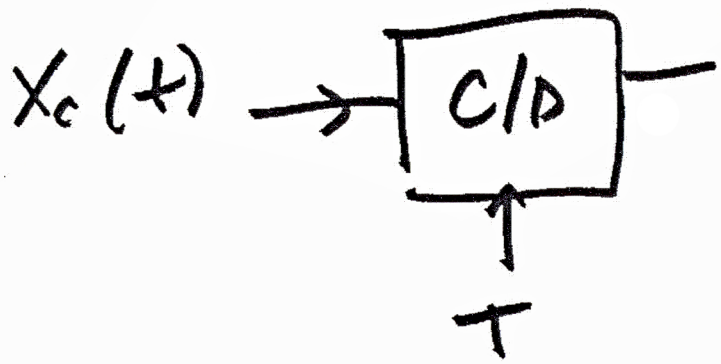
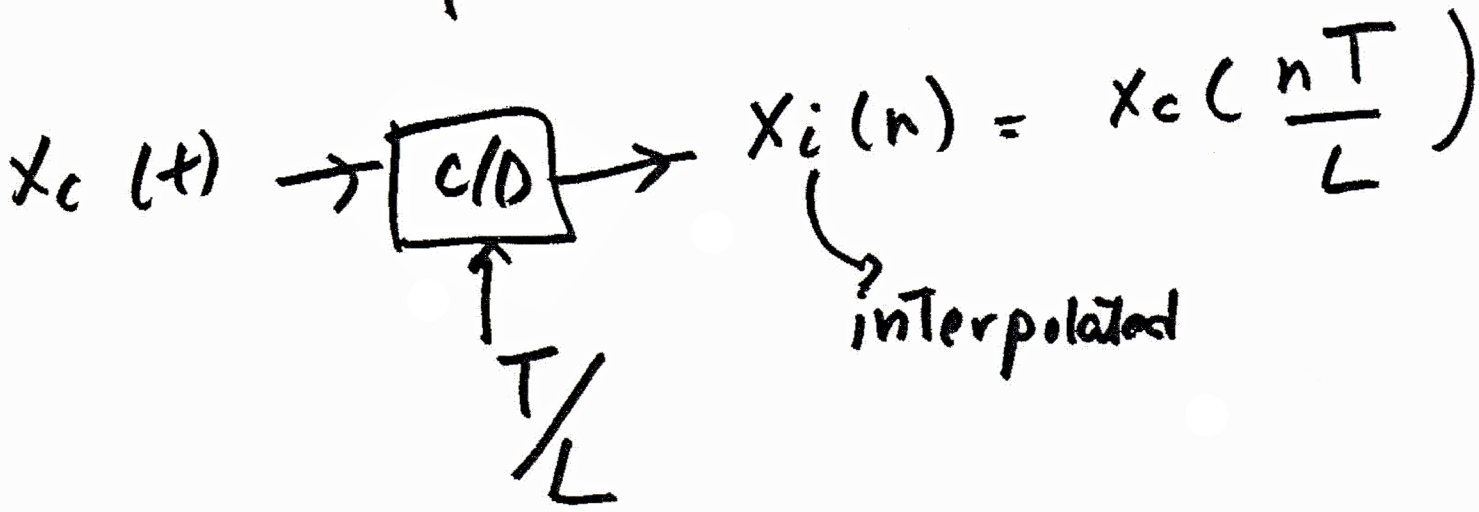
11/13/08

