Discussion Section 9: Midterm 2 Review

Giulia Fanti

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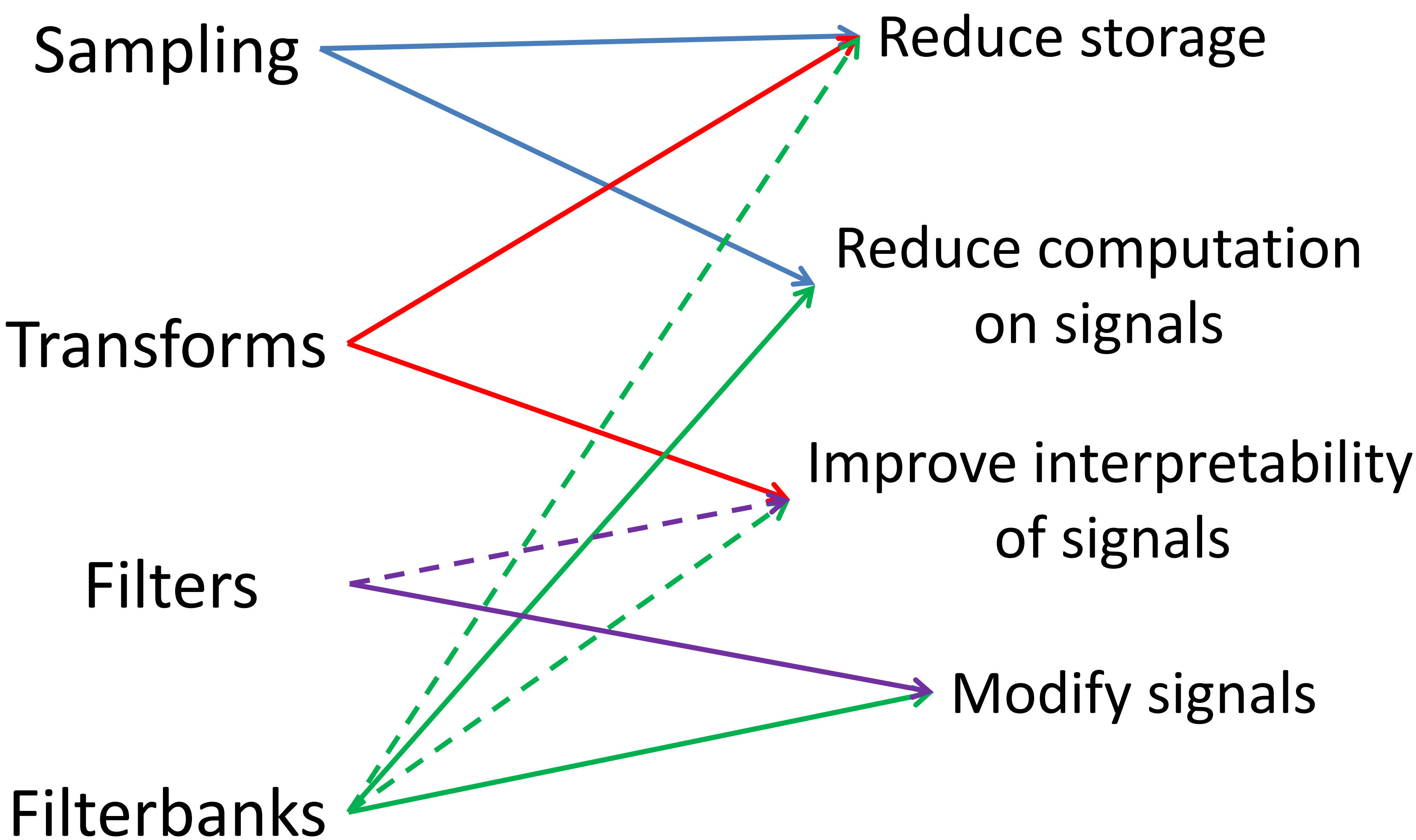
Topics

- Transforms
  - STFT
    - Window types
  - DWT
- Sampling
  - Upsampling/downsampling
- Filterbanks
  - Polyphase decomposition
  - Multirate processing
- 2D signals

Tools we have studied | **Things we want**



Draw the time-frequency tiling for the STFT and the DWT.

What are the relative advantages of one transform over the other?

Methods for computing the DWT

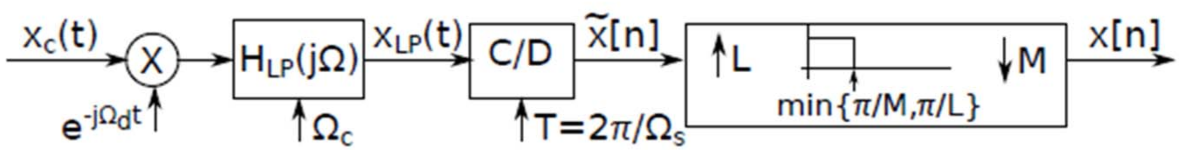
- Use the formula/project the signal onto each scaled and shifted wavelet
- Pyramidal decomposition
- Multiply by the DWT matrix (like a DFT matrix)

Sampling

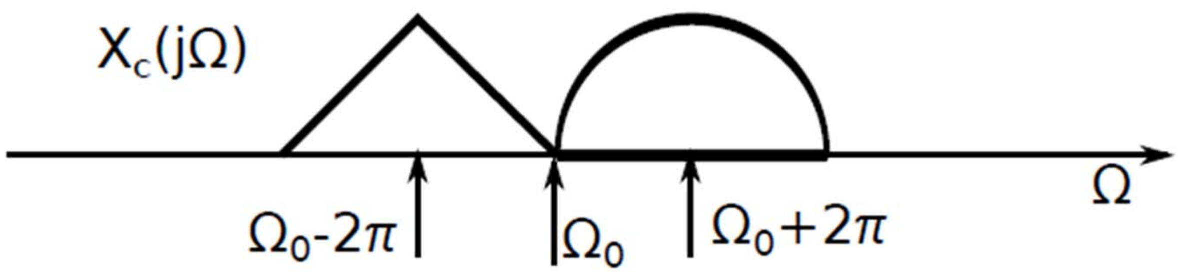
- We have covered this topic extensively in class and discussion
- C/D sampling: Generates copies of the spectrum
- Downsampling: Stretches the spectrum
- Upsampling: Squeezes the spectrum

Practice problem:

Consider a continuous to digital system that consists of a demodulation by Ω_d, ideal analog low-pass filter with a cutoff Ω_c, ideal C/D (without LP filter) with a sampling rate $T = \frac{1}{\Omega_s}$, and an ideal digital $\frac{L}{M}$ resampling (with an appropriate $\min\{\frac{\pi}{M}, \frac{\pi}{L}\}$ LP filter).

