EECS 16A Designing Information Devices and Systems I
Spring 2017 Babak Ayazifar, Vladimir Stojanovic **Discussion 4A**

1. **Mechanical Problems**

   (a) Compute the determinant of $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

   (b) Compute the determinant of $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

   (c) Compute the determinant of $\begin{bmatrix} -4 & 0 & 0 & 0 \\ 0 & 17 & 0 & 0 \\ 0 & 0 & -31 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

2. **Row Operations and Determinants**

   In this question we explore the effect of row operations on the determinant of a matrix. Note that scaling a row by a will increase the determinant by a, and adding a multiple of one row to another will not change the determinant. Swapping two rows of a matrix and computing the determinant is equivalent to multiplying the determinant of the original matrix by -1. The determinant of an identity matrix is 1. Feel free to prove these properties to convince yourself that they hold for general square matrices.

   (a) An upper triangular matrix is a matrix with zeros below its diagonal. For example a 3×3 upper triangular matrix is :

   $$\begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{bmatrix}$$

   By considering row operations and what they do to a determinant, argue that the determinant of a general $n \times n$ upper-triangular matrix is the product of its diagonal entries, if they are non-zero. For example, the determinant of the 3×3 matrix above is $a_1 \times b_2 \times c_3$ if $a_1, b_2, c_3 \neq 0$.

   (b) If the diagonal of an upper-triangular matrix has a zero entry, argue that its determinant is still the product of its diagonal entries.

3. Steady State Reservoir Levels We have 3 reservoirs: A, B and C. The pumps system between the reservoirs is depicted in Figure **??**.

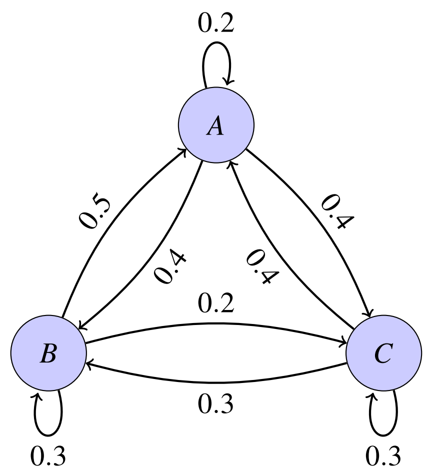

Figure 1: Reservoir pumps system

(a) Write the transition matrix representing the pumps system in the problem.

(b) Assuming you start the pumps with water levels $A_0 = 129, B_0 = 109, C_0 = 0$ (in kiloliters). What would be the steady state water levels (in kiloliters) according to the pumps system described in the problem?

Hint: If $\vec{x_{ss}} = \begin{bmatrix} A_{ss} \\ B_{ss} \\ C_{ss} \end{bmatrix}$ is a vector describing the steady state levels of water in the reservoirs (in kiloliters), what happens if you fill the reservoirs A, B and C with A_{ss}, B_{ss} and C_{ss} kiloliters of water, respectively and apply the pumps once?

Hint II: Note that the pumps system preserves the total amount of water in the reservoirs. That is, no water is lost or gained by applying the pumps.