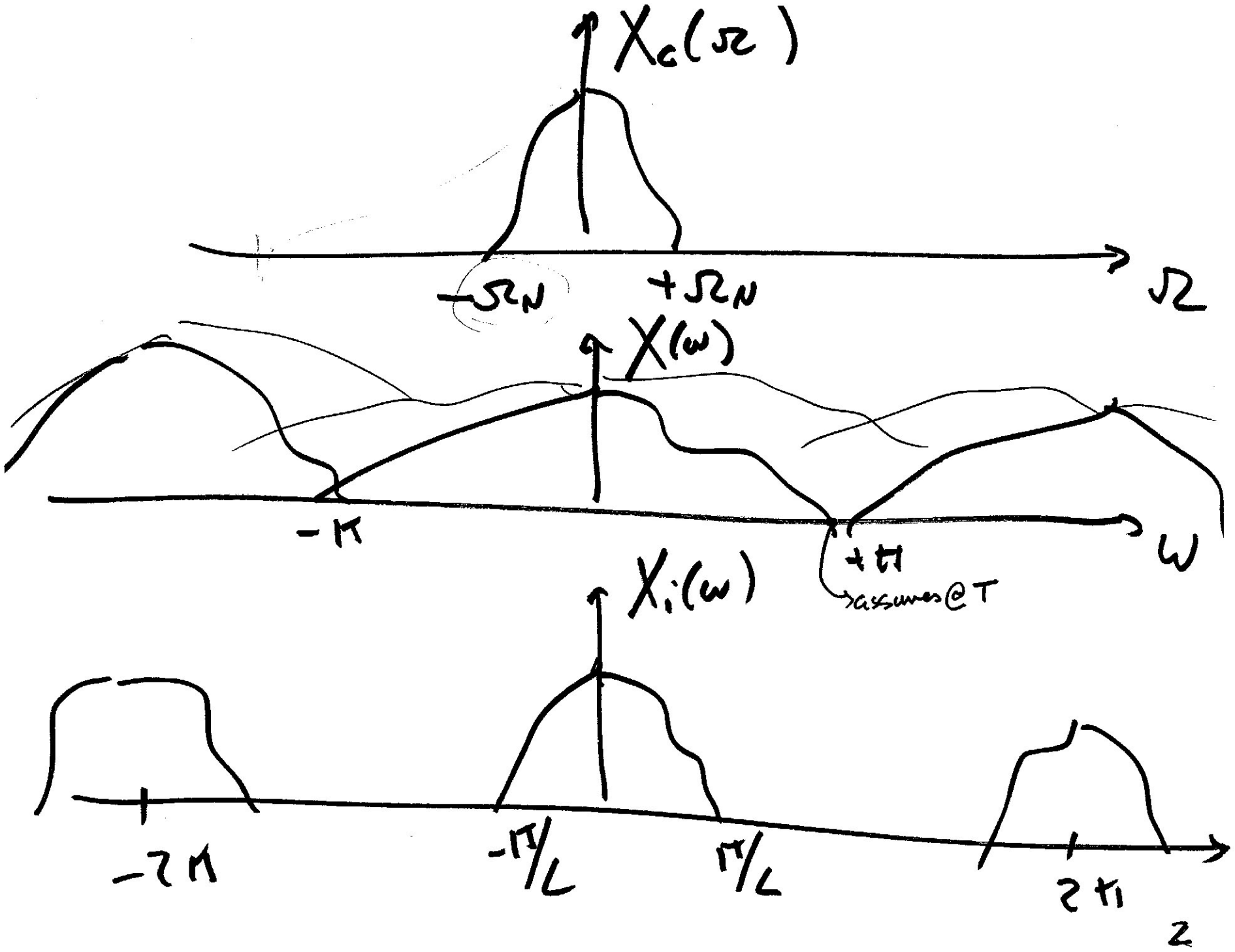
Upsampling:

- Increasing sampling rate by an integ factor

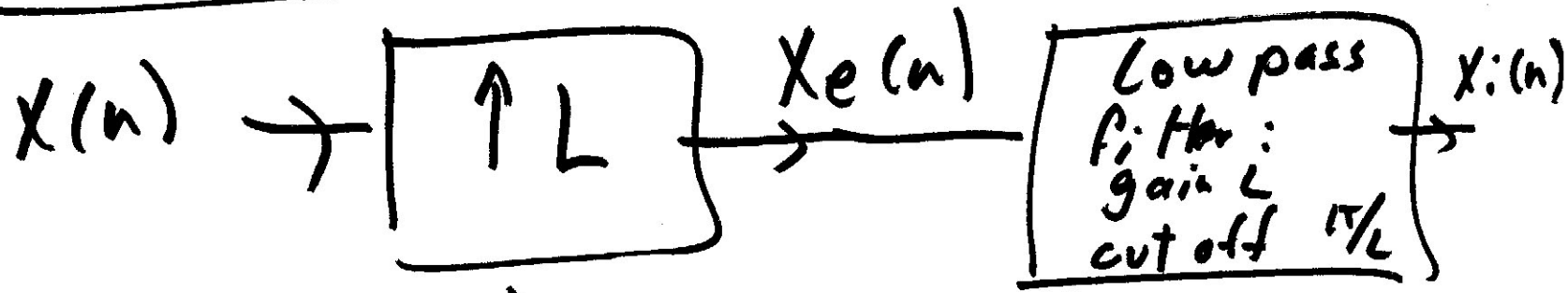




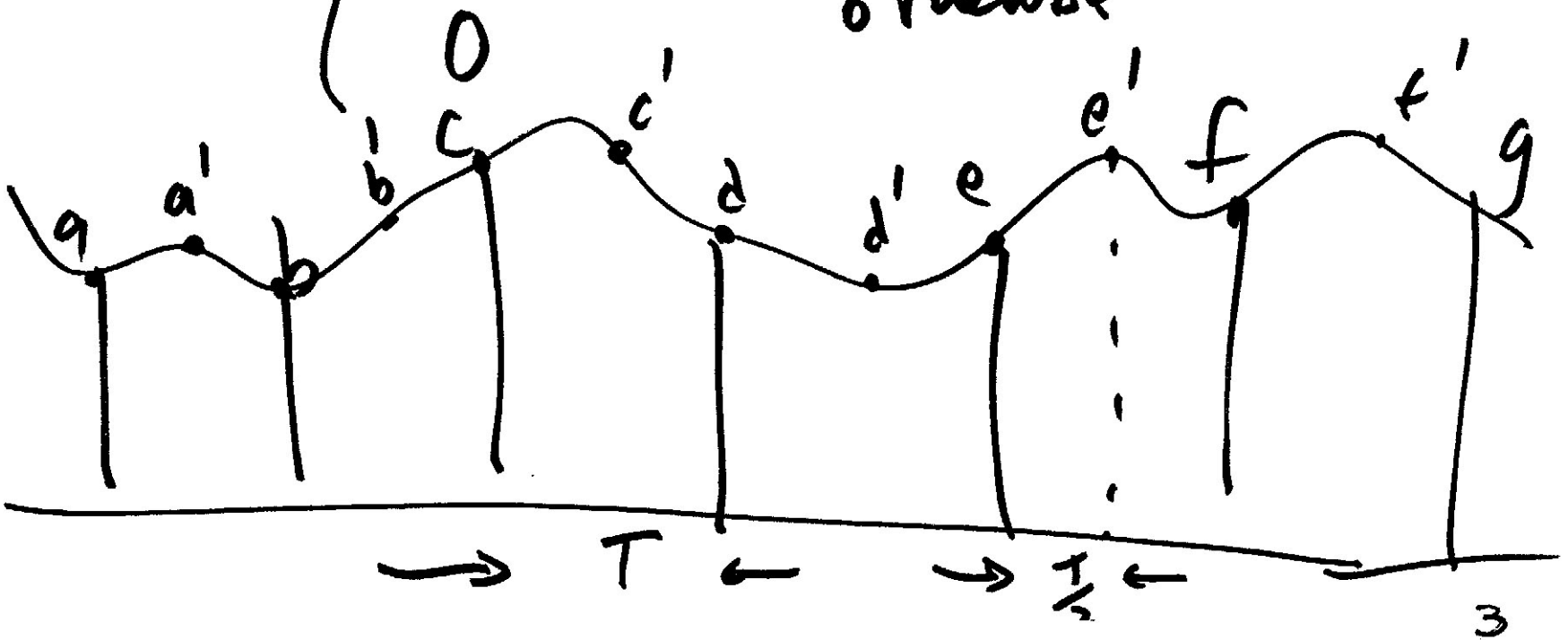
Q: can I obtain x_i from x?