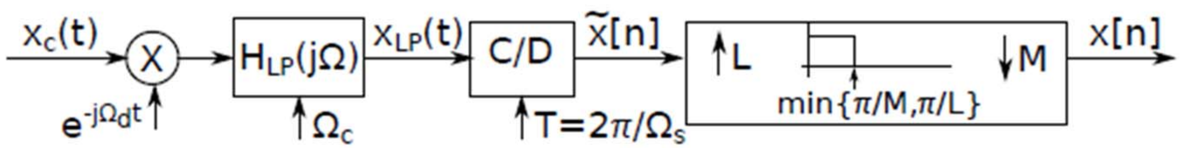


Also consider a signal with an unknown gain, $x_c(t)$, with the following spectrum, $X_c(j\Omega)$:



For each of the following possible outputs of the system, find the parameters Ω_d, Ω_c, Ω_s and L/M that will produce it. Also, plot qualitatively the intermediate spectrums $X_{\text{LP}}(j\Omega)$ and $\tilde{X}(e^{j\omega})$. It is possible that a parameter combination that produces the output does not exist. In that case, mark the appropriate box and explain why.

The analog H_{LP} filter has limits: $2\pi \leq \Omega_c \leq 8\pi$. When given a choice, choose the **lowest** Ω_c



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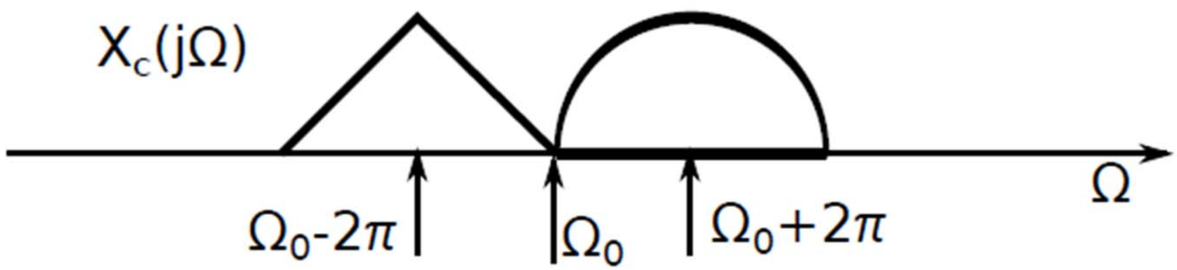
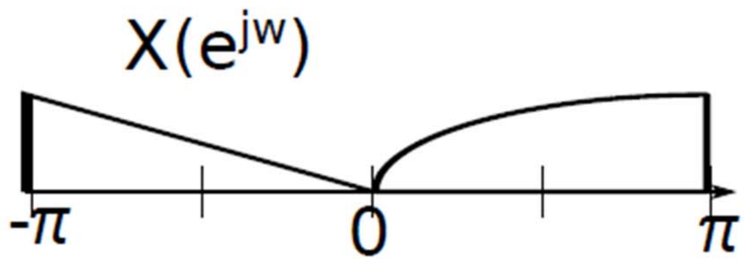


a) The spectrum $X(e^{j\omega})$ is:

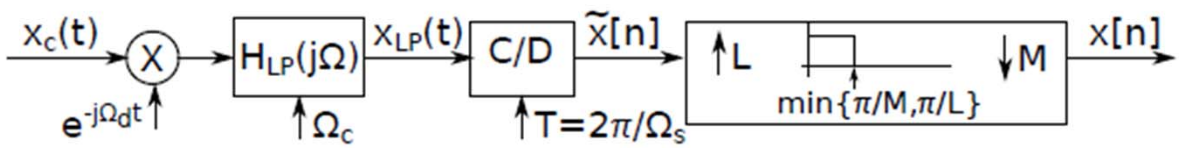


find the parameters Ω_d, Ω_c, Ω_s and L/M

Solution:

$X(e^{j\omega})$ is centered at center so Ω_d is simply Ω_0.

We then crop the spectrum at 2π to get the desired shape and sample at the Nyquist rate, $\Omega_s = 2 \cdot 2\pi$ to get the result. We don't need any resampling.



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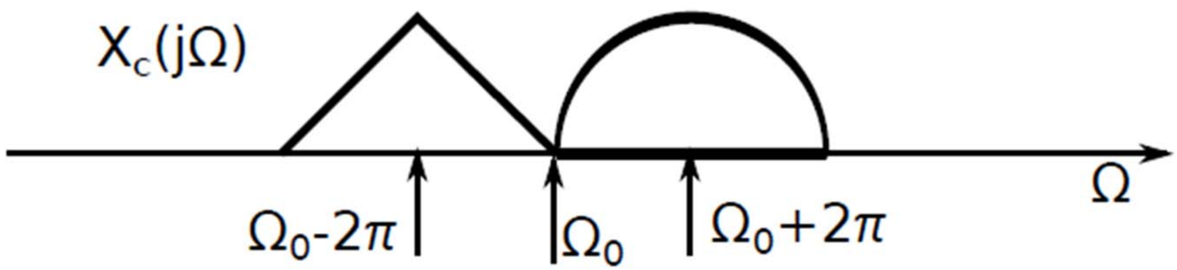
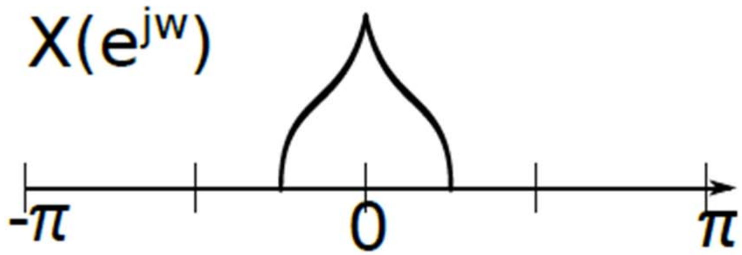


