UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 8
Spring 2019

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1. **Random Walk on the Cube**

   Consider the symmetric random walk on the vertices of the 3-dimensional unit cube where two vertices are connected by an edge if and only if the line connecting them is an edge of the cube. In other words, this is the random walk on the graph with 8 nodes each written as a string of 3 bits, so that the vertex set is $\{0,1\}^3$, and where two vertices are connected by an edge if and only if their corresponding bit strings differ in exactly one location.

   This random walk is modified so that the nodes 000 and 111 are made absorbing.

   (a) What are the communicating classes of the resulting Markov chain? For each class, determine its period, and whether it is transient or recurrent.

   (b) For each transient state, what is the probability that the modified random walk started at that state gets absorbed in the state 000?

2. **Random Walk on an Undirected Graph**

   Consider a random walk on an undirected connected finite graph (that is, define a Markov chain where the state space is the set of vertices of the graph, and at each time step, transition to a vertex chosen uniformly at random out of the neighborhood of the current vertex). What is the stationary distribution?

3. **Reversible Markov Chains**

   Let $(X_n)_{n\in\mathbb{N}}$ be an irreducible Markov chain on a finite set $\mathcal{X}$, with stationary distribution π and transition matrix P. The **graph** associated with the Markov chain is formed by taking the transition diagram of the Markov chain, removing the directions on the edges (making the graph undirected), removing self-loops, and removing duplicate edges. Show that if the graph associated with the Markov chain is a tree, then the Markov chain is reversible.

   Hint: To solve this problem, try induction on the size of $\mathcal{X}$:

   (a) Use the fact that every tree has a leaf node x connected to only one neighbor y, and show that detailed balance holds for the edge (x, y) connecting the leaf with its single neighbor.

   (b) Then, argue that if you remove the leaf x from the Markov chain and increase the probability of a self-transition at state y by $P(y, x)$, then the stationary distribution of the original chain (when restricted to $\mathcal{X} \setminus \{x\}$) is the stationary distribution for the new chain, and use this to conclude the inductive proof.