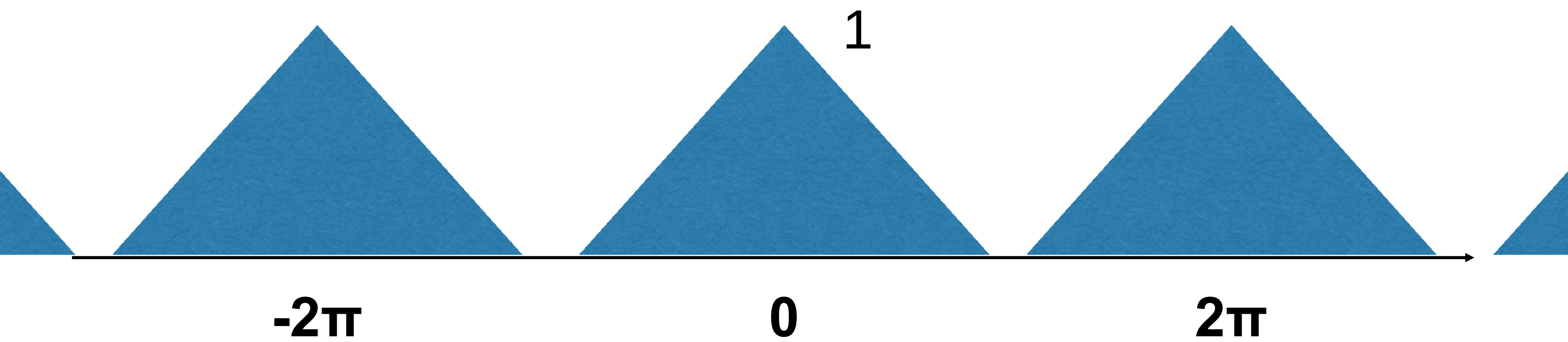


Effect of discrete time undersampling in spectral domain

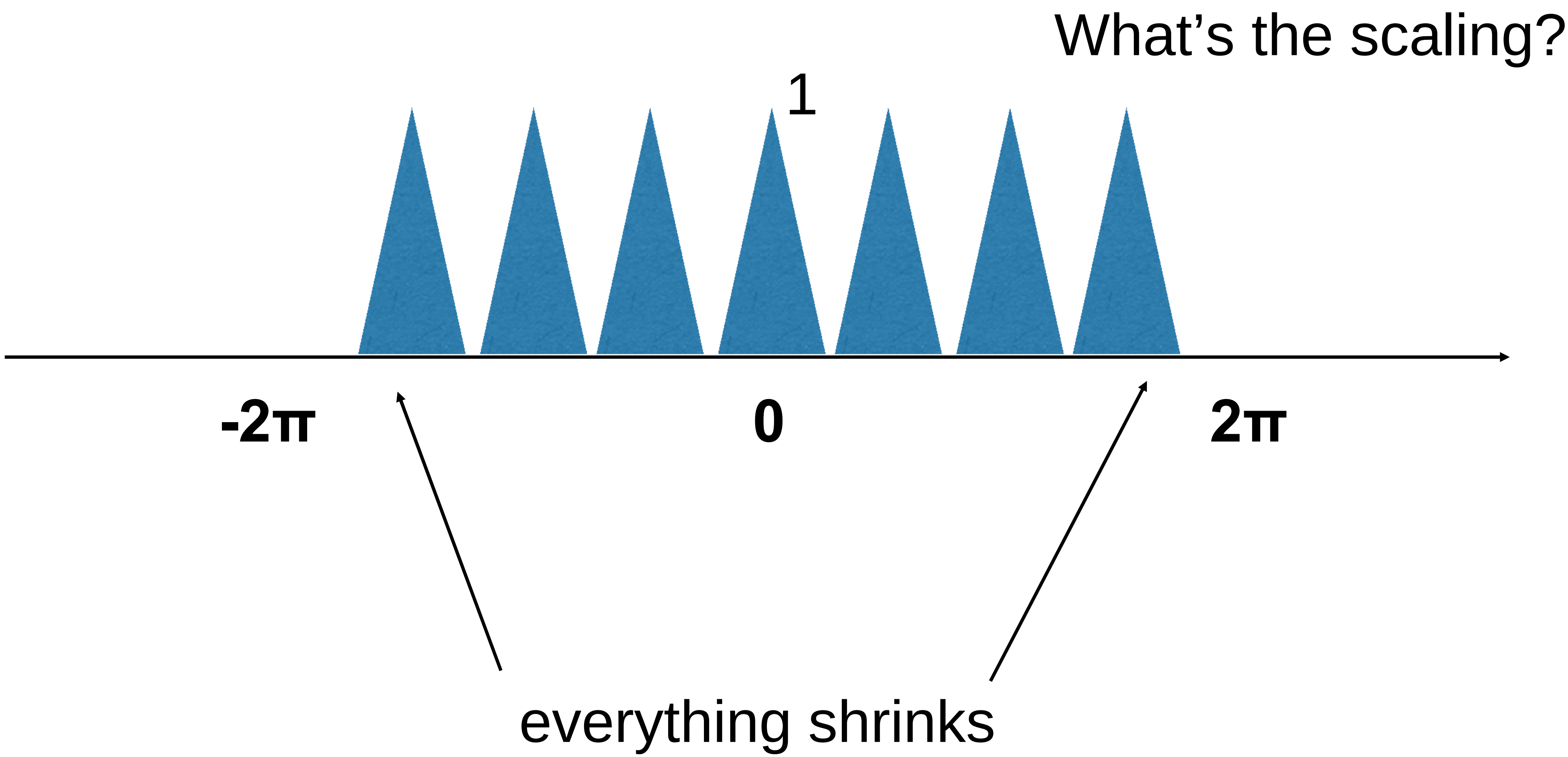
What's the scaling?