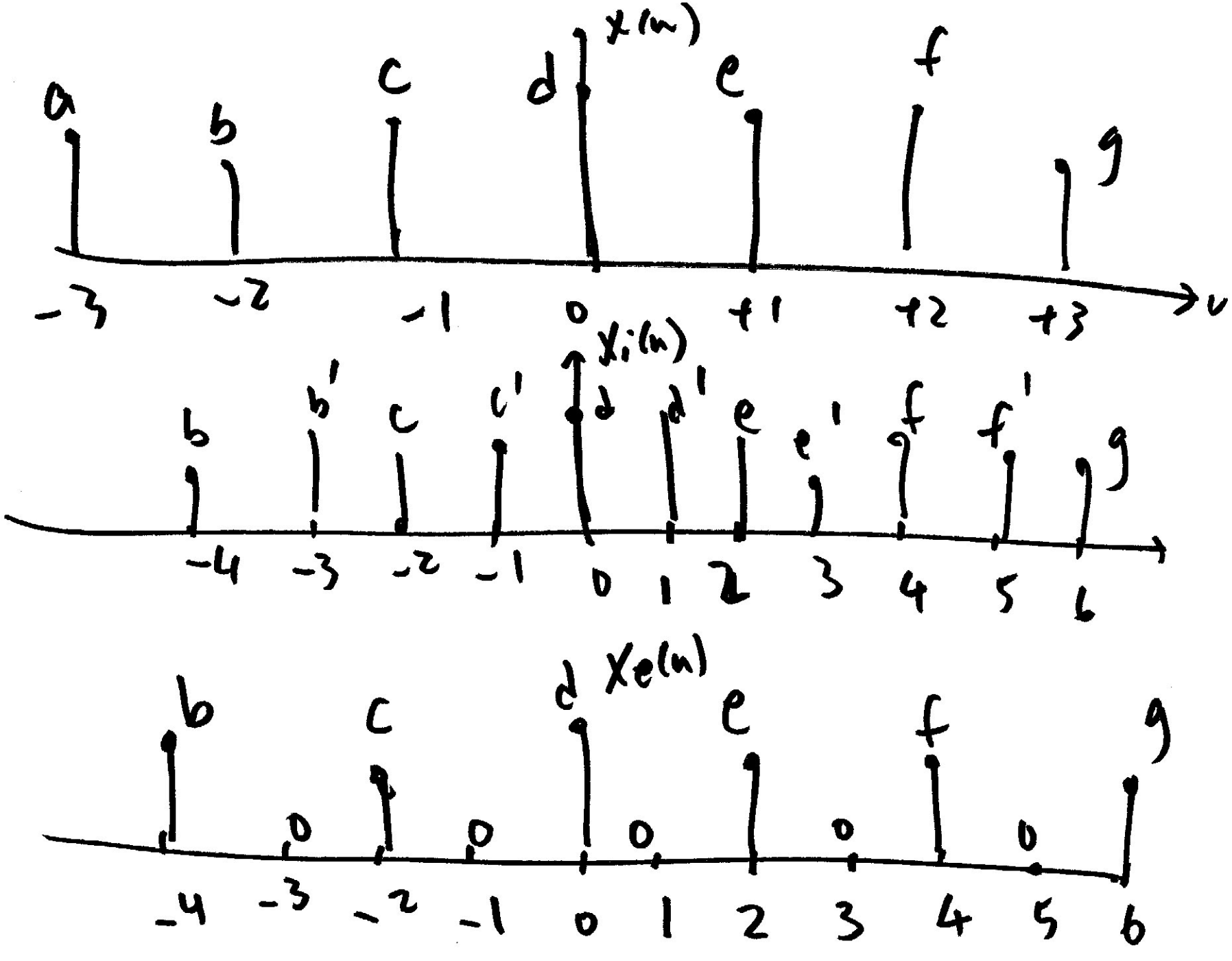

$X_c(\Omega)$
$-\Omega_N$
$+\Omega_N$
Ω
$X(\omega)$
$-\pi$
$+\pi$
ω
assumes @ T
$X_i(\omega)$
-2π
$-\pi/L$
π/L
2π

Proposed soln



$$X_e(n) \triangleq \begin{cases} x\left(\frac{n}{L}\right) & n = 0, \pm L, \pm 2L, \pm 3L \\ 0 & \text{otherwise} \end{cases}$$




x(n)
a
b
c
d
e
f
g
-3
-2
-1
0
+1
+2
+3
n
xi(n)
b
b'
c
c'
d
d'
e
e'
f
f'
g
-4
-3
-2
-1
0
1
2
3
4
5
6
Xe(n)
b
c
d
e
f
g
0
0
0
0
0
0
-4
-3
-2
-1
0
1
2
3
4
5
6

