UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 11
Spring 2019

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1. **Statistical Estimation**

   Given $X \in \{0, 1\}$, the random variable Y is exponentially distributed with rate $3X + 1$.

   (a) Assume $\mathbb{P}(X = 1) = p \in (0, 1)$ and $\mathbb{P}(X = 0) = 1 - p$. Find the MAP estimate of X given Y.

   (b) Find the MLE of X given Y.

2. **Poisson Process MAP**

Customers arrive to a store according to a Poisson process of rate 1. The store manager learns of a rumor that one of the employees is sending 1/2 of the customers to the rival store. Refer to hypothesis $X = 1$ as the rumor being true, that one of the employees is sending every other customer arrival to the rival store and hypothesis $X = 0$ as the rumor being false, where each hypothesis is equally likely. Assume that at time 0, there is a successful sale. After that, the manager observes $S_1, S_2, \ldots, S_n$ where n is a positive integer and S_i is the time of the ith subsequent sale for $i = 1, \ldots, n$. Derive the MAP rule to determine whether the rumor was true or not.

3. **Laplace Prior & ℓ^1-Regularization**

Suppose you draw n i.i.d. data points $(x_1, y_1), \ldots, (x_n, y_n)$, where n is a positive integer and the true relationship is $Y = WX + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. (That is, Y has a linear dependence on X, with additive Gaussian noise.) Further suppose that W has a prior distribution with density

$$f_W(w) = \frac{1}{2\beta} \mathrm{e}^{-|w|/\beta}, \qquad \beta > 0.$$

(This is known as the **Laplace distribution**.) Show that finding the MAP estimate of W given the data points $\{(x_i, y_i) : i = 1, \ldots, n\}$ is equivalent to minimizing the cost function

$$J(w) = \sum_{i=1}^{n} (y_i - wx_i)^2 + \lambda |w|$$

(you should determine what λ is). This is interpreted as a one-dimensional ℓ^1-regularized least-squares criterion, also known as LASSO.