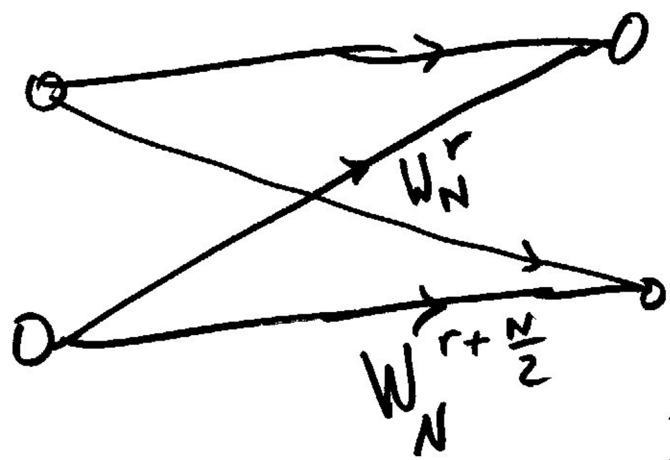
Oct. 26, 2006

Discovered: FFT flowgraph is made of butterflies of the following form.

2 input



2 output

observe: $W_N^{r+N/2} \triangleq e^{-j\frac{2\pi}{N}\left(r+\frac{N}{2}\right)}$

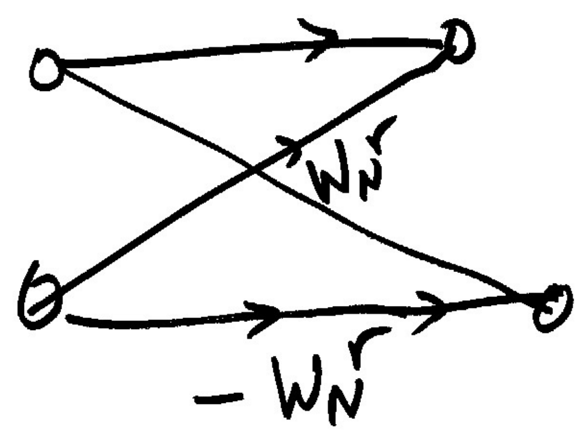
$= e^{-j\frac{2\pi r}{N}} e^{-j\frac{2\pi N}{2N}}$

$= - e^{-j\frac{2\pi r}{N}}$

$= - W_N^r$

$$W_N^{r+N/2} = - W_N^r$$

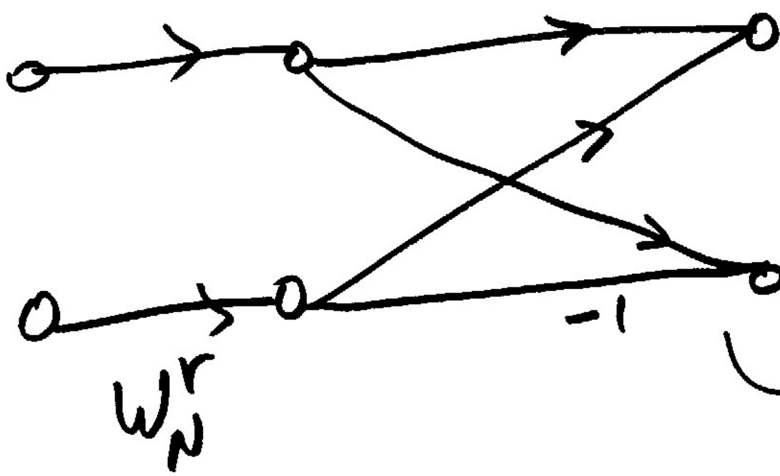
2 inputs



2 outputs.

2 mults
2 adds.

2 inpt



2 outputs

1 mult
2 adds.

new improved more efficient butterfly.

show Fig 9.10

From 9.10, observations

(1) In place computation.
start with an array of N #s
and keep ~~re~~ re-using it, over-writing it.

(2) Embarrasingly parallel.

(3) $\log_2 N$ stage.

(4) Each stage has $N/2$ butterflies
with 2 inputs and 2 outputs

(5) each processor do a one of the $N/2$
butterflies in each stage.
parallelize it to $N/2$ processors.

(6) Each butterfly 2 adds
1 mult.

Total: Each stage.