Relate $D.T.F.T\{x_e(n)\}$ to $D.T.F.T.\{x(n)\}$

$$x_e(n) = \sum_k x(k)\,\delta(n-kL)$$

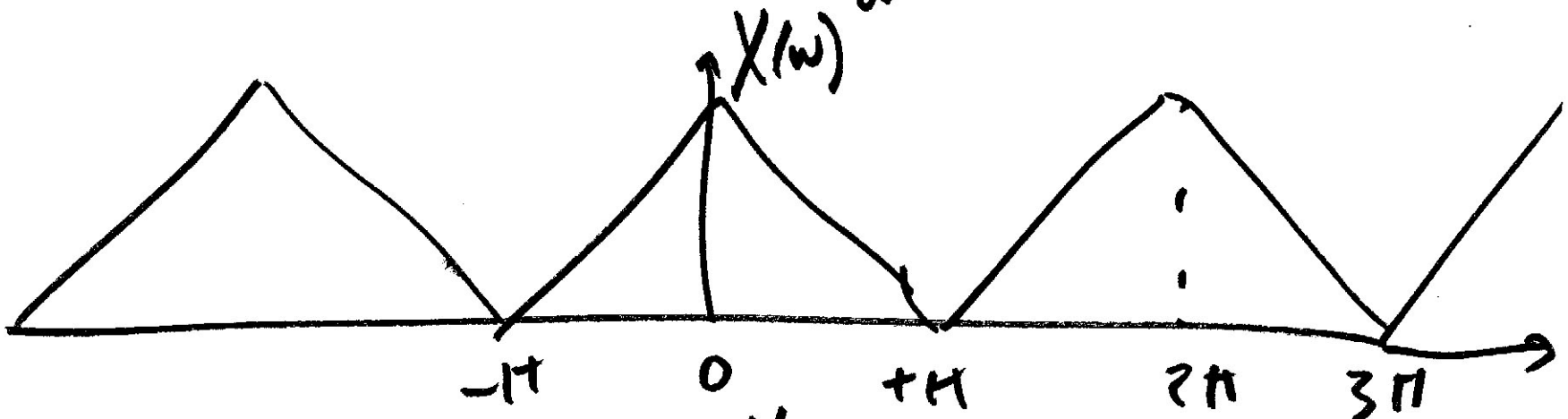
$$X_e(\omega) = \sum_n \left(\sum_k x(k)\,\delta(n-kL) \right) e^{-j\omega n}$$

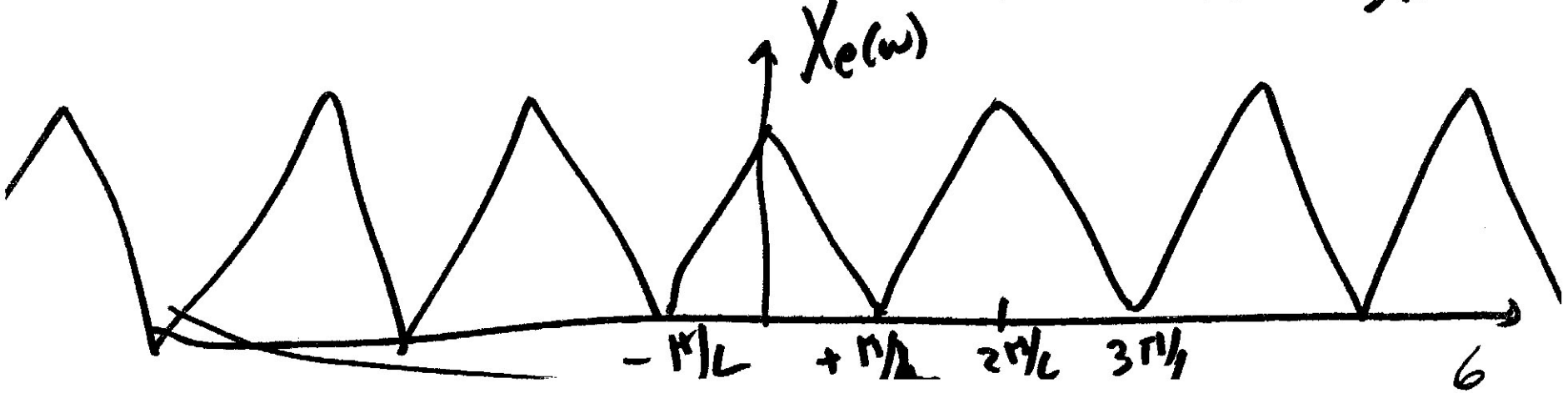
$$X_e(\omega) = \sum_{k=-\infty}^{+\infty} x(k) \underbrace{\sum_{n=-\infty}^{+\infty} \delta(n-kL)\, e^{-j\omega n}}$$

