UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: PROBABILITY AND RANDOM PROCESSES

Discussion 4
Spring 2019

---

1. Revisiting Facts Using Transforms

(a) Let $X \sim \text{Poisson}(\lambda)$, $Y \sim \text{Poisson}(\mu)$ be independent. Calculate the MGF of $X + Y$ and use this to show that $X + Y \sim \text{Poisson}(\lambda + \mu)$.

(b) Calculate the MGF of $X \sim \text{Exponential}(\lambda)$ and use this to find all of the moments of X.

2. Office Hours

In an EE 126 office hour, students bring either a difficult-to-answer question with probability $p = 0.2$ or an easy-to-answer question with probability $1 - p = 0.8$. A GSI takes a random amount of time to answer a question, with this time duration being exponentially distributed with rate $\mu_D = 1$ (questions per minute)—where D denotes "difficult"—if the problem is difficult, and $\mu_E = 2$ (questions per minute)—where E denotes "easy"—if the problem is easy.

(a) You visit office hours and find a GSI answering the question of another student. Conditioned on the fact that the GSI has been busy with the other student's question for $T > 0$ minutes, let q be the conditional probability that the problem is difficult. Find the value of q.

(b) Conditioned on the information above, find the expected amount of time you have to wait from the time you arrive until the other student's question is answered.

3. Is this true?

If X_1, X_2, X_3 are i.i.d. continuous RV, is the following argument correct? If it is correct, can you use generalize this argument to find the probability that $X_1 > \max_{2 \le i \le n} X_i$ (i.e. the probability that X_1 is the max of n i.i.d copies of itself)? If not, where does it break and what would be the right answer?

- For any arbitrary x, we have that $P(x > X_2 \cap x > X_3) = P(x > X_2)P(x > X_3)$ since X_2 and X_3 are independent.
- Thus, for any distribution X_1 independent of X_2 and X_3 we have that $P(X_1 > X_2 \cap X_1 > X_3) = P(X_1 > X_2)P(X_1 > X_3)$
- $P(X_1 > X_2)P(X_1 > X_3) = P(X_1 > X_2)^2$ since X_2 and X_3 have the same distribution and are independent from X_1
- $P(X_1 > X_2)^2 = (1/2)^2$ since $P(X_1 = X_2) = 0$, and by symmetry $P(X_1 > X_2) = P(X_2 > X_1)$, and they sum up to 1.

4. First Time to Decrease

Let $X_1, X_2, \ldots, X_n, \ldots$ be a sequence of independent and identically distributed (i.i.d.) continuous random variables with common PDF f.

(a) Argue that $\mathbb{P}(X_i = X_j) = 0$ for $i \neq j$.

(b) Calculate $\mathbb{P}(X_1 \leq X_2 \leq \cdots \leq X_{n-1})$.

(c) Let N be a random variable which is equal to the first time that the sequence of the random variables will decrease, i.e.

$$N = \min\{n \in \mathbb{Z}_{\geq 2} \mid X_{n-1} > X_n\}.$$

Calculate $\mathbb{E}[N]$.