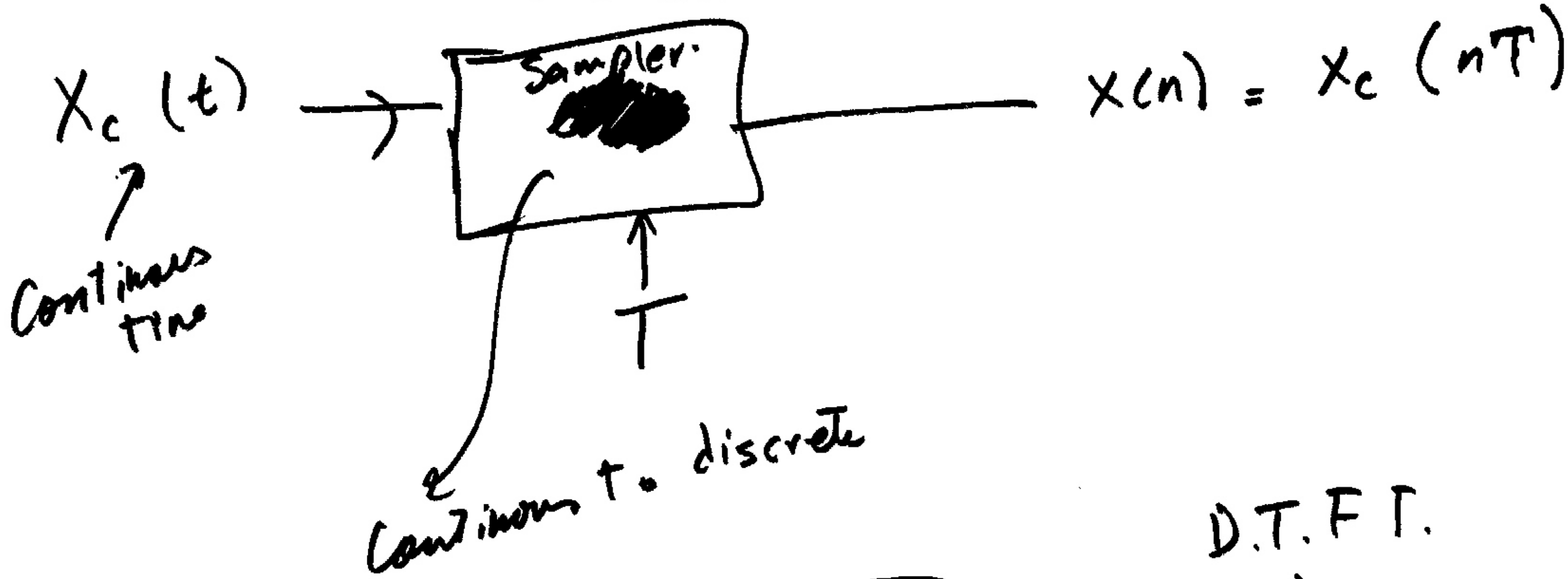
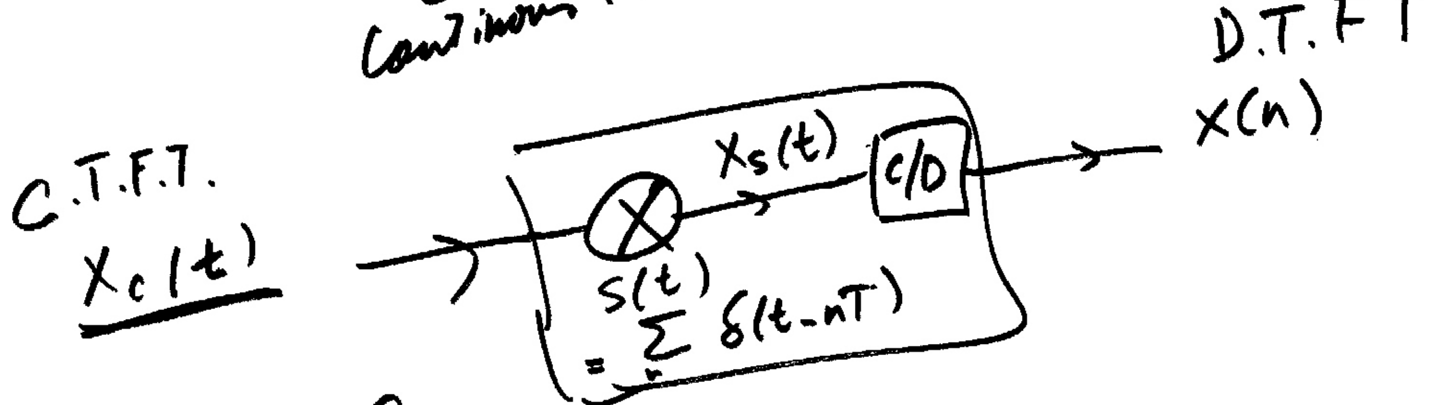