Effect of discrete time upsampling in spectral domain

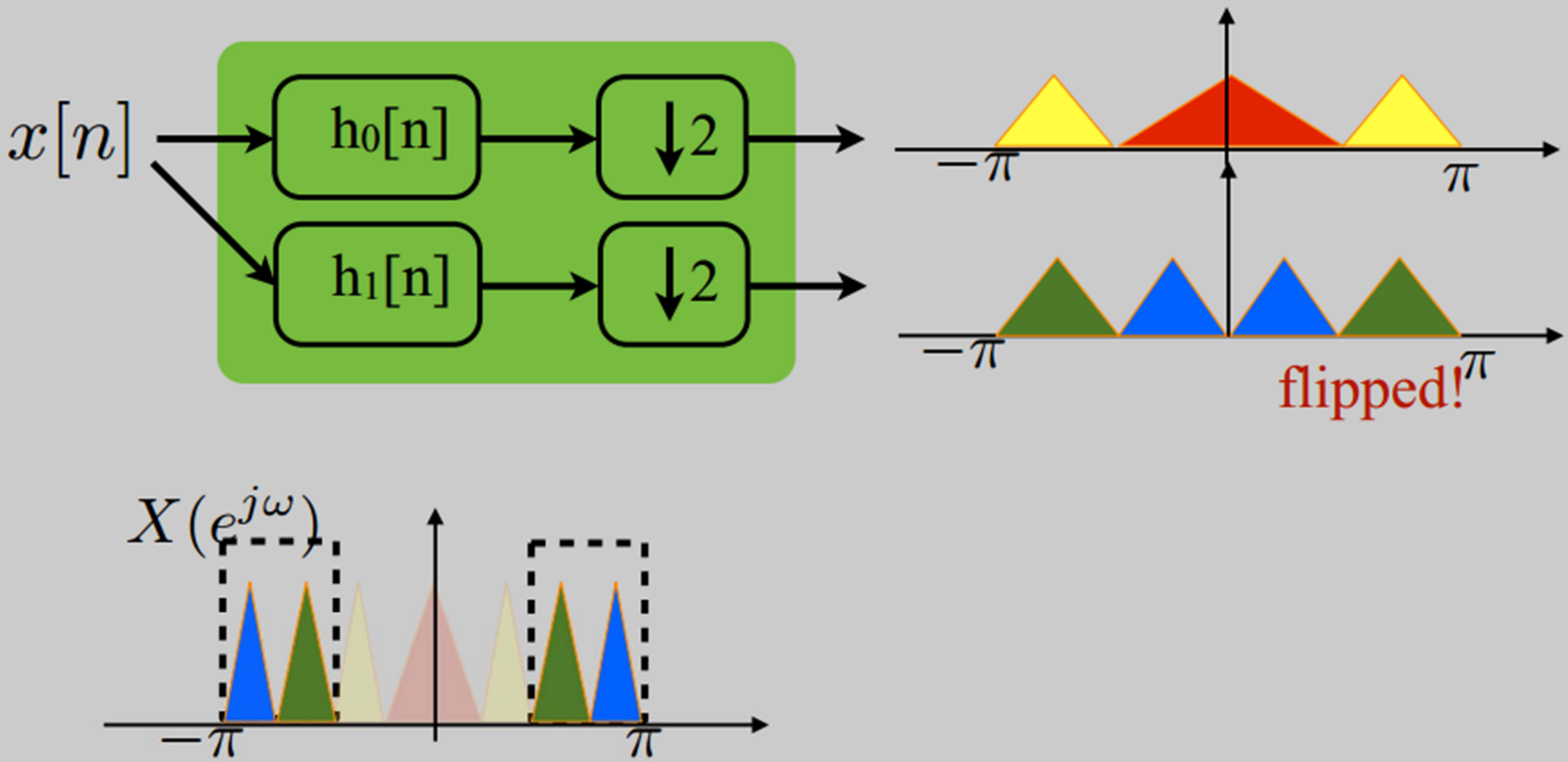


Effect of discrete time upsampling in spectral domain



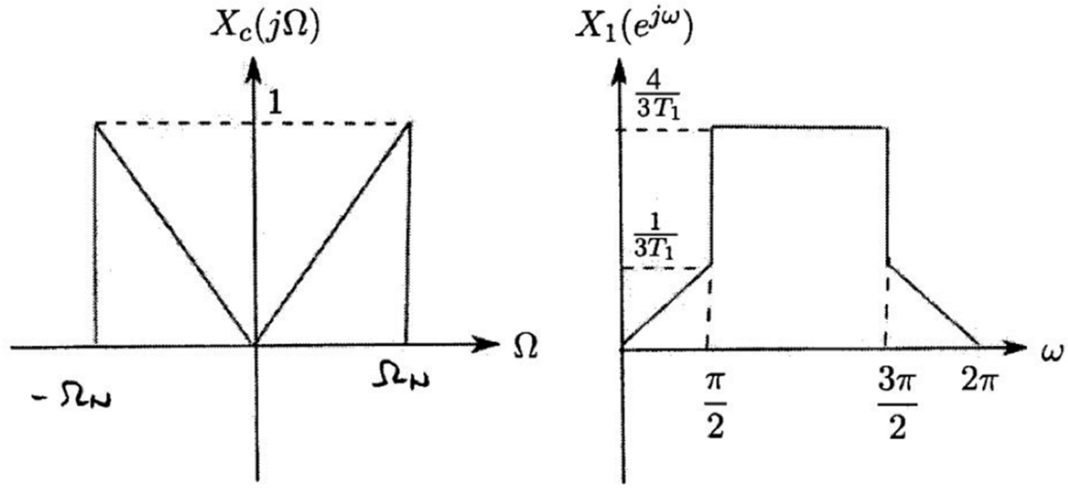
Subtleties in Time-Freq Tiling

- Assume h_0, h_1 are ideal low,high pass filters



Question 1

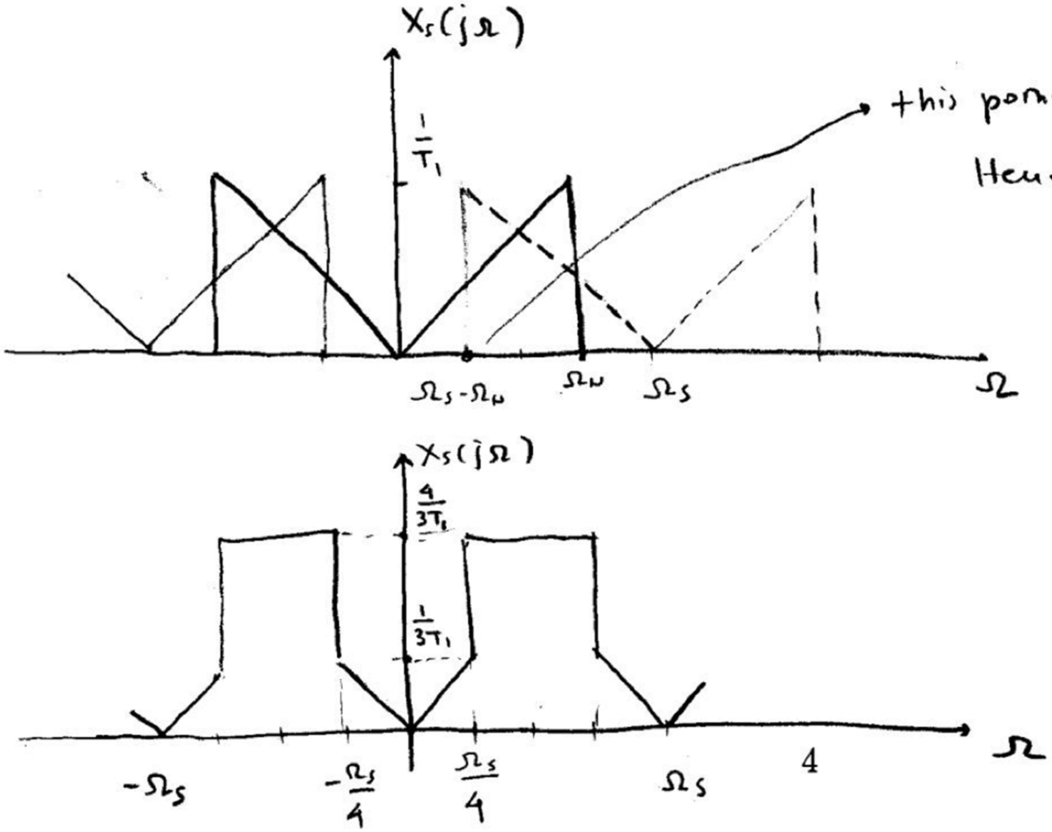
2. A continuous time signal $x_c(t)$ with the spectrum $X_c(j\Omega)$ depicted below is sampled with period T_1, resulting in a discrete sequence $x_1[n]$ with the DTFT $X_1(e^{j\omega})$ below.



a) (15 points) Determine the largest sampling period T_2 that would avoid aliasing, and express it in terms of T_1. Sketch the DTFT of the sequence $x_2[n]$, sampled with period T_2.

b) (15 points) Draw the block diagram of a post-processing unit that down-samples $x_2[n]$ by a factor of T_2/T_1. Sketch the DTFT of the output and compare it with $X_1(e^{j\omega})$ above.

Solution 1



→ this point maps to $\frac{\pi}{2}$.

Hence, $\Omega_s - \Omega_N = \frac{\Omega_s}{4}$

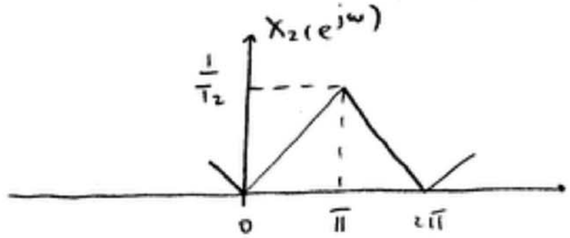
$\Rightarrow \Omega_s = \frac{4}{3}\Omega_N$