$$X_e(\omega) = \sum_{k=-\infty}^{+\infty} x(k)\, e^{-j\omega Lk} \Longrightarrow$$

$$X(\omega) = \sum_{k=-\infty}^{+\infty} x(k)\, e^{-j\omega k}$$

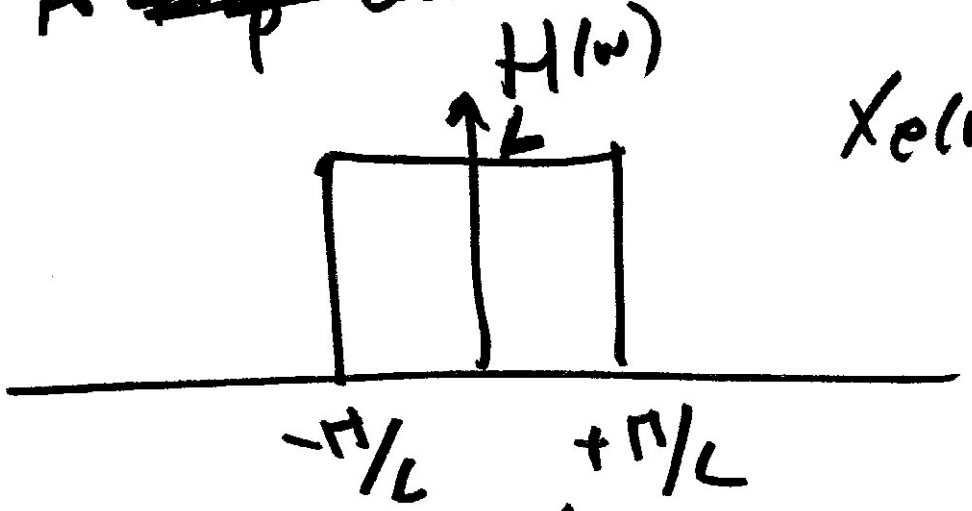
$$X_e(\omega) = \left[X(\omega) \right]_{\omega = \omega L} = X(\omega L)$$

