DCT & its relation to DFT

Proposal 1:

1. $x(n)$ N pT sequence, assue it is real
2. Take $\underline{2N}$ pt DFT $\longrightarrow X(k)$
3. $2\,\mathrm{Re}\left\{ X(k)\, e^{-j\frac{\pi k}{2N}} \right\} \overset{?}{=} X^{C_2}(k)$

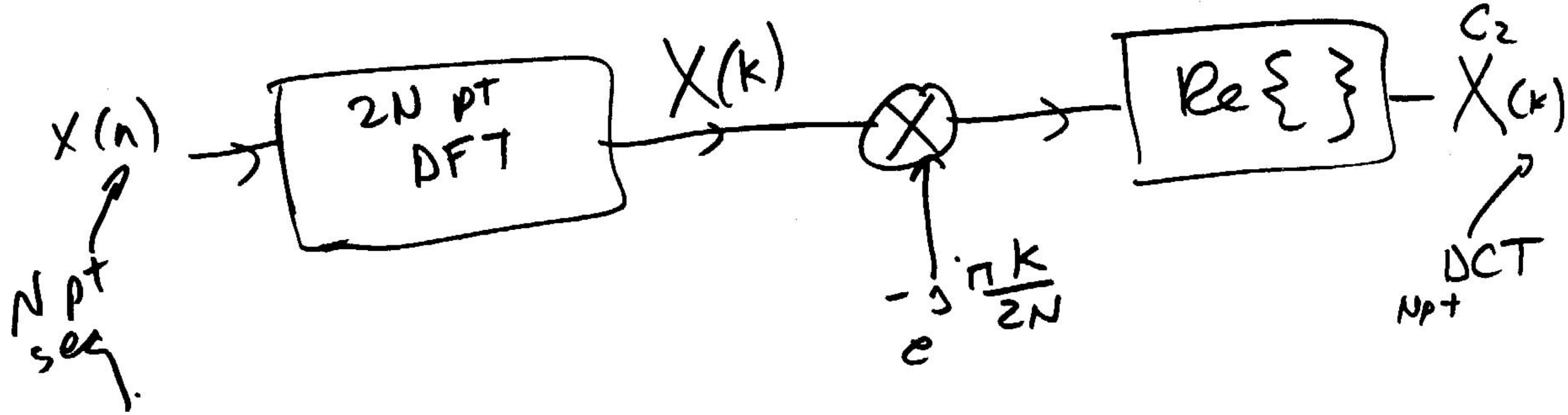
Proof:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n k}{2N}}$$

$$X(k)\, e^{-j\frac{\pi k}{2N}} = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{\pi k(2n+1)}{2N}}$$

$$2\,\mathrm{Re}\left\{ \downarrow \right\} = 2 \sum_{n=0}^{N-1} x(n) \cos \frac{\pi k (2n+1)}{2N}$$

$$2\,\mathrm{Re}\left\{ X(k)\, e^{-j\frac{\pi k}{2N}} \right\} = X^{C_2}(k) \quad \Rightarrow \text{QED.}$$



Aliter: Relating DFT to DCT:

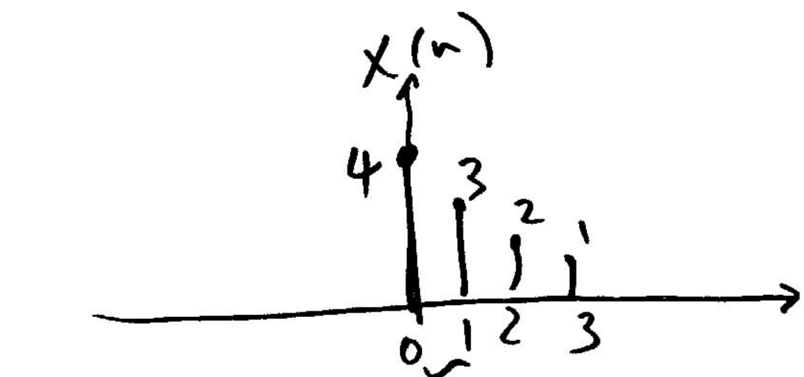
Proposal:

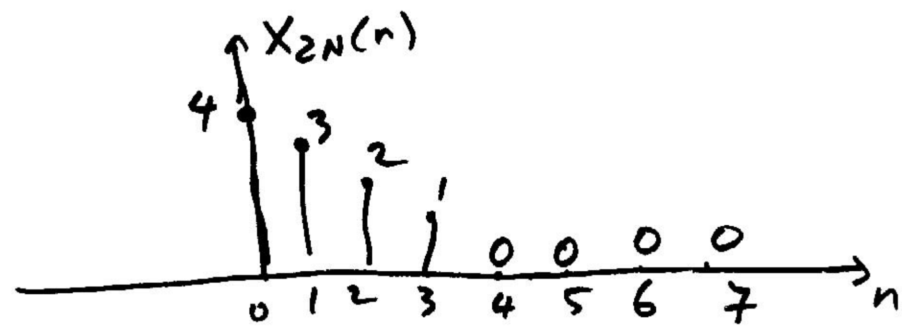
1. Start with N pt real sequence $x(n)$.
2. Pad it with N zeros $\longrightarrow x_{2N}(n)$
3. Form a periodic sequence