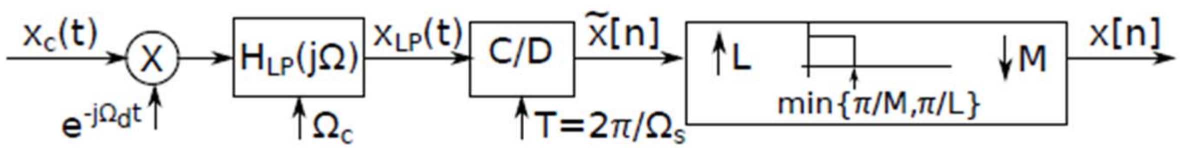
b) The spectrum $X(e^{j\omega})$ is:



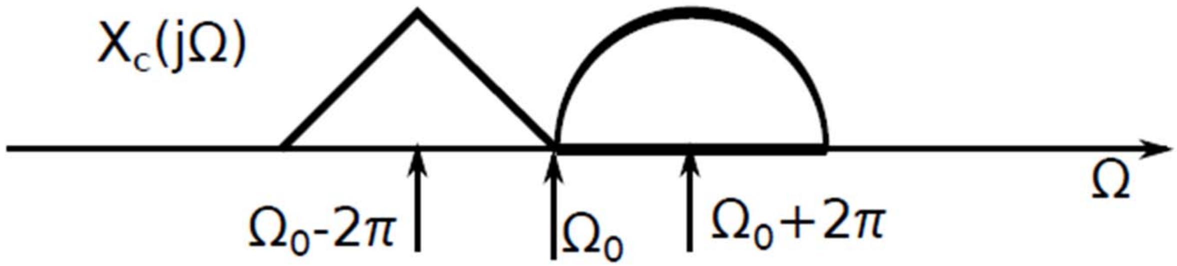
find the parameters Ω_d, Ω_c, Ω_s and L/M

Solution:

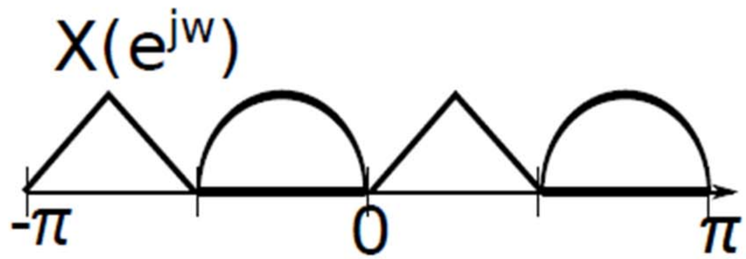
$X(e^{j\omega})$ looks like the aliasing of the triangle and circle so we first want to shift the spectrum to the center of one of the shapes. We don't want to crop anything, so the lowest Ω_c is 6π. To create aliasing, we sample at the same rate as the bandwidth, which is 4π. We then upsample it by 4 to shrink the spectrum.



The analog $H_{\rm LP}$ filter has limits: $2\pi \le \Omega_c \le 8\pi$. When given a choice, choose the **lowest** Ω_c



c) The spectrum $X(e^{j\omega})$ is:

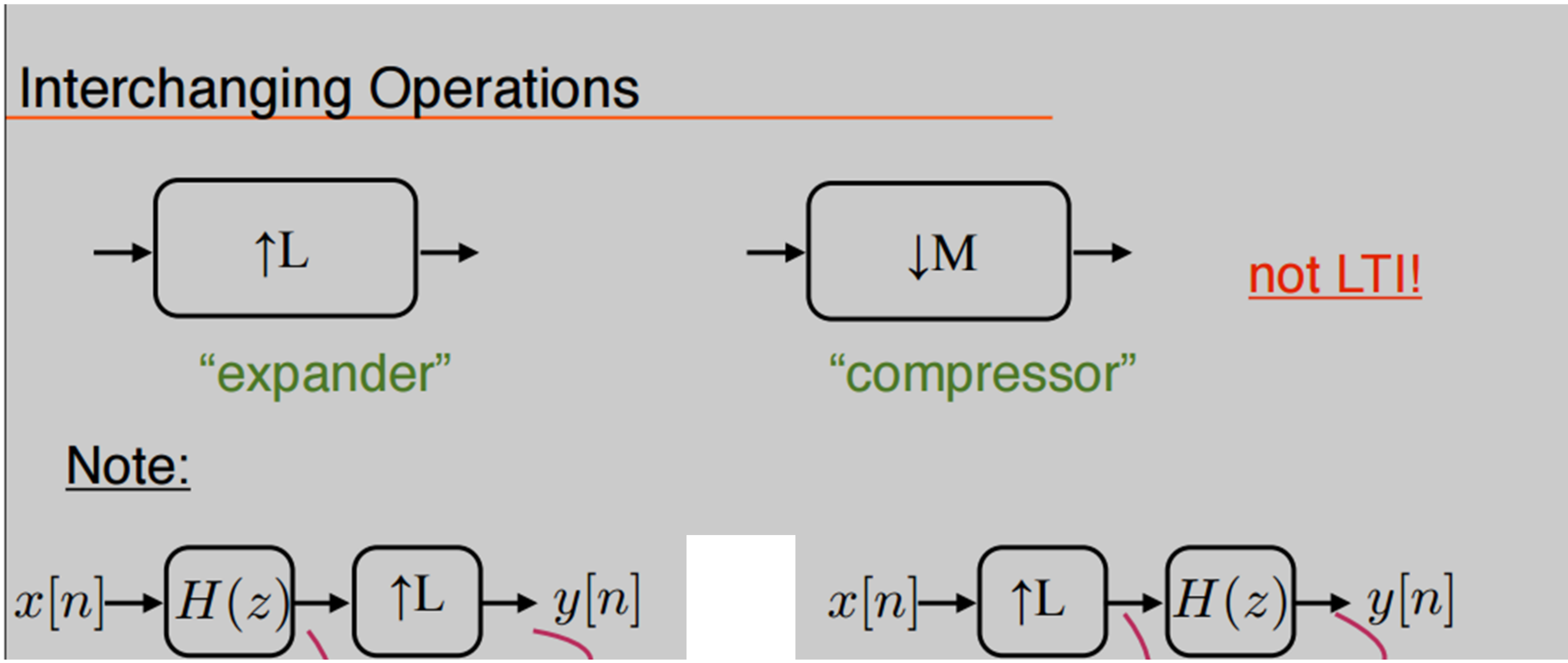


find the parameters Ω_d, Ω_c, Ω_s and L/M

Solution:

We cannot get $X(e^{j\omega})$ because the ideal resampling always applies a filter to crop out the extra copies. We can flip the spectrum following part d, but there is no way to squeeze two copies in $-\pi$ to π.

Filterbanks, Polyphase



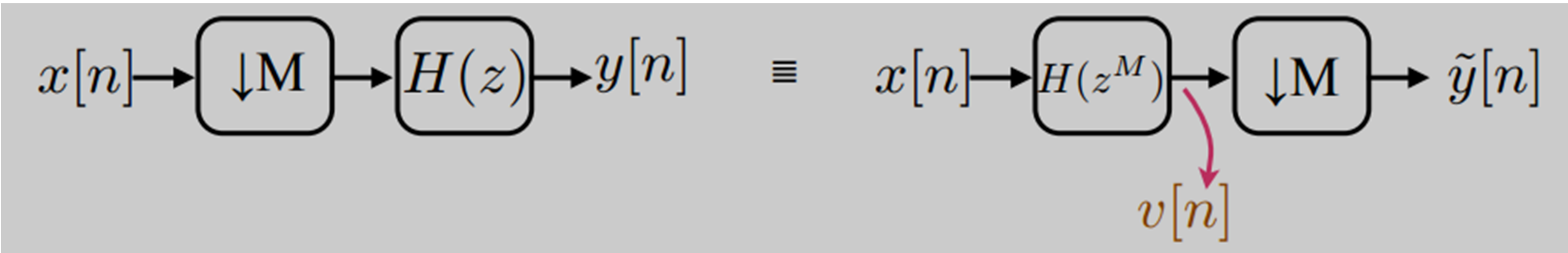
Interchanging Filter Expander

- Q: Can we move expander from Left to Right (with xform)?

Do you agree with this statement?

Should say if $H(z^{1/L})$ is rational!

What are the conditions on this statement?

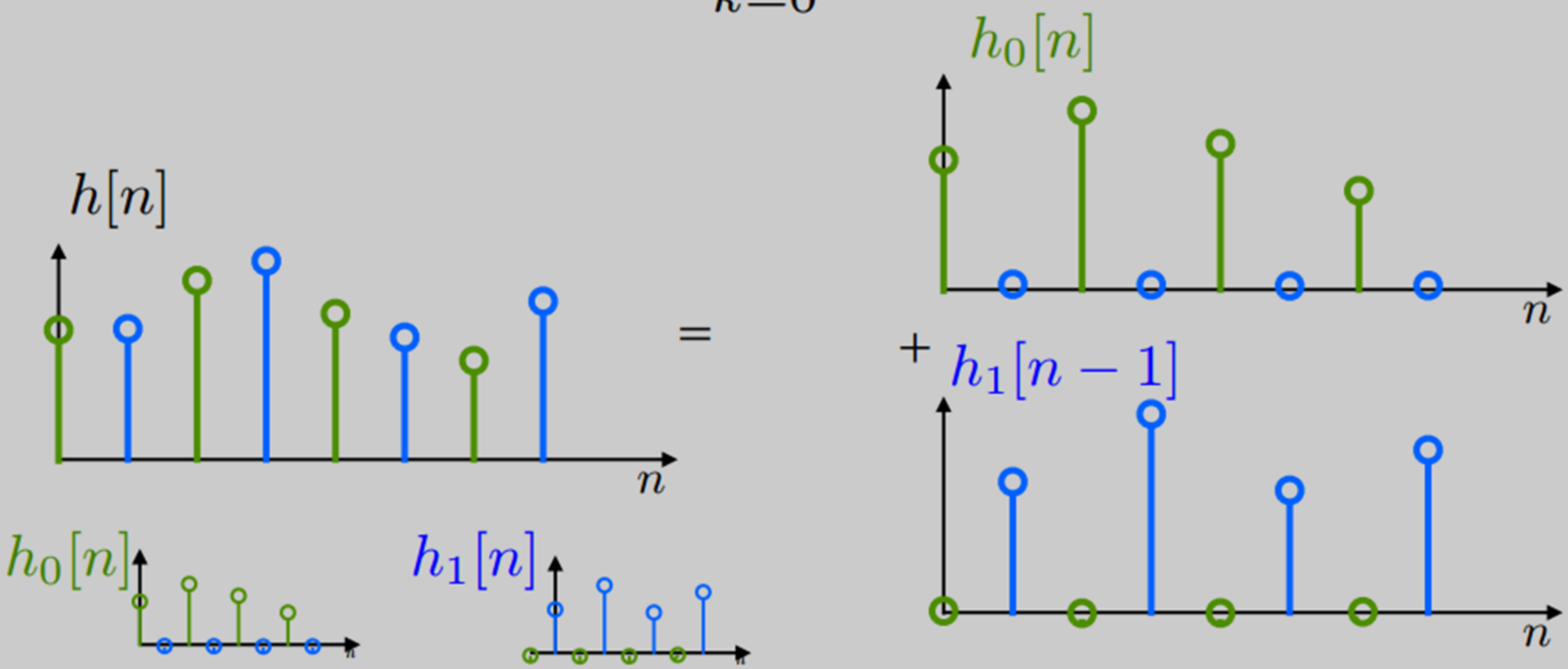


This always holds. How would you prove it?

