EECS 16A Designing Information Devices and Systems I
Spring 2017 Babak Ayazifar, Vladimir Stojanovic **Discussion 13A**

1. **The Order of Gram-Schmidt**

   If we are performing the Gram-Schmidt method on a set of vectors, does the order in which we take the vectors matter? Consider the set of vectors

$$\left\{v_1,\ v_2,\ v_3\right\} = \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix},\ \begin{bmatrix}1\\1\\0\end{bmatrix},\ \begin{bmatrix}1\\1\\1\end{bmatrix} \right\} \tag{1}$$

   Perform Gram-Schmidt on these vectors first in the order $v_1,\ v_2,\ v_3$ and then in the order $v_3,\ v_2,\ v_1$. Do you get the same answer?

2. **Orthogonal Polynomials**

   So far we've applied most of the linear algebra tools learned to vector spaces over real or complex numbers, that is $\mathbb{R}^n$ or $\mathbb{C}^n$. However the Gram-Schmidt process and least squares can be applied to other vector spaces as well. Here we consider the polynomial vectors space $\mathbb{P}^n$.

   Suppose we are operating in $\mathbb{P}^3$, that is polynomials of degree 3 or less.

   (a) Represent the polynomial x^3+5x^2+3x+2 as a linear combination of the standard basis vectors.

   (b) There are other basis for the $\mathbb{P}^3$. Represent x^3+5x^2+3x+2 as a linear combination of the following basis: $\{x^3+x^2+x+1, x^2+x+1, x+1, 1\}$.

   (c) We cannot apply the same inner product we use over $\mathbb{R}^n$ over $\mathbb{P}^n$. Instead, we have to create a new inner product. One inner product is: $\langle f(x), g(x)\rangle = \int_{-1}^{1} f(x)g(x)dx$. Show that this inner product satisfies the properties below:

$$\langle f(x), g(x)\rangle = \langle g(x), f(x)\rangle$$
$$\langle cf(x), g(x)\rangle = c\langle f(x), g(x)\rangle$$
$$\langle f(x)+g(x), h(x)\rangle = \langle f(x), h(x)\rangle + \langle g(x), h(x)\rangle$$

   (d) With the definition of inner product shown above, we can find an orthogonormal basis for polynomials using the Gram-Schmidt process. Find an orthonormal basis for $\mathbb{P}^3$.

   (e) We can also use this definition to find a least squares approximation. Suppose we want to find a polynomial approximation for the function $f(x) = e^x$. Find the best approximation for e^x in $\mathbb{P}^3$ You may find the following integrals useful:

$$\int xe^x dx = (x-1)e^x$$
$$\int x^2e^x dx = (x^2-2x+2)e^x$$
$$\int x^3e^x dx = (x^3-3x^2+6x-6)e^x$$

3. Gram-Schmidt Procedure and QR Factorization

Compute the QR Factorization of the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$