$\Omega_s = \frac{2\pi}{T_1}$

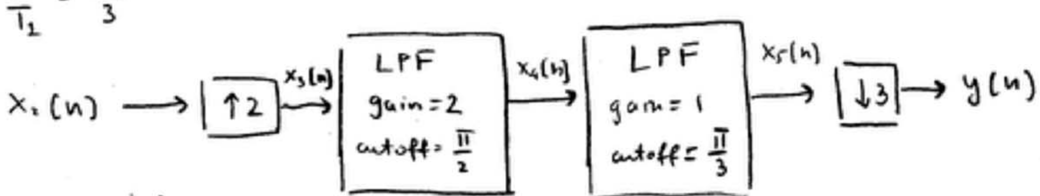
$\Rightarrow \boxed{\Omega_N = \frac{3}{4} \cdot \frac{2\pi}{T_1}}$

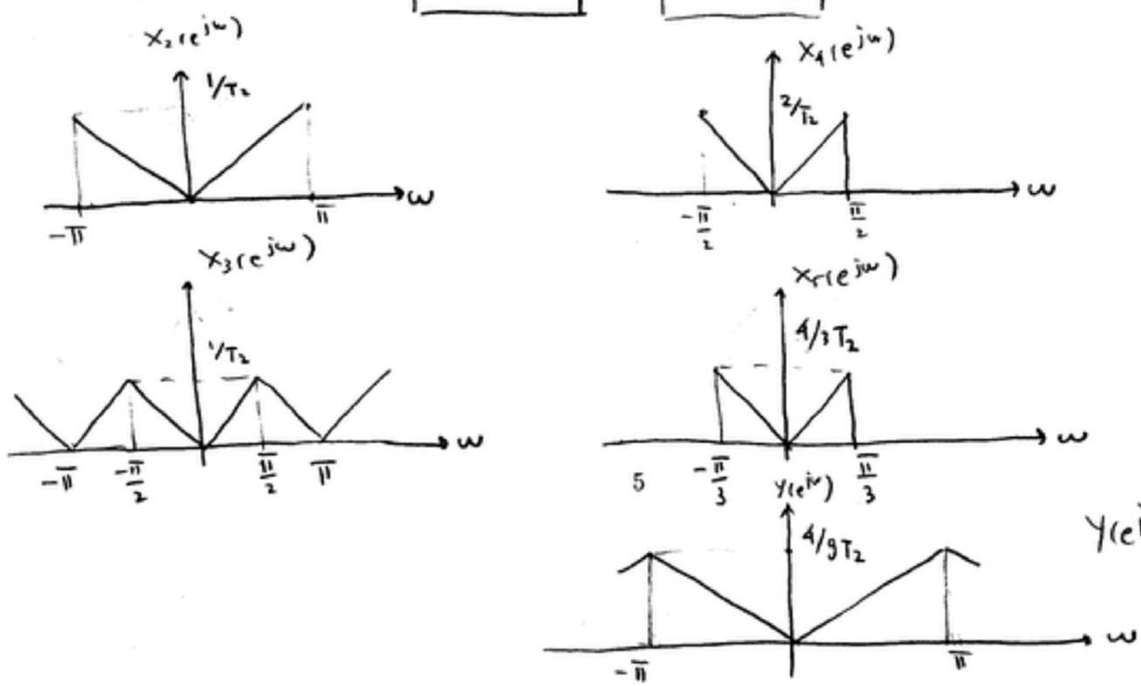
To avoid aliasing, we need

$$\frac{2\pi}{T_2} = 2\Omega_N = \frac{3\pi}{T_1} \Rightarrow \boxed{T_2 = \frac{2}{3} \cdot T_1}$$



b) $\frac{T_2}{T_1} = \frac{2}{3}$





$Y(e^{j\omega}) \neq X_1(e^{j\omega})$

This problem is ambiguously worded. Why?

Question 2 a,b

4.46. Consider the system in Figure P4.46-1 with $H_0(z)$, $H_1(z)$, and $H_2(z)$ as the system functions of LTI systems. Assume that $x[n]$ is an arbitrary stable complex signal without any symmetry properties.