Polyphase Decomposition

- We can decomposed an impulse response to:

$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$



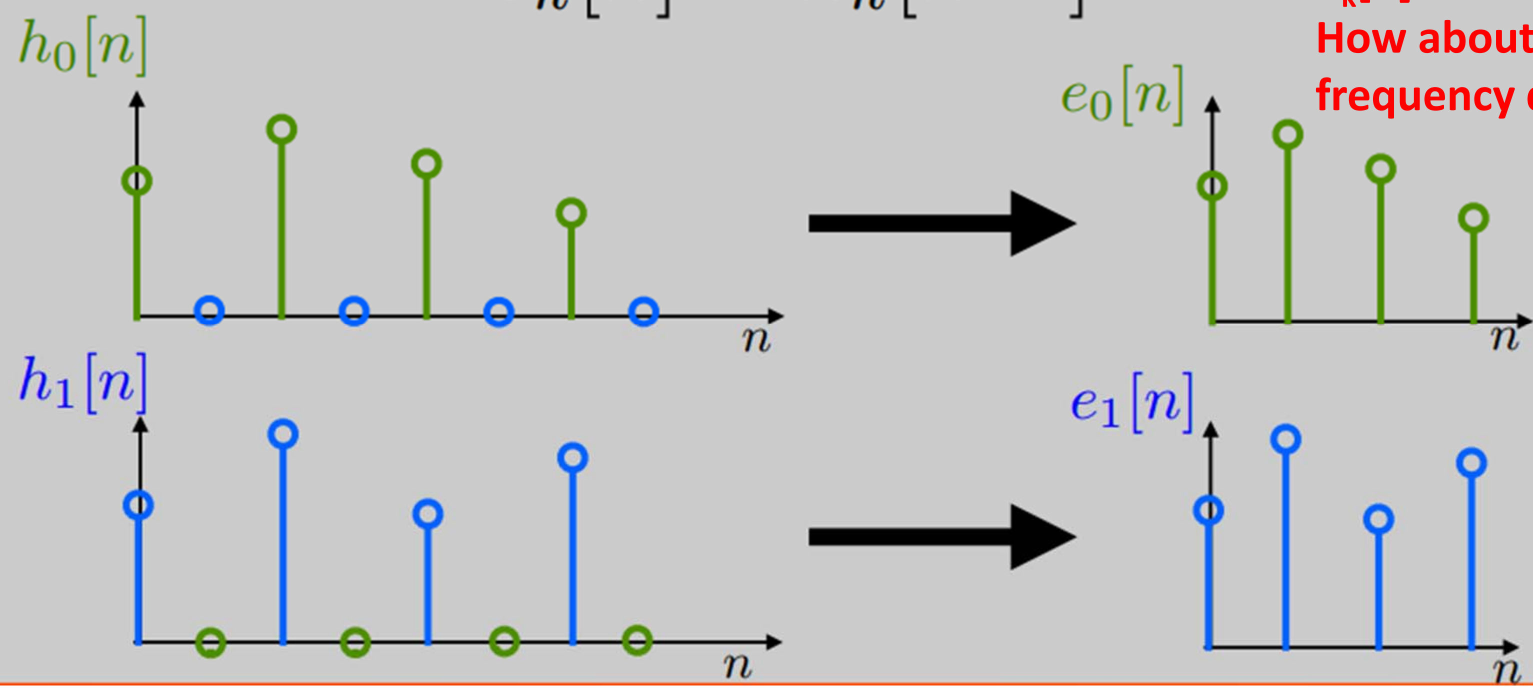
Polyphase Decomposition

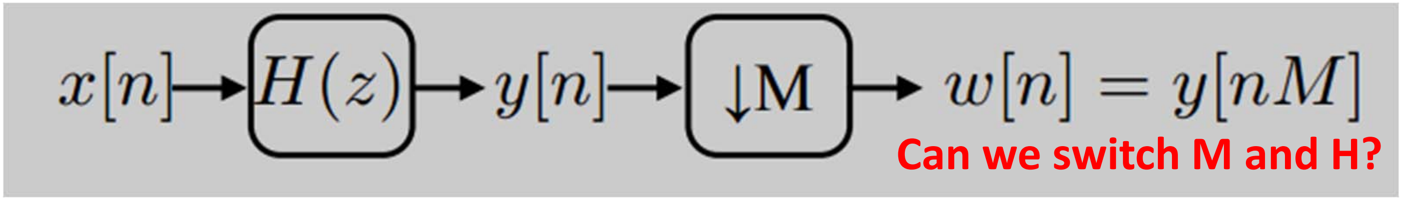
- Define:

$$h_k[n] \longrightarrow \boxed{\downarrow \mathrm{M}} \longrightarrow e_k[n]$$

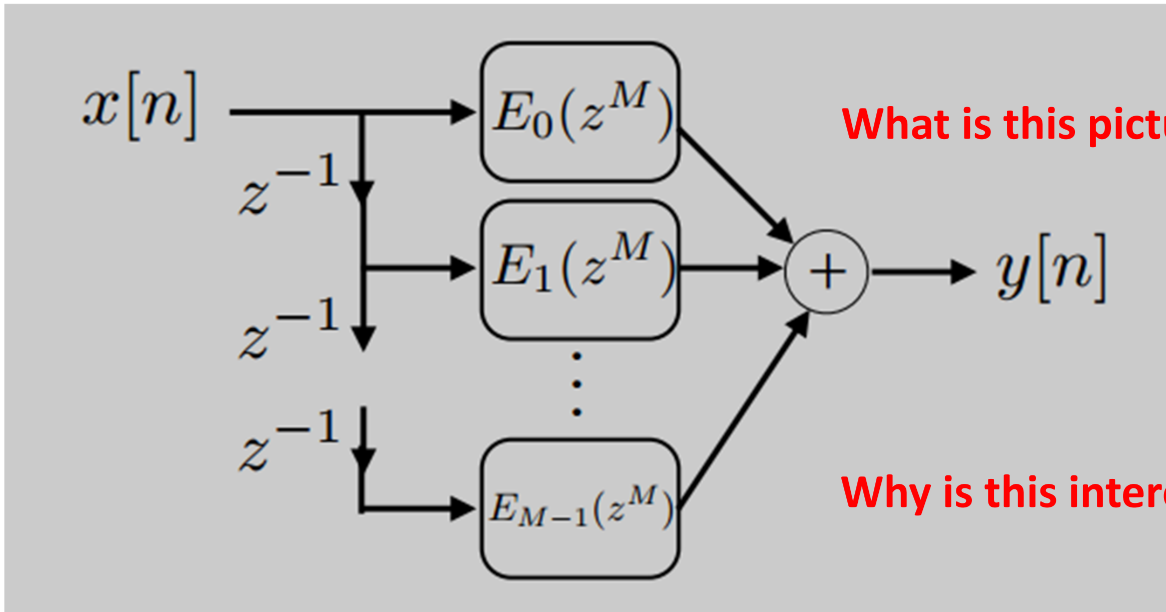
So how do we get back from $e_k[n]$ to $h_k[n]$?
How about in frequency domain?

$$e_k[n] = h_k[nM]$$



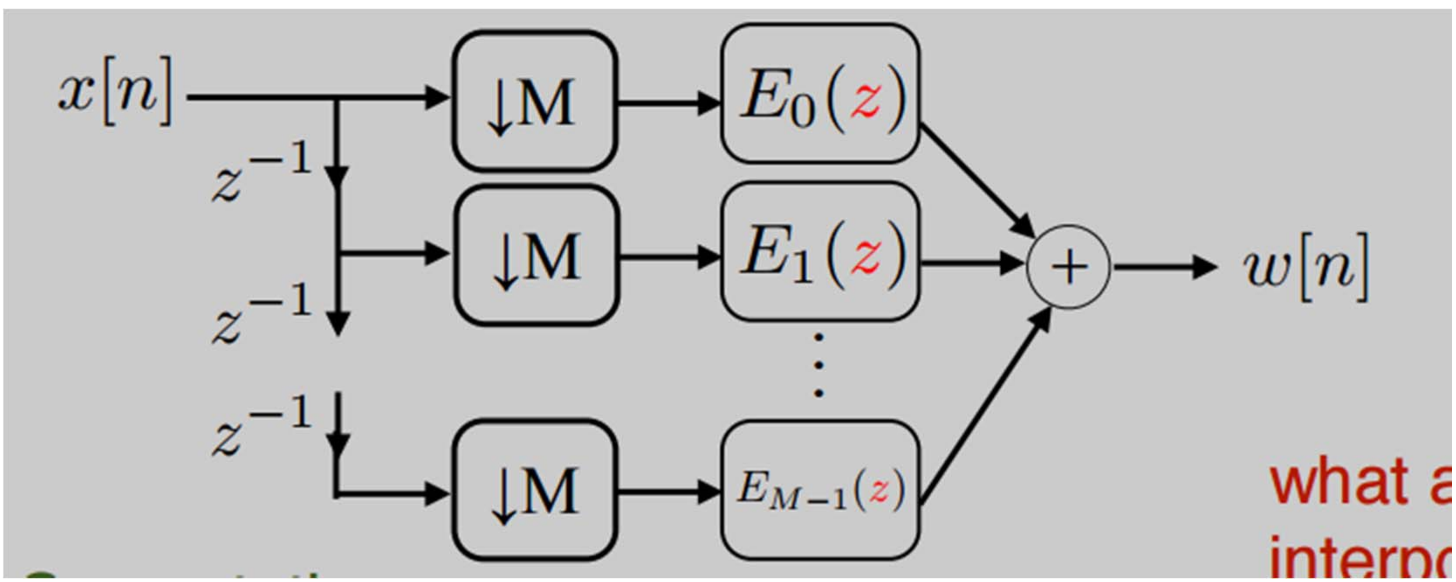


Can we switch M and H?



What is this picture representing?

Why is this interesting?



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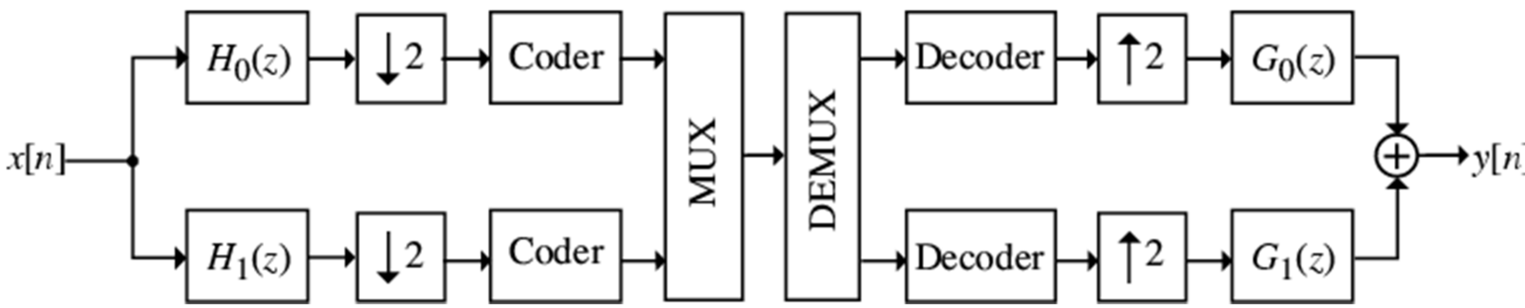
Reconstruction: Quadrature Mirror Filterbanks

- What is a Quadrature mirror filterbank?