$N/2$ butterflies
each butter 1 mult
2 add

$\rightarrow$ $N/2$ mults
N adds

$\log_2 N$ stages

$$\Longrightarrow \boxed{\begin{array}{ll} \frac{N}{2}\log_2 N & \text{mults.} \\ N \log_2 N & \text{adds} \end{array}} \longleftarrow \text{Total.}$$

much better N^2

Ex $N = 64 \times 10^6$

$$N^2 = 64 \times 64 \times 10^{12}$$

$$N \log_2 = 64 \times 10^6 \times (6 + 20) = 64 \times 26 \times 10^6$$

$$\frac{N^2}{N \log_2 N} = \frac{\cancel{64} \times 64 \times 10^{12}}{\cancel{64} \times 26 \times 10^6} \approx 2 \times 10^6$$

$$\frac{2000000}{4000} = 500 \text{ hours}$$

= 3 weeks

1 sec

$N \log N$

Fig 9.10 → input order is messed up.

Bit Reversing

$x(0)$	0	000	000	0
$x(4)$	1	001	100	4
$x(2)$	2	010	010	2
$x(6)$	3	011	110	6
$x(1)$	4	100	001	1
$x(5)$	5	101	101	5
$x(3)$	6	110	011	3
$x(7)$	7	111	111	7

FFT: Decimation in Frequency:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}} \qquad 0 \le k < N$$

Divide computation 2 parts: even indicies of K $k=2r$
odd " " " $k=2r+1$

(1) K even $k=2r$

$$X(2r) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n 2r}{N}} \qquad 0 \le r < N/2$$

$$X(2r) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi rn}{N/2}} + \sum_{n=N/2}^{N-1} x(n)\, e^{-j\frac{2\pi rn}{N/2}}$$

$n = m + N/2$

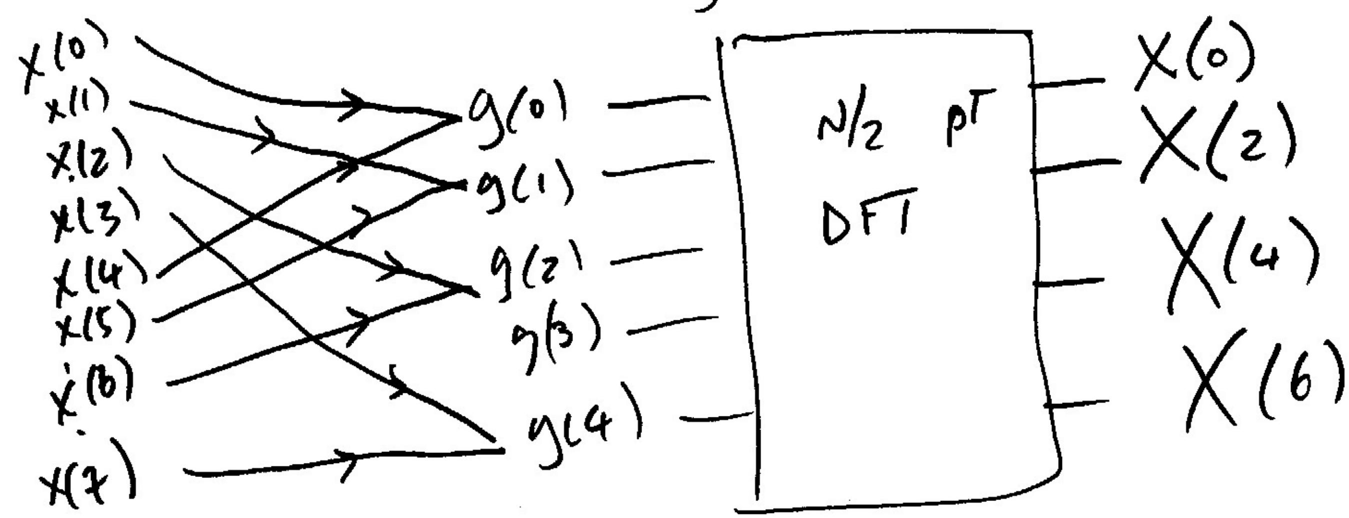
$$X(2r) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi r n}{N/2}} + \sum_{m=0}^{N/2-1} x\left(m+\frac{N}{2}\right) e^{-j\frac{2\pi r}{N/2}\left(m+\frac{N}{2}\right)}$$

$$X(2r) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi r n}{N/2}} + \sum_{n=0}^{N/2-1} x\left(n+\frac{N}{2}\right) e^{-j\frac{2\pi r n}{N/2}}$$

$$X(2r) = \sum_{n=0}^{N/2-1} \underbrace{\left[x(n) + x\left(n+\frac{N}{2}\right)\right]}_{g(n)} e^{-j\frac{2\pi r n}{N/2}} \qquad 0 \le r < N/2$$

