

Discussion 7
Spring 2019

1. Markov Chain Practice

Consider a Markov chain with three states 0, 1, and 2. The transition probabilities are $P(0, 1) = P(0, 2) = 1/2$, $P(1, 0) = P(1, 1) = 1/2$, and $P(2, 0) = 2/3$, $P(2, 2) = 1/3$.

- (a) Classify the states in the chain. Is this chain periodic or aperiodic?
- (b) In the long run, what fraction of time does the chain spend in state 1?
- (c) Suppose that X_0 is chosen according to the steady state distribution. What is $\mathbb{P}(X_0 = 0 \mid X_2 = 2)$?
- (d) Suppose that $X_0 = 0$, and let T denote the first time by which the process has visited all the states. Find $\mathbb{E}[T]$.

2. Running Sum of a Markov Chain

Let $(X_n)_{n \in \mathbb{N}}$ be a Markov chain with two states, -1 and 1 , and transition probabilities $P(-1, 1) = P(1, -1) = a$ for $a \in (0, 1)$. Define

$$Y_n = X_0 + X_1 + \cdots + X_n.$$

For what values of a is $(Y_n)_{n \in \mathbb{N}}$ a Markov chain?

3. Finite Random Walk

- (a) Find the steady-state probabilities π_0, \dots, π_{k-1} for the Markov chain in Figure 1. Here, k is a positive integer and $p \in (0, 1)$. Express your answer in terms of the ratio $\rho = p/q$, where $q = 1 - p$. Pay particular attention to the special case $\rho = 1$.
- (b) Find the limit of π_0 as k approaches infinity; give separate answers for $\rho < 1$, $\rho = 1$, and $\rho > 1$. Find limiting values of π_{k-1} for the same cases.

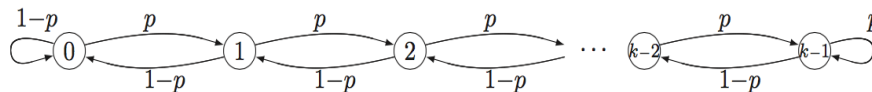


Figure 1: Markov chain for Problem 2.