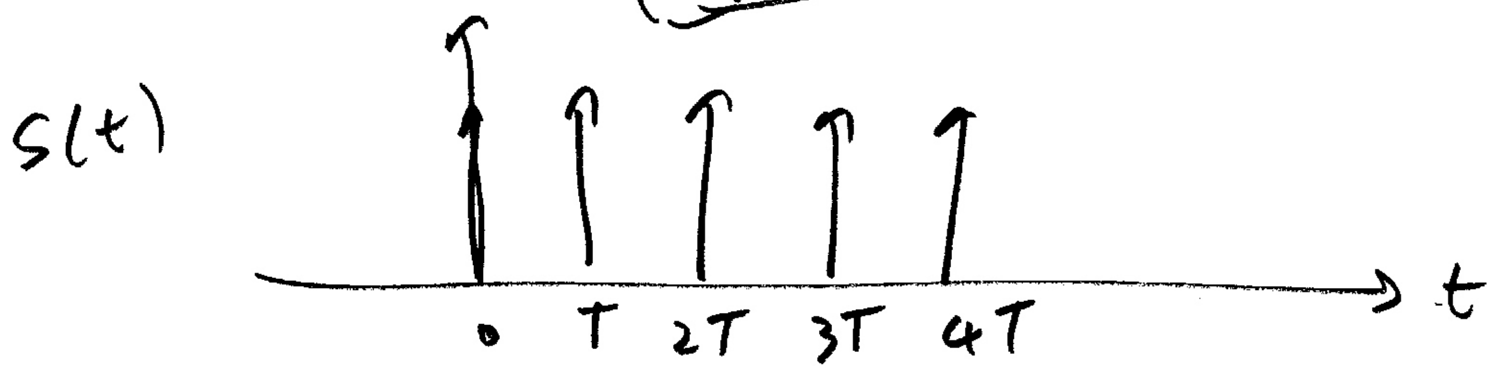
Sampling