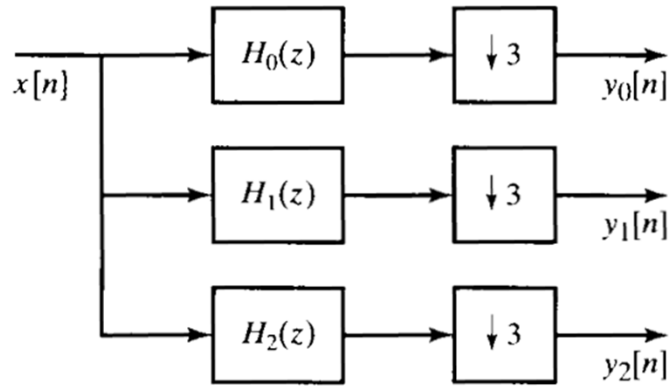


Figure P4.46-1

(a) Let $H_0(z) = 1$, $H_1(z) = z^{-1}$, and $H_2(z) = z^{-2}$. Can you reconstruct $x[n]$ from $y_0[n]$, $y_1[n]$, and $y_2[n]$? If so, how? If not, justify your answer.

(b) Assume that $H_0(e^{j\omega})$, $H_1(e^{j\omega})$, and $H_2(e^{j\omega})$ are as follows:

$$H_0(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \pi/3, \\ 0, & \text{otherwise}, \end{cases}$$

$$H_1(e^{j\omega}) = \begin{cases} 1, & \pi/3 < |\omega| \leq 2\pi/3, \\ 0, & \text{otherwise}, \end{cases}$$

$$H_2(e^{j\omega}) = \begin{cases} 1, & 2\pi/3 < |\omega| \leq \pi, \\ 0, & \text{otherwise}. \end{cases}$$

Can you reconstruct $x[n]$ from $y_0[n]$, $y_1[n]$, and $y_2[n]$? If so, how? If not, justify your answer.

Solution 2 a,b

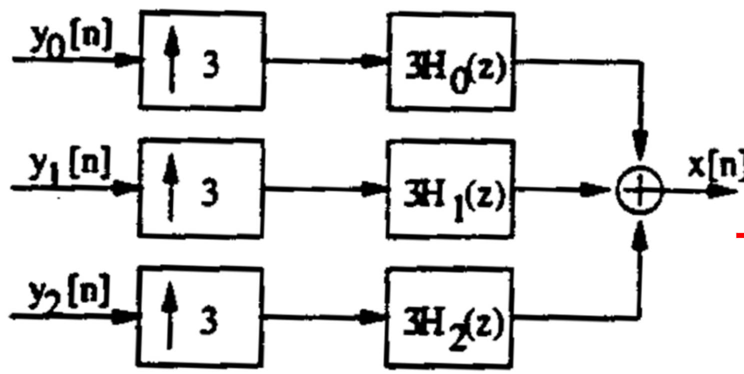
(a) Notice that

$$\begin{aligned} y_0[n] &= x[3n] \\ y_1[n] &= x[3n+1] \\ y_2[n] &= x[3n+2], \end{aligned}$$