- Quadrature mirror filters:
  - $G_0(z)$ and $G_1(z)$ are mirror images of $H_0(z)$ and $H_1(z)$, mirrored around pi/2
  - What kinds of filters are these?

Practice problem

4.53. Consider the analysis–synthesis system shown in Figure P4.53-1. The lowpass filter $h_0[n]$ is identical in the analyzer and synthesizer, and the highpass filter $h_1[n]$ is identical in the analyzer and synthesizer. The Fourier transforms of $h_0[n]$ and $h_1[n]$ are related by

$$H_1(e^{j\omega}) = H_0(e^{j(\omega+\pi)}).$$

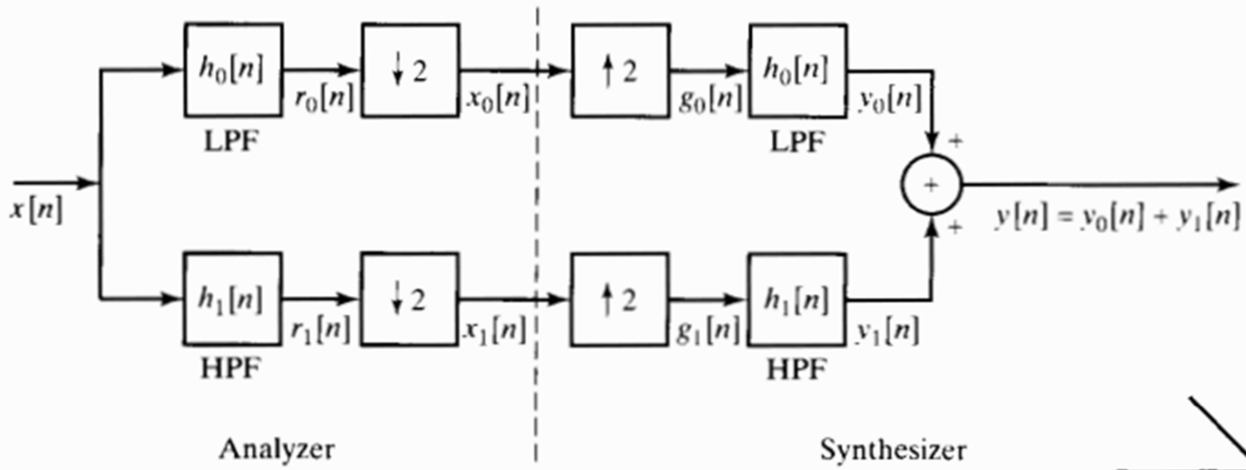
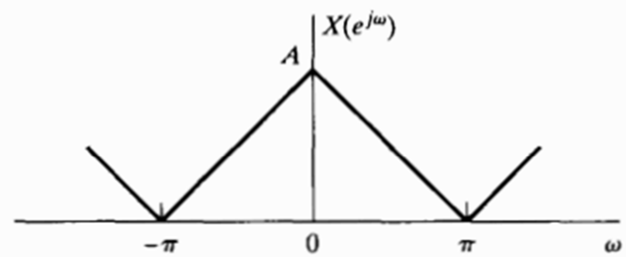


Figure P4.53-1



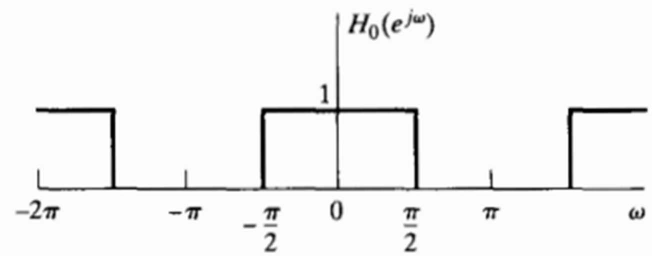
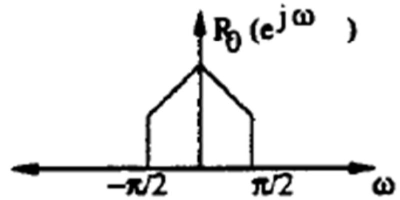


Figure P4.53-2

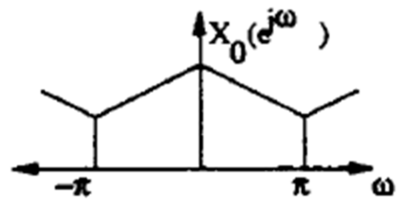
(a) If $X(e^{j\omega})$ and $H_0(e^{j\omega})$ are as shown in Figure P4.53-2, sketch (to within a scale factor) $X_0(e^{j\omega})$, $G_0(e^{j\omega})$, and $Y_0(e^{j\omega})$.

(a) First, $X(e^{j\omega})$ is plotted.

The lowpass filter cuts off at $\frac{\pi}{2}$.



The downsampler expands the frequency axis. Since $R_0(e^{j\omega})$ is bandlimited to $\frac{\pi}{M}$, no aliasing occurs.



The upsampler compresses the frequency axis by a factor of 2.

The lowpass filter cuts off at $\frac{\pi}{2} \Rightarrow Y_0(e^{j\omega}) = R_0(e^{j\omega})$ as sketched above.

(b) Write a general expression for $G_0(e^{j\omega})$ in terms of $X(e^{j\omega})$ and $H_0(e^{j\omega})$. Do *not* assume that $X(e^{j\omega})$ and $H_0(e^{j\omega})$ are as shown in Figure 4.53-2.