9/9/05







Q: what is the relationship between CTFT of $X_c(t)$ and DTFT of $x(n)$?

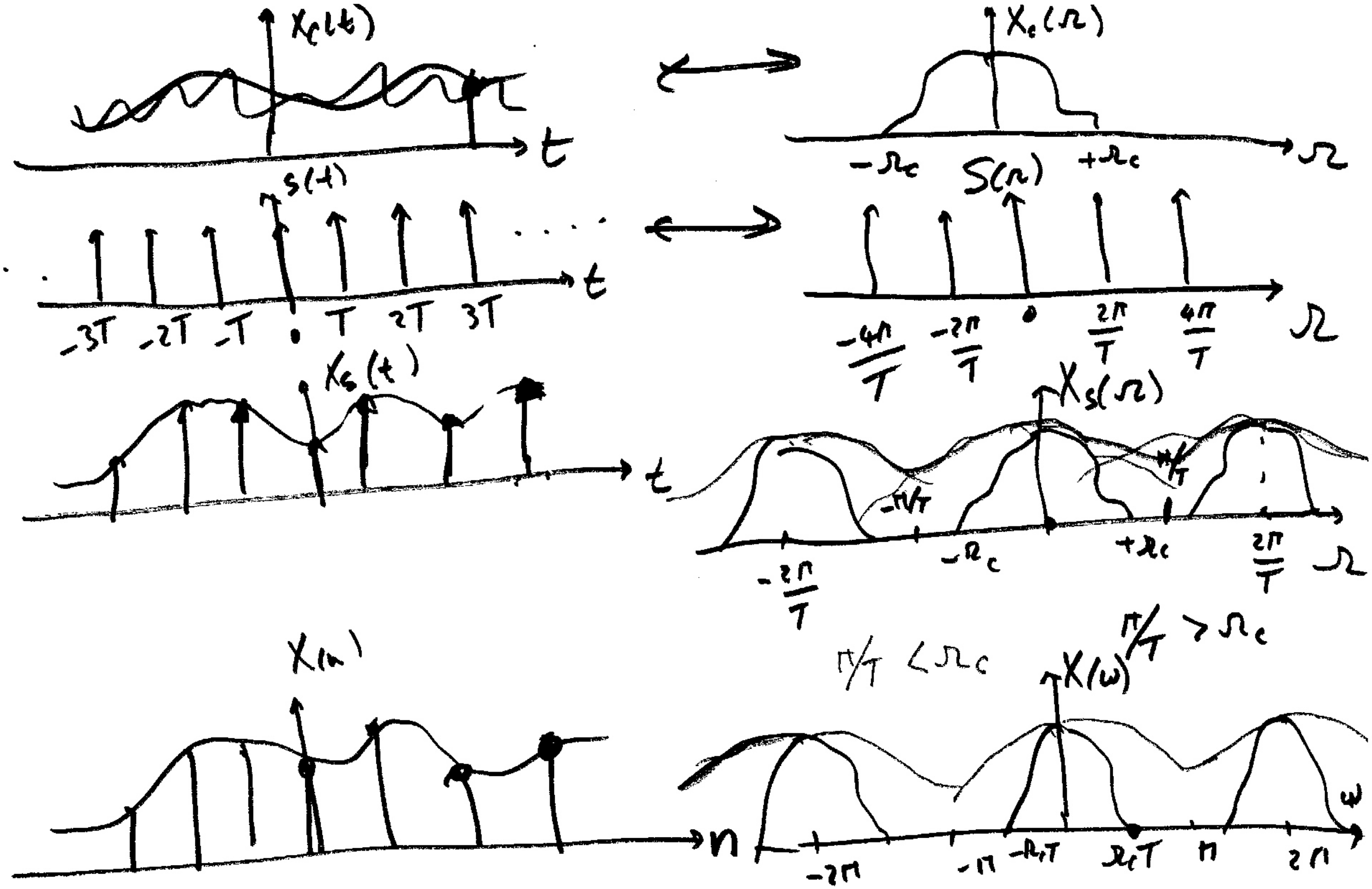
$$X_c(\Omega) = \text{C.T.F.T}\{x(n)\} = \int_{-\infty}^{+\infty} x_c(t)\, e^{-j\Omega t}\, dt$$

continuous Time domain.

$$X(\omega) = \text{DTFT}\{x(n)\} = \sum_{n=-\infty}^{+\infty} x(n)\, e^{-j\omega n}$$

Can be shown:

$$X(\omega) = \frac{1}{T} \sum_{K=-\infty}^{+\infty} X_c\left(\frac{\omega - 2\pi K}{T}\right)$$



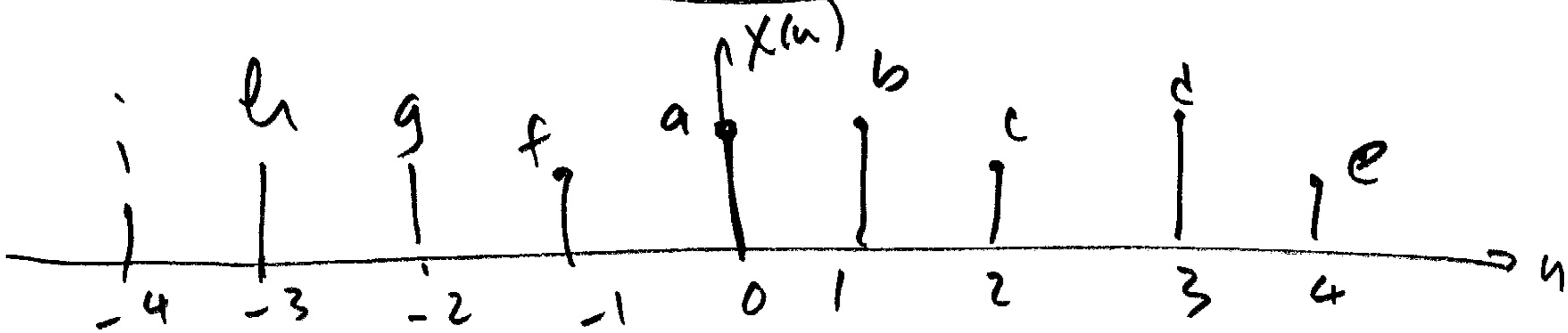
If $\Omega_c < \pi/T \Rightarrow$ no aliasing