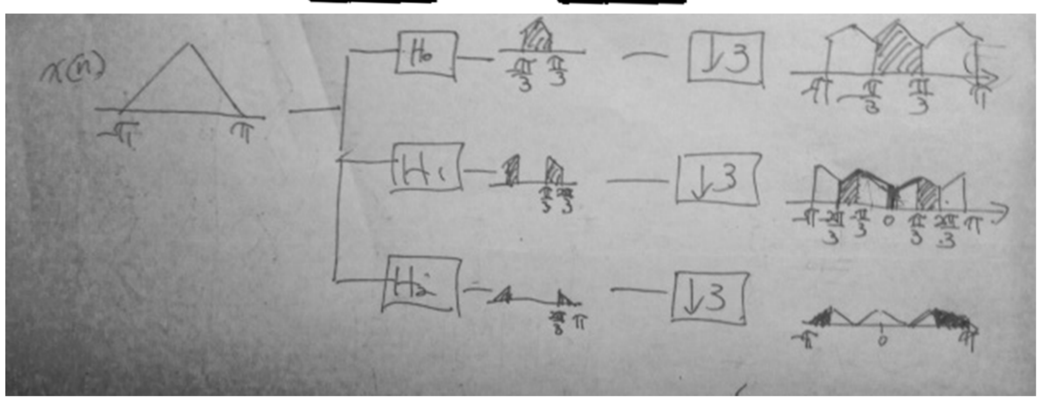
and therefore,

$$x[n] = \begin{cases} y_0[n/3], & n = 3k \\ y_1[(n-1)/3], & n = 3k+1 \\ y_2[(n-2)/3], & n = 3k+2 \end{cases}$$

(b) Yes. Since the bandwidth of the filters are $2\pi/3$, there is no aliasing introduced by downsampling. Hence to reconstruct $x[n]$, we need the system shown in the following figure:



This solution has an error. What is it?



Question 2c

Now consider the system in Figure P4.46-2. Let $H_3(e^{j\omega})$ and $H_4(e^{j\omega})$ be the frequency responses of the LTI systems in this figure. Again, assume that $x[n]$ is an arbitrary stable complex signal with no symmetry properties.