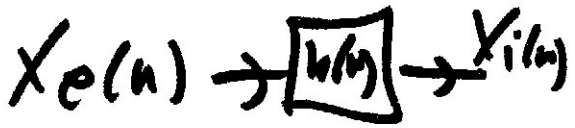


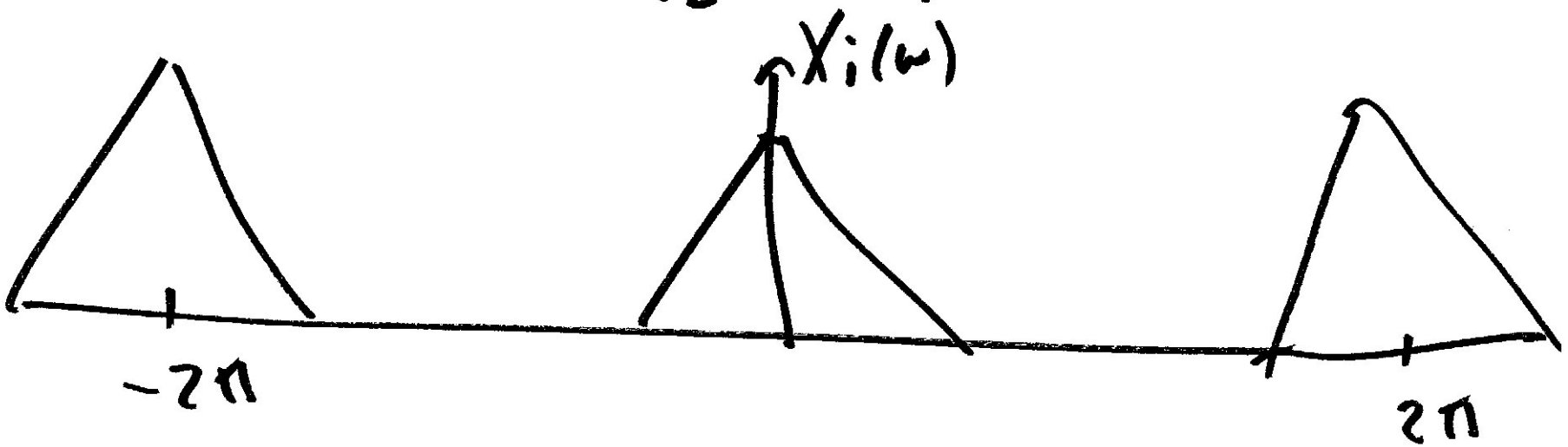


to get $X_i(w)$ from $X_e(w)$,

Ideal LPF ~~Fc Rectangular~~.



