EECS 16A Designing Information Devices and Systems I
Spring 2017 Babak Ayazifar, Vladimir Stojanovic Discussion 1B

1. **Visualizing Matrices as Operators** This problem is going to help you visualize matrices as operations. For example, when we multiply a vector by a "rotation matrix," we will see it "rotate" in the true sense here. Similarly, when we multiply a vector by a "reflection matrix," we will see it be "reflected." The way we will see this is by applying the operation to all the vertices of a polygon, and seeing how the polygon changes.

   Your TA will now show you how a unit square can be rotated, scaled or reflected using matrices!

   Part 1: Rotation Matrices as Rotations

   (a) We are given matrices T_1 and T_2, and we are told that they will rotate the unit square by 15 degrees and 30 degrees, respectively. Design a procedure to rotate the unit square by 45 degrees using only T_1 and T_2, and plot the result in the iPython notebook. How would you rotate the square by 60 degrees?

   (b) Try to rotate the unit square by 60 degrees using only one matrix. What does this matrix look like?

   (c) T_1, T_2, and the matrix you used in part c) are called "rotation matrices". They rotate any vector by an angle, θ. Show that a rotation matrix has the following form:

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

   where θ is the angle of rotation. (Hint: Use your trigonometric identities!)

   (d) Now, we want to get back the original unit square from the rotated square in part b). What matrix should we use to do this? *Don't use inverses!*

   (e) Use part d) to obtain the "inverse" rotation matrix for a matrix that rotates a vector by θ. Multiply the inverse rotation matrix with the rotation matrix, and vice-versa. What do you get?

   Part 2: Commutativity of Operators A natural next question to ask is the following: Does the *order* in which you apply these operations matter? Follow your TA to obtain the answers to the following questions!

   (a) Let's see what happens to the unit square when we rotate the matrix by 60 degrees, and then reflect it along the y-axis.

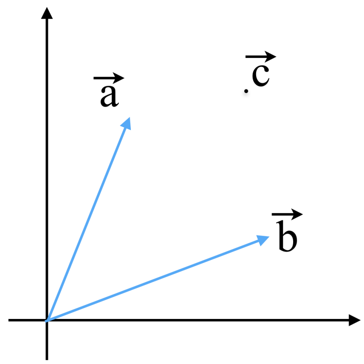
   (b) Now, let's see what happens to the unit square when we first reflect it along the y-axis, then rotate the matrix by 60 degrees.

   (c) Try to do steps a) and b) by multiplying the reflection and rotation matrices together (in the correct order for each case). What does this tell you?

   (d) If you reflected the unit square twice (along any pair of axes), do you think the order in which you applied the reflections would matter? Why/why not?

2. Visualizing Span

We are given a point $\vec{c}$ that we want to get to, but we can only move in two directions: $\vec{a}$ and $\vec{b}$. We know that to get to $\vec{c}$, we can travel along $\vec{a}$ for some amount α, then change direction and travel along $\vec{b}$ for some amount β. We want to find these two scalars α and β such that we reach point $\vec{c}$. That is, $\alpha\vec{a}+\beta\vec{b}=\vec{c}$.



(a) First, consider the case where $\vec{a}=\begin{bmatrix}1\\0\end{bmatrix}$, $\vec{b}=\begin{bmatrix}1\\1\end{bmatrix}$, and $\vec{c}=\begin{bmatrix}-2\\2\end{bmatrix}$. Find the two scalars α and β such that we reach point $\vec{c}$. What if $\vec{a}=\begin{bmatrix}1\\1\end{bmatrix}$, $\vec{b}=\begin{bmatrix}2\\1\end{bmatrix}$?

(b) Now formulate the general problem as a system of linear equations, and write it in matrix form.