$N/2$ pT DFT of $g(n)$

$N=8$



Consider $k = 2r+1$ i.e. odd indices of k.

$$X(2r+1) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n(2r+1)}{N}} \qquad 0 \le r < N/2$$

$$X(2r+1) = \underbrace{\sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi n(2r+1)}{N}}} + \sum_{n=N/2}^{N-1} x(n)\, e^{-j\frac{2\pi n}{N}(2r+1)}$$

$$n = m + N/2$$

$$X(2r+1) = \quad \downarrow \quad + \sum_{m=0}^{N/2-1} x\left(m+\frac{N}{2}\right) e^{-j\frac{2\pi}{N}\left(m+\frac{N}{2}\right)(2r+1)}$$

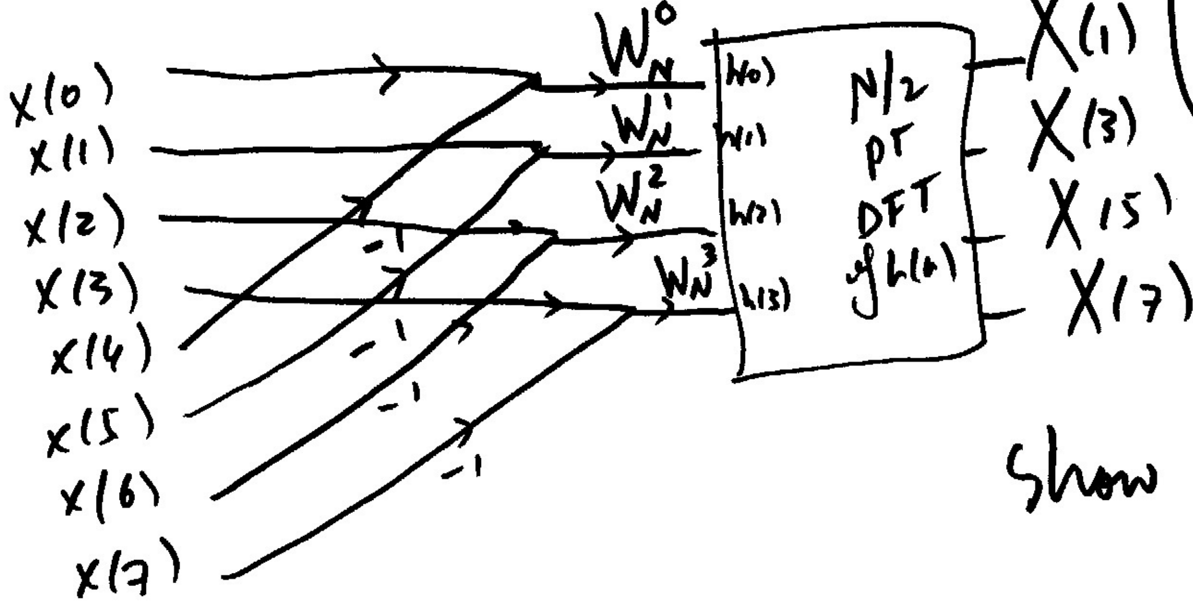
$$X(2r+1) = \quad \downarrow \quad + \underbrace{e^{-j\frac{2\pi}{N}\frac{N}{2}(2r+1)}}_{-1} \sum_{m=0}^{N/2-1} x\left(m+\frac{N}{2}\right) e^{-j\frac{2\pi m}{N}(2r+1)}$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi n}{N}(2r+1)} - \sum_{n=0}^{N/2-1} x\left(n+\frac{N}{2}\right) e^{-j\frac{2\pi n}{N}(2r+1)}$$

$$X(2r+1) = \underbrace{\sum_{n=0}^{N/2-1} \left[x(n) - x\left(n+\frac{N}{2}\right)\right] e^{-j\frac{2\pi n}{N}}}_{h(n)} \; e^{-j\frac{2\pi n r}{N/2}}$$

$N/2$ pt DFT of $h(n)$

$e^{-j\frac{2\pi n}{N}} \triangleq W_N^n$



Show 9.17, 9.18, 9.19

9.20

Talk about Transposition of Decim Freq → Decim Time