$\Rightarrow$ recover $x_c(t)$, $X_c(\Omega)$ exactly from $x[n]$ or $X(\omega)$

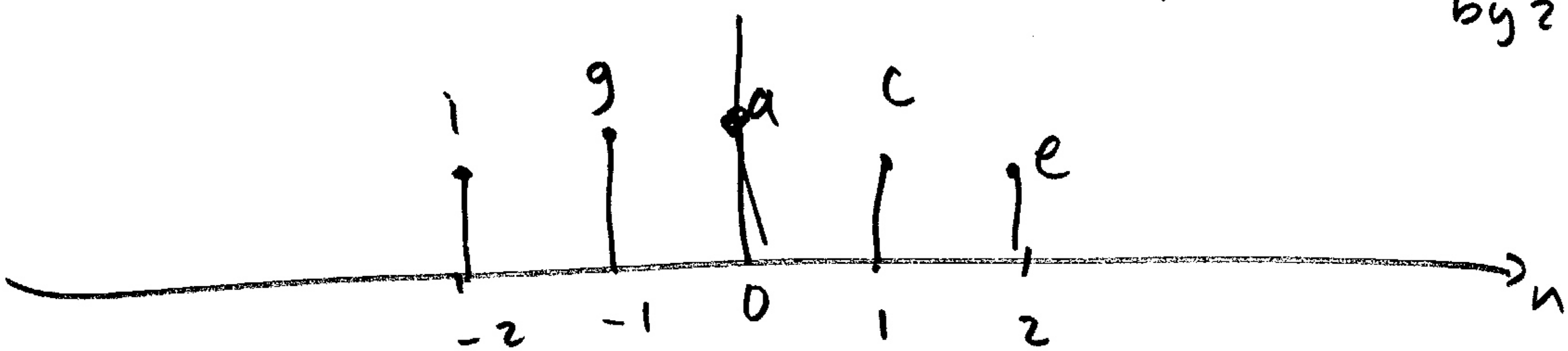
If $\Omega_c > \pi/T$

There is aliasing.

Down Sampling:



$\downarrow 2$ = downsampling by 2.



Intuitively: keep one out of every N sample

N = Downsampling factors.

inter downsampling. Fractional downsampling, keep M drop N.

$x_c(t)$ = Continuous Time Signal

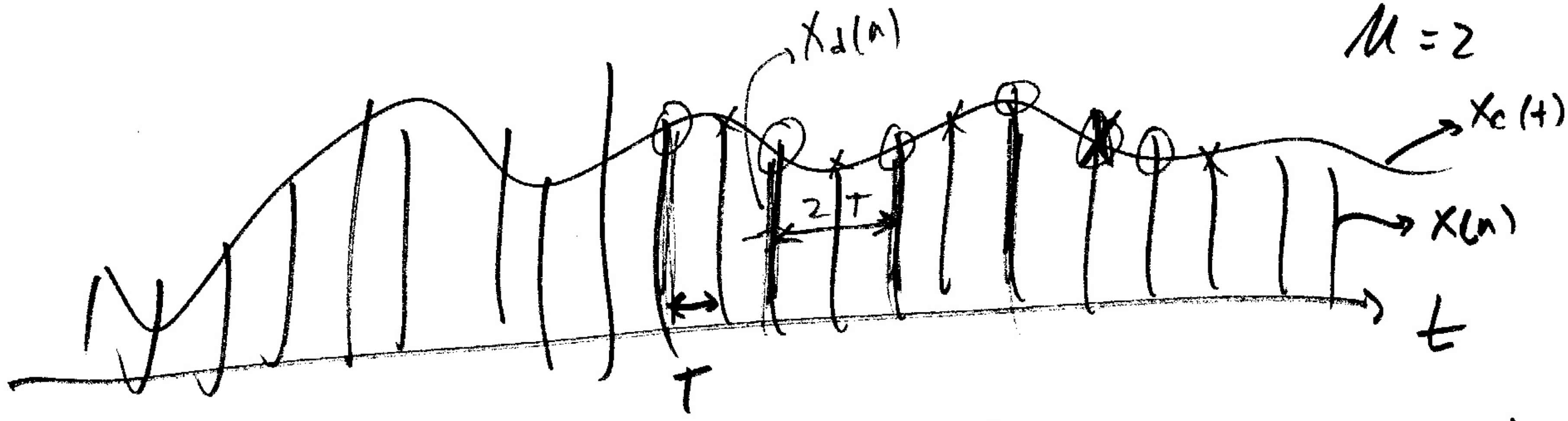
Sample at 2 rates:

(1) $\underset{\text{c.t. signal}}{x_c(t)} \longrightarrow \boxed{C/D} \longrightarrow x(n) = x_c(nT)$, with T

(2) $x_c(t) \longrightarrow \boxed{C/D} \longrightarrow x_d(n) = x_c(nMT)$, with MT

$x_d(n)$: downsampled signal by a factor of M

<u>Goal</u>: Relate DTFT of $x(n)$ to DTFT of $x_d(n)$



$$X(\omega) = DTFT\{x(n)\} = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)$$

$$X_d(\omega) = DTFT \text{ of } \{x_d(n)\} = \frac{1}{MT}\sum_{r=-\infty}^{+\infty} X_c\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)$$

change of variable $\quad r = i + kM$

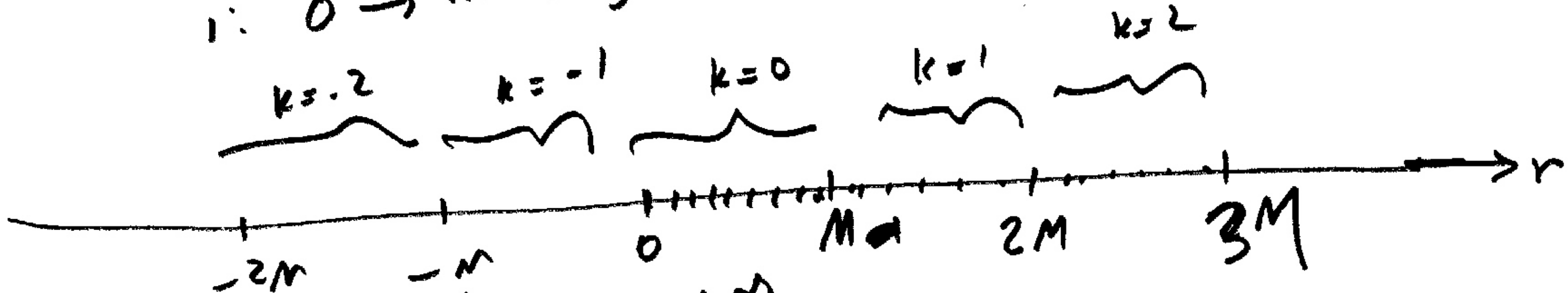
$$-\infty < k < \infty$$

$$0 \leq i \leq M-1$$

$$\left.\begin{array}{l} k=0 \\ i: 0 \to M-1 \end{array}\right\} \to r: \quad 0 \longrightarrow M-1$$

$$\left.\begin{array}{l} k=1 \\ i: 0 \to M-1 \end{array}\right\} \longrightarrow r: \quad M \longrightarrow 2M-1$$

$$\left.\begin{array}{l} k=2 \\ i: 0 \to M-1 \end{array}\right\} \longrightarrow r: \quad 2M \longrightarrow 3M-1$$



$$X_d(\omega) = \frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\frac{\omega}{MT} - \frac{2\pi k}{T} - \frac{2\pi i}{MT}\right)$$

$$X_d(\omega) = \frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\frac{\omega - 2\pi i}{MT} - \frac{2\pi k}{T}\right)$$

