UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 9
Spring 2019

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1. **Poisson Practice**

   Let $(N(t), t \geq 0)$ be a Poisson process with rate λ. Let T_k denote the time of k-th arrival, for $k \in \mathbb{N}$, and given $0 \leq s < t$, we write $N(s,t) = N(t) - N(s)$. Compute:

   (a) $\mathbb{P}(N(1) + N(2,4) + N(3,5) = 0)$.

   (b) $\mathbb{E}(N(1,3) \mid N(1,2) = 3)$.

   (c) $\mathbb{E}(T_2 \mid N(2) = 1)$.

2. **Illegal U-Turns**

   Each morning, as you pull out of your driveway, you would like to make a U-turn rather than drive around the block. Unfortunately, U-turns are illegal and police cars drive by according to a Poisson process with rate λ. You decide to make a U-turn once you see that the road has been clear of police cars for $\tau > 0$ units of time. Let N be the number of police cars you see before you make a U-turn.

   (a) Find $\mathbb{E}[N]$.

   (b) Let n be a positive integer ≥ 2. Find the conditional expectation of the time elapsed between police cars $n-1$ and n, given that $N \geq n$.

   (c) Find the expected time that you wait until you make a U-turn.

3. **Continuous-Time Markov Chains: Introduction**

   Consider the continuous-time Markov process with state space $\{1, 2, 3, 4\}$ and the rate matrix

$$Q = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 0 & -3 & 2 & 1 \\ 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

   (a) Find the stationary distribution p of the Markov process.

   (b) Find the stationary distribtion π of the jump chain, i.e., the discrete-time Markov chain which only keeps track of the jumps of the CTMC. Formally, if the CTMC $(X(t))_{t \geq 0}$ jumps at times $T_1, T_2, T_3, \ldots$, then the DTMC is defined as $(Y_n)_{n=1}^{\infty}$ where $Y_n := X_{T_n}$.

   (c) Suppose the chain starts in state 1. What is the expected amount of time until it changes state for the first time?

   (d) Again assume the chain starts in state 1. What is the expected amount of time until the chain is in state 4?