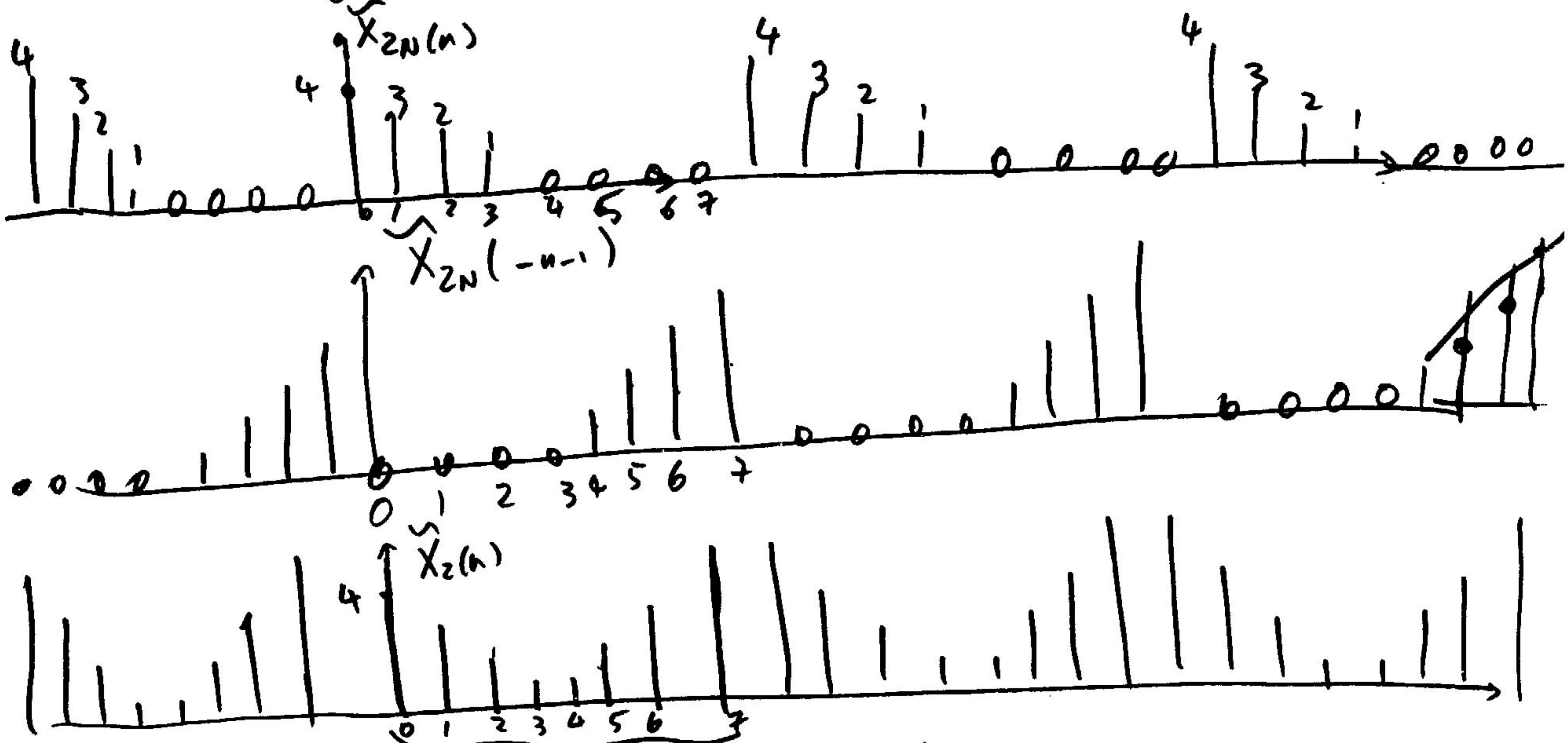
$$\tilde{x}_2(n) = \tilde{x}_{2N}(n) + \tilde{x}_{2N}(-n-1)$$

4. Take 2N pt DFT of one period of $\tilde{x}_2(n) \longrightarrow \bar{X}_2(k)$

5. Relate $X_2(k)$ to $X^{C_2}(k)$.







2N pT i.e. 8pT DFT.

<u>Q</u> How is $X_2(k)$ related to $X^{C_2}(k)$?

Let $\bar{X}(k) =$ 2N pt DFT of $x(n)$.

$$X_2(k) = \bar{X}(k) + e^{j\frac{2\pi k}{2N}} \bar{X}^*(k)$$

$$= e^{j\frac{\pi k}{2N}} \left[\bar{X}(k) e^{-j\frac{\pi k}{2N}} + \bar{X}^*(k) e^{j\frac{\pi k}{2N}} \right]$$

$$X_2(k) = e^{j\frac{\pi k}{2N}} \underbrace{2 \, \mathrm{Re}\left\{ \bar{X}(k) e^{-j\frac{\pi k}{2N}} \right\}}_{X^{C_2}(k) \leftarrow \text{proposal}}$$

$$\boxed{X_2(k) e^{-j\frac{\pi k}{2N}} = X^{C_2}(k)}$$

Fig 8.29

FFT = Fast Fourier Transform:

* merely a way of computing DFT faster.
  not a new Transform.

* Two flavors
  - Decimation in Time.
  - Decimation in freq.

* Motivation
  Real life sys $\rightarrow$ LTI $\rightarrow$ convolution $\rightarrow$ DFT $\rightarrow$ FFT

DFT: $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j 2 \frac{\pi n k}{N}}$ $\quad 0 \leq k < N$

<u>Direct Computation</u>:

For each k: $(N-1)$ adds (complex)

N complex mults

N values of k

$\Rightarrow$ # add $N(N-1) \simeq N^2$

mults N^2

$\underline{\underline{O(N^2)}}$

<u>FFT</u> $N \log_2 N$

$N = 10^6$

Direct comput $O(10^{12})$

FFT $\Rightarrow 10^6 \log_2 10^6 \simeq 2 \times 10^7$ $\Bigg\} \Rightarrow$

5 orders of magnitude