$$\left[X(\omega)\right]_{\omega \leftarrow \frac{\omega - 2\pi i}{M}}$$

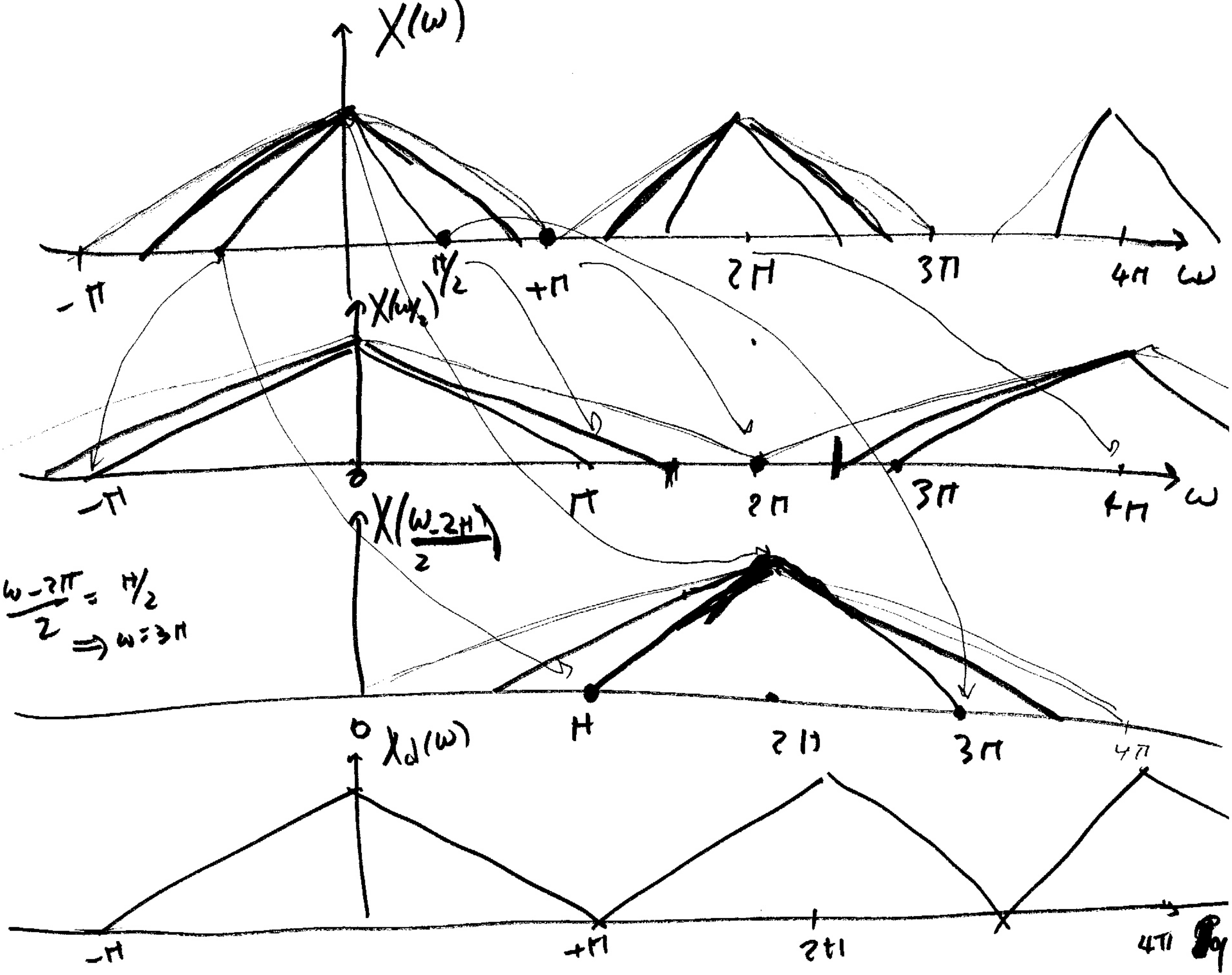
$$\boxed{X_d(\omega) = \frac{1}{M} \sum_{i=0}^{M-1} X\left(\frac{\omega - 2\pi i}{M}\right)}$$

Relationship between DTFT of $x(n)$ and $x_d(n)$

$M = 2$

$$X_d(\omega) = \frac{1}{2} \sum_{i=0}^{1} X\left(\frac{\omega}{2} - \frac{2\pi i}{2}\right)$$

$$= \frac{1}{2}\left[X\left(\frac{\omega}{2}\right) + X\left(\frac{\omega}{2} - \pi\right)\right]$$



$$\frac{\omega-2\pi}{2}=\frac{\pi}{2} \Rightarrow \omega=3\pi$$