UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

**Discussion 3**
Spring 2019

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1. **Triangle Density**

   Consider random variables $X$ and $Y$ which have a joint PDF uniform on the triangle with vertices at $(0,0),(1,0),(0,1)$.

   (a) Find the joint PDF of $X$ and $Y$.

   (b) Find the marginal PDF of $Y$.

   (c) Find the conditional PDF of $X$ given $Y$.

   (d) Find $\mathbb{E}[X]$ in terms of $\mathbb{E}[Y]$.

   (e) Find $\mathbb{E}[X]$.

2. **Change of Variables**

   (a) Suppose that $X$ has the **standard normal distribution**, that is, $X$ is a continuous random variable with density function

   $$f(x) = \frac{1}{\sqrt{2\pi}} \exp\Big(-\frac{x^2}{2}\Big).$$

   What is the density function of $\exp X$? (The answer is called the **lognormal distribution**.)

   (b) Suppose that $X$ is a continuous random variable with density $f$. What is the density of $X^2$?

   (c) What is the answer to the previous question when $X$ has the standard normal distribution? (This is known as the **chi-squared distribution**.)

3. **Order Statistics**

   For $n$ a positive integer, let $X_1, \ldots, X_n$ be i.i.d. continuous random variables with common PDF $f$ and CDF $F$. For $i = 1, \ldots, n$, let $X^{(i)}$ be the $i$th smallest of $X_1, \ldots, X_n$, so we have $X^{(1)} \le \cdots \le X^{(n)}$. $X^{(i)}$ is known as the **$i$th order statistic**.

   (a) What is the CDF of $X^{(i)}$?

   (b) Differentiate the CDF to obtain the PDF of $X^{(i)}$.

   (c) Can you obtain the PDF of $X^{(i)}$ directly?