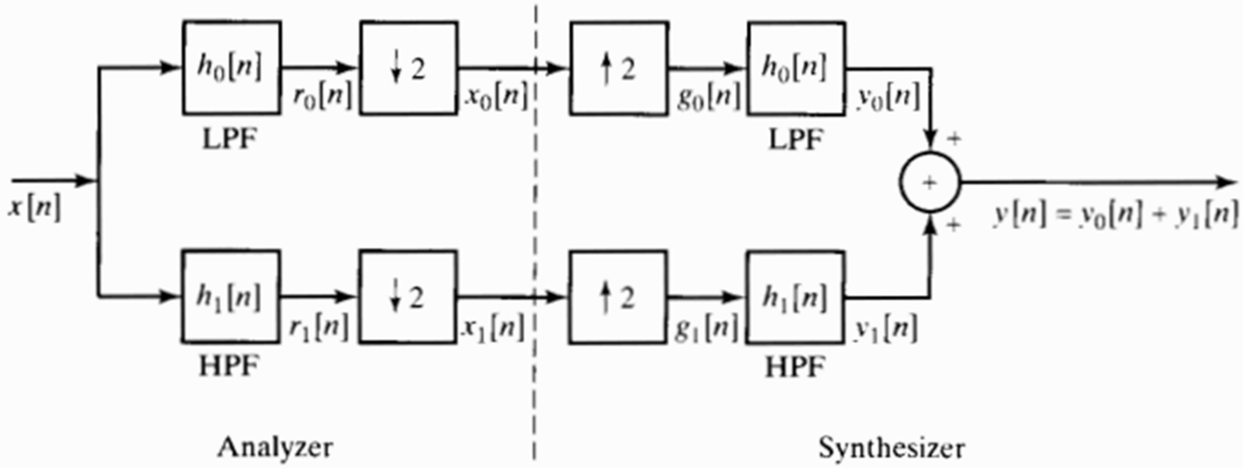


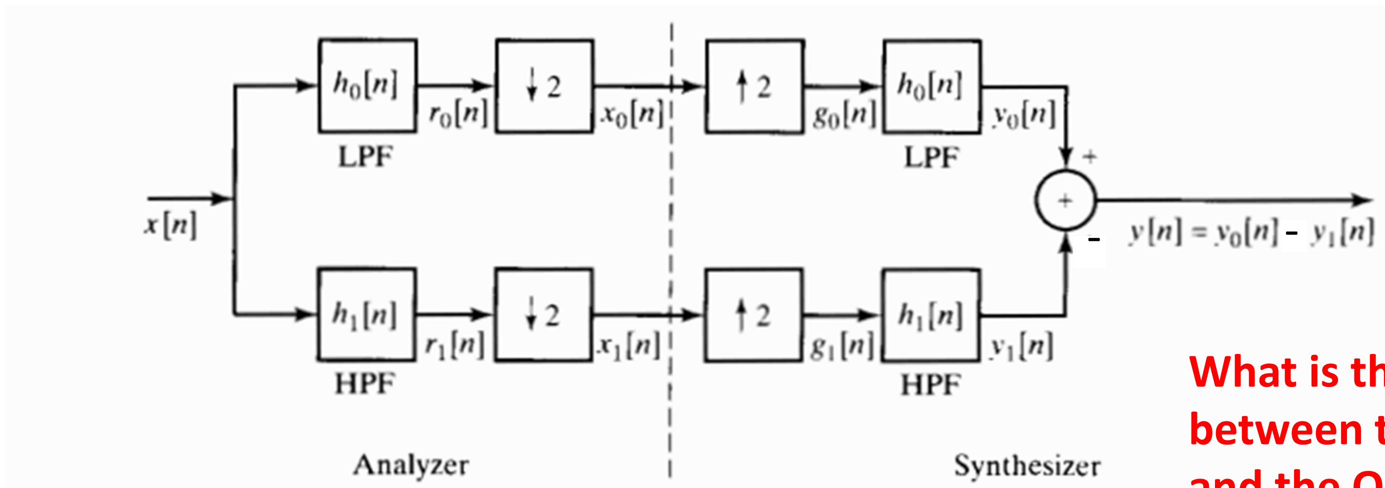
Figure P4.53-1

(b) $G_0(e^{j\omega}) = \frac{1}{2}\left(X(e^{j\omega})H_0(e^{j\omega}) + X(e^{j(\omega+\pi)})H_0(e^{j(\omega+\pi)})\right)$

(c) Determine a set of conditions on $H_0(e^{j\omega})$ that is as general as possible and that will guarantee that $y[n]$ is proportional to $x[n-n_d]$ for any stable input $x[n]$.

Note: Analyzer–synthesizer filter banks of the form developed in this problem are very similar to quadrature mirror filter banks. For further reading, see Crochiere and Rabiner (1983), pp. 378–392.

$$H_1(e^{j\omega}) = H_0(e^{j(\omega+\pi)}).$$



What is the relation between this figure and the QMF we have seen in class?

(c)

$$
\begin{aligned}
Y_0(e^{j\omega}) &= \frac{1}{2}H_0(e^{j\omega})\left(X(e^{j\omega})H_0(e^{j\omega}) + X(e^{j(\omega+\pi)})H_0(e^{j(\omega+\pi)})\right)\\
Y_1(e^{j\omega}) &= \frac{1}{2}H_1(e^{j\omega})\left(X(e^{j\omega})H_1(e^{j\omega}) + X(e^{j(\omega+\pi)})H_1(e^{j(\omega+\pi)})\right)\\
Y(e^{j\omega}) &= Y_0(e^{j\omega}) - Y_1(e^{j\omega})\\
&= \frac{1}{2}X(e^{j\omega})\left[H_0^2(e^{j\omega}) - H_1^2(e^{j\omega})\right]\\
&\quad + \frac{1}{2}X(e^{j(\omega+\pi)})\underbrace{\left[H_0(e^{j\omega})H_0(e^{j(\omega+\pi)}) - H_1(e^{j\omega})H_1(e^{j(\omega+\pi)})\right]}_{=0}
\end{aligned}
$$

The aliasing terms always cancel. $Y(e^{j\omega})$ is proportional to $X(e^{j\omega})$ if $\{H_0^2(e^{j\omega}) - H_1^2(e^{j\omega})\}$ is a constant.

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