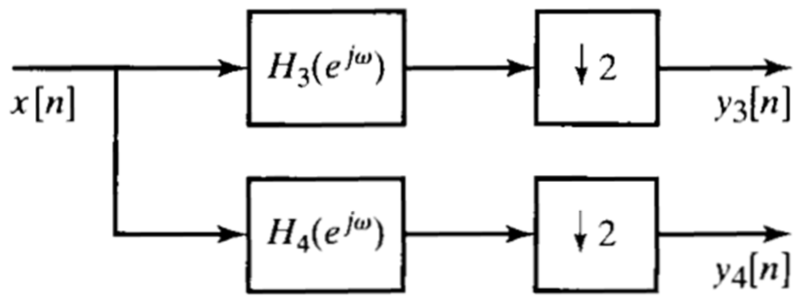


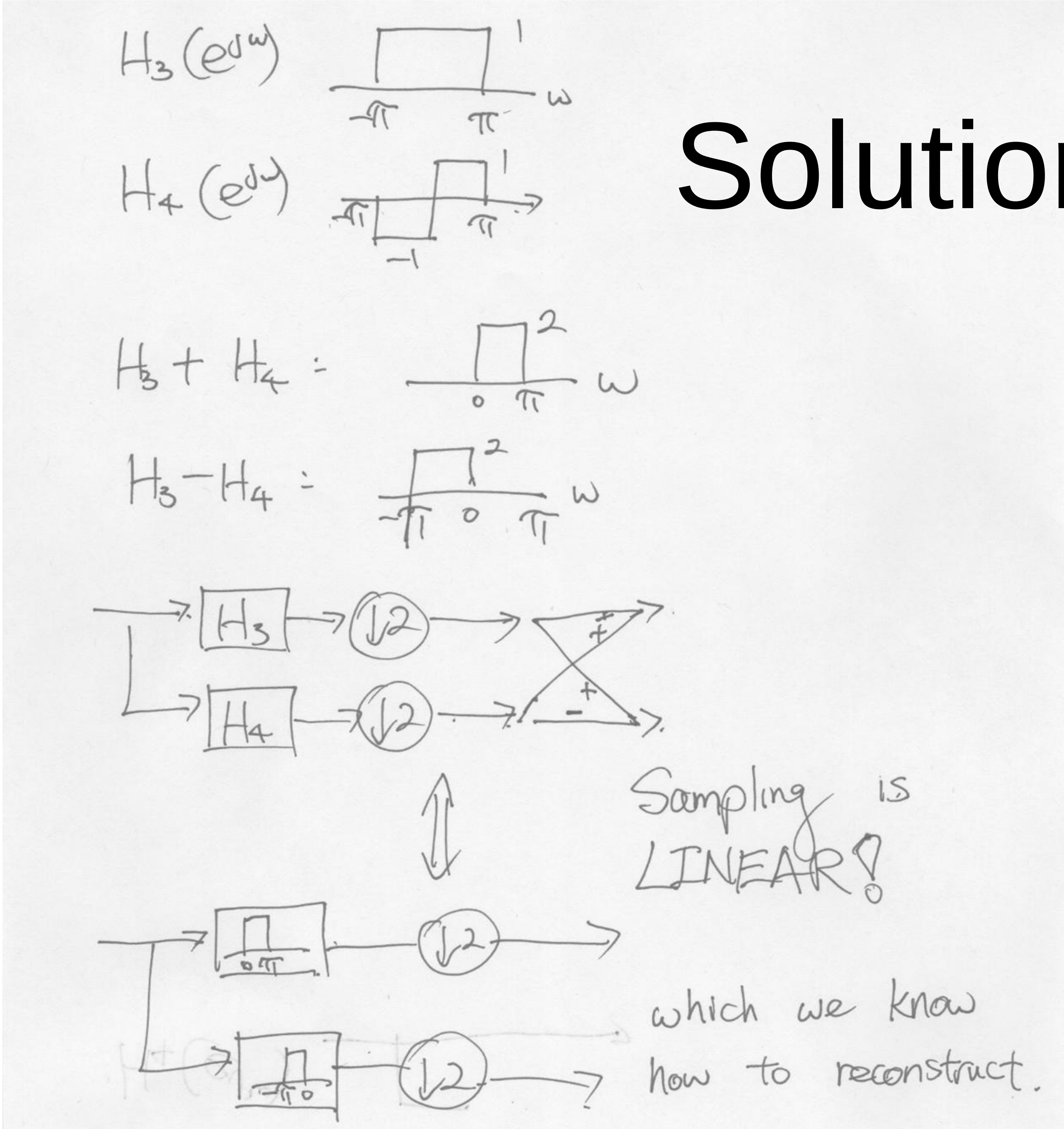
Figure P4.46-2

(c) Suppose that $H_3(e^{j\omega}) = 1$ and

$$H_4(e^{j\omega}) = \begin{cases} 1, & 0 \le \omega < \pi, \\ -1, & -\pi \le \omega < 0. \end{cases}$$

Can you reconstruct $x[n]$ from $y_3[n]$ and $y_4[n]$? If so, how? If not, justify your answer.

Solution 2c


H3(e^jω)
H4(e^jω)
H3 + H4 :
H3 − H4 :
Sampling is LINEAR!
which we know how to reconstruct.

Question 3

As discussed in Section 4.8.3, if the quantization interval Δ is small compared with changes in the level of the input sequence, we can assume that the output of the quantizer is of the form

$$y[n] = x[n] + e[n],$$

where $e[n] = Q(x[n]) - x[n]$ and $e[n]$ is a stationary random process with a first-order probability density uniformly distributed between $-\Delta/2$ and $\Delta/2$, uncorrelated from sample to sample and uncorrelated with $x[n]$, so that $\mathcal{E}\{e[n]x[m]\} = 0$ for all m and n.

Let $x[n]$ be a stationary white-noise process with zero mean and variance σ_x^2.

(a) Find the mean, variance, ~~and autocorrelation sequence~~ of $e[n]$.

(b) What is the signal-to-quantizing-noise ratio σ_x^2/σ_e^2?

Solution 3

(a)

$$E(e) = \int ep(e)de = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} ede = \frac{e^2}{2\Delta}\Big|_{-\Delta/2}^{\Delta/2} = 0$$

$$\sigma_e^2 = E(e^2 - 0) = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e^2 de = \frac{e^3}{3\Delta}\Big|_{-\Delta/2}^{\Delta/2} = \frac{\Delta^2}{12}$$

(b)

$$\mathrm{SNR} = \frac{\sigma_x^2}{\sigma_e^2} = \frac{12\sigma_x^2}{\Delta^2}$$