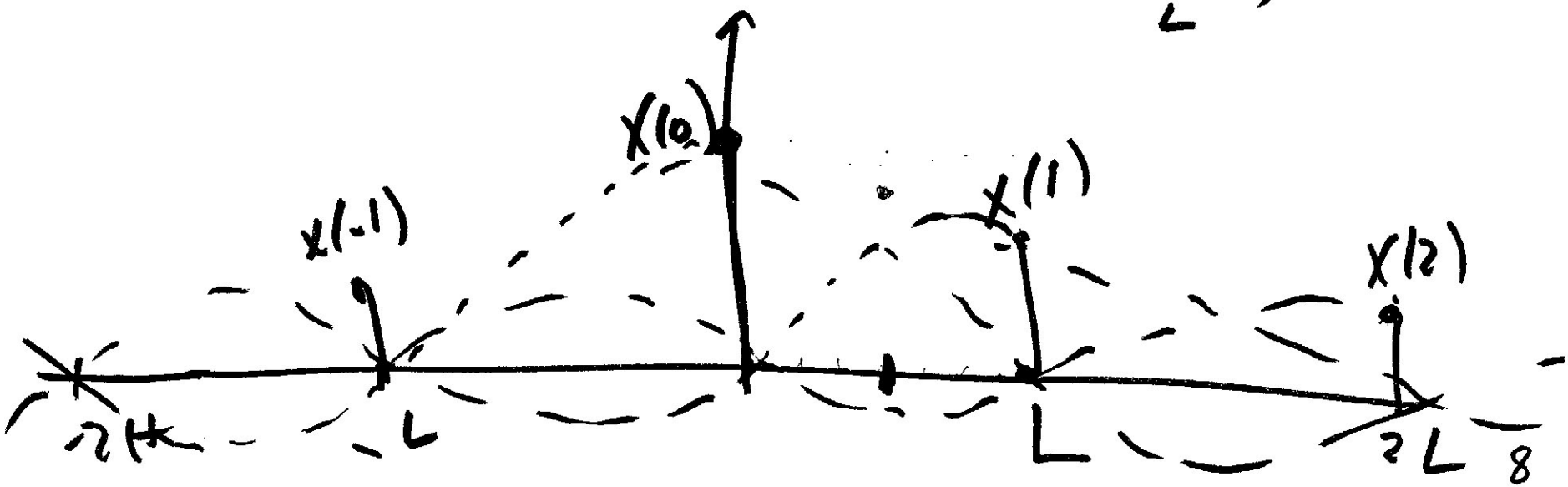


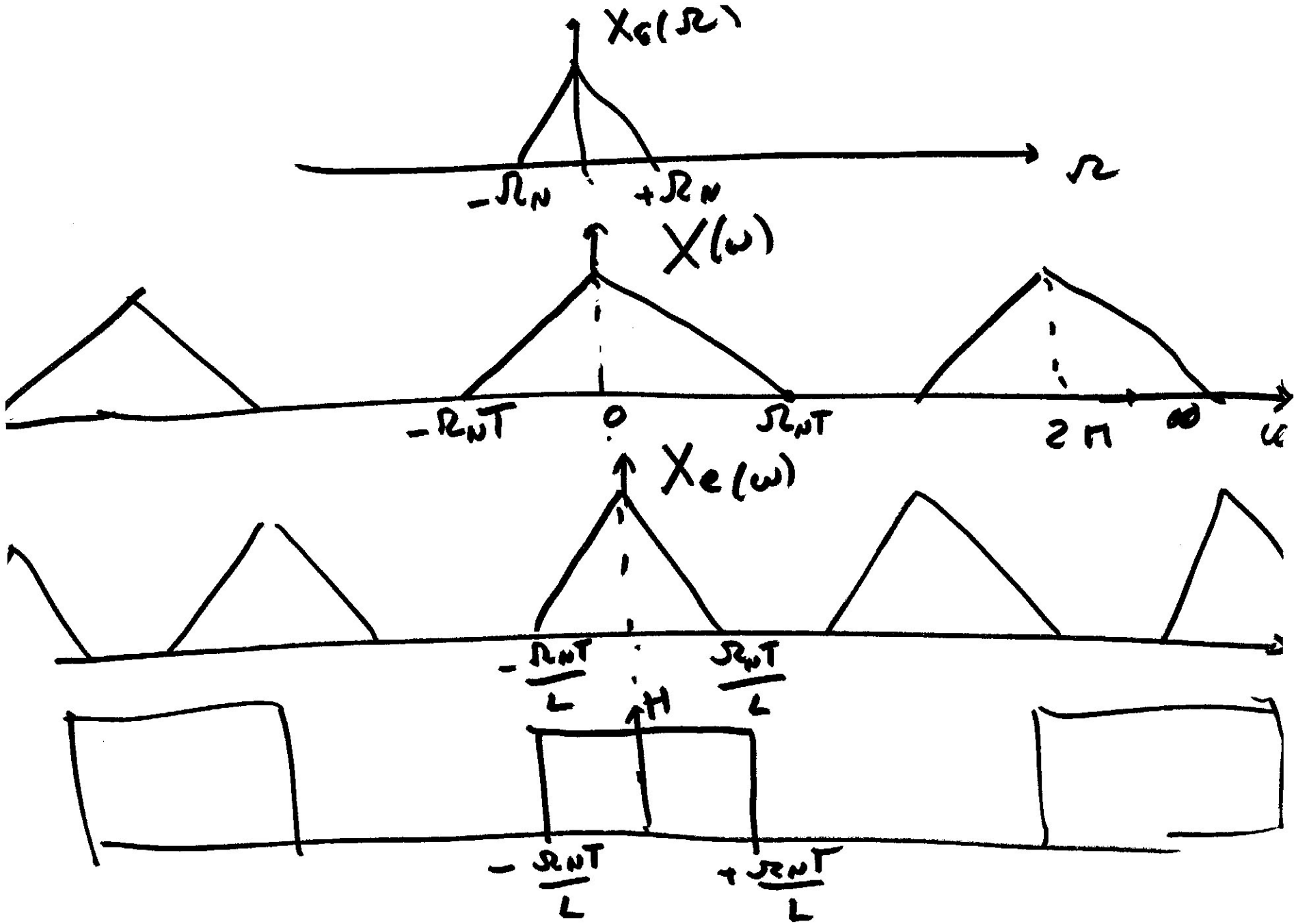


Impulse response: $h(n) = \dfrac{\sin \pi n/L}{\pi n/L}$

$$X_i(n) = x_e * h = \sum_k x_e(k)\, h(n-k)$$

$$X_i(n) = \sum_k x(k) \; \frac{\sin \frac{\pi(n-k)}{L}}{\frac{\pi(n-k)}{L}}$$




$X_c(\Omega)$
Ω
$-\Omega_N$
$+\Omega_N$
$X(\omega)$
$-\Omega_N T$
0
$\Omega_N T$
2π
ω
$X_c(\omega)$
$-\frac{\Omega_N T}{L}$
$\frac{\Omega_N T}{L}$
H
$-\frac{\Omega_N T}{L}$
$+\frac{\Omega_N T}{L}$