This homework is due Monday, April 10, 2017, at 23:59.
Self-grades are due Thursday, April 13, 2017, at 23:59.

Submission Format

Your homework submission should consist of **one** file.

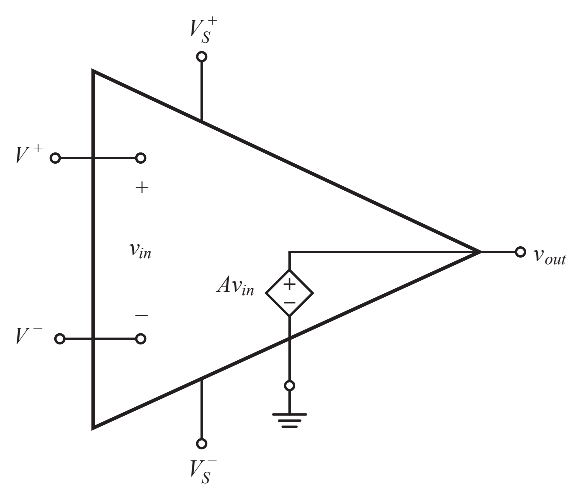
- `hw10.pdf`: A single pdf file that contains all your answers (any handwritten answers should be scanned). If you used an IPython notebook, it should also be saved as a pdf.

  If you do not attach a pdf of your IPython notebook, you will not receive credit for problems that involve coding. Make sure your results and plots are showing.

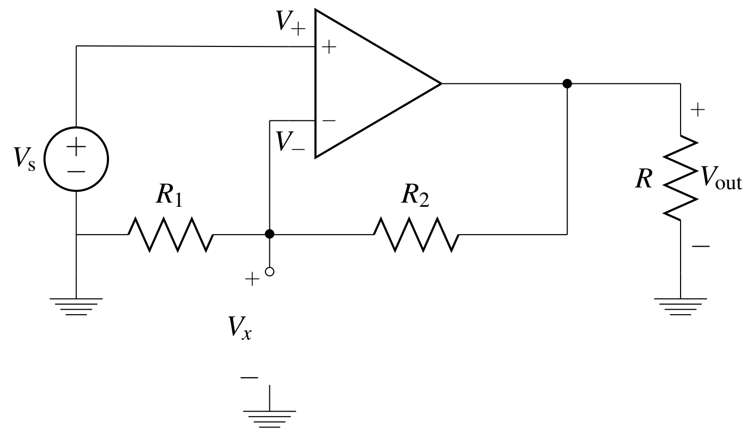
Submit this file to the appropriate assignment in Gradescope.

1. Op-Amp Golden Rules

In this question we are going to show that the golden rules for op-amps hold by analyzing equivalent circuits and then taking the limit as the open-loop gain approaches infinity. Below is a picture of the equivalent model of an op-amp we are using for this question.



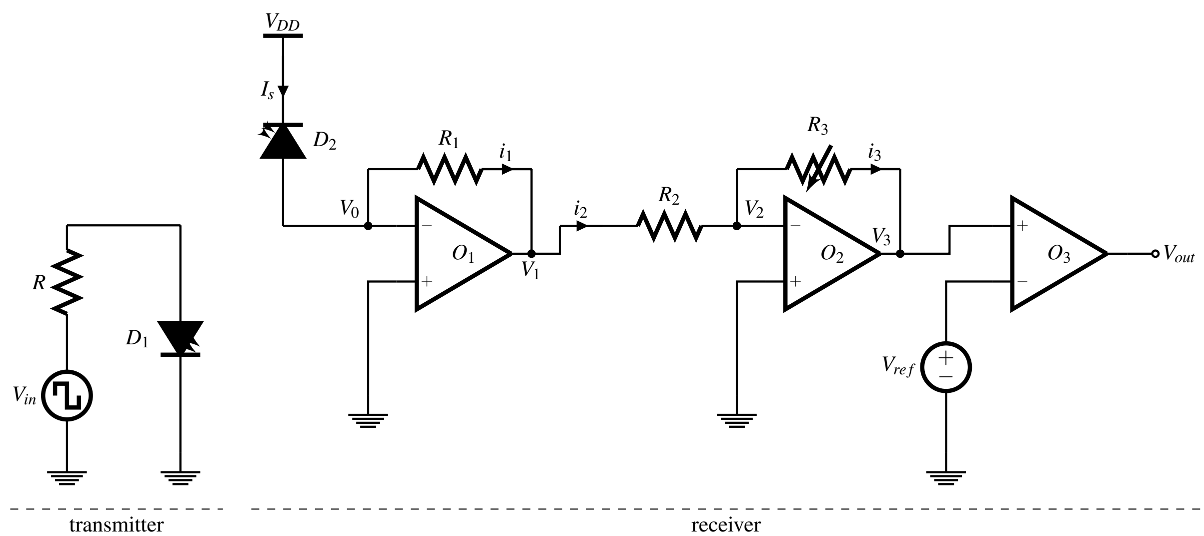
(a) Now consider the circuit below. Draw an equivalent circuit using the op-amp model shown above and calculate V_{out} and V_x in terms of A, V_s, R_1, R_2 and R. Is the magnitude of V_x larger or smaller than the magnitude of V_s? Do these values depend on R?



(b) Using your solution to part (a), calculate the limits of V_{out} and V_x as $A \to \infty$. Do you get the same answers if you apply the golden rules ($V_+ = V_-$ when there is negative feedback)?

2. Wireless Communication With An LED

In this question, we are going to analyze the system shown in the figure below. It shows a circuit that can be used as a wireless communication system using visible light (or infrared, very similar to remote controls).



The element D_1 in the transmitter is a light-emitting diode (LED in short). An LED is an element that emits light and whose brightness is controlled by the current flowing through it. You can recall controlling the light emitted by an LED using your MSP430 in touchscreen lab 1. In our circuit, the current across the LED, hence its brightness, can be controlled by choosing the applied voltage V_{in} and the value of the resistor R. In the receiver, the element labeled as D_2 is a reverse biased solar cell. You can recall using a reverse-biased solar cell in imaging labs 1 to 3 as a light-controlled current source. We will denote the current supplied by the solar cell by I_S. In this circuit, the LED D_1 is used as a means for transmitting information with light, and the reverse-biased solar cell D_2 is used as a light receiver to see if anything was transmitted.

Remark: In imaging lab 3, we talked about how non-idealities, such as background light, affect the performance of a system that does light measurements. In this question we assume ideal conditions, that is, there is no source of light around except for the LED.

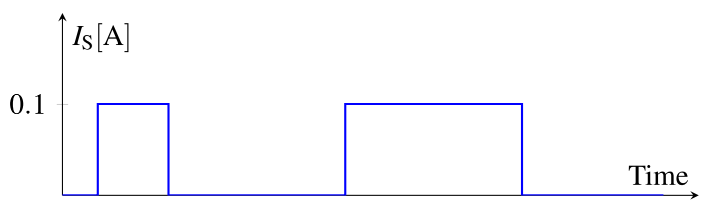
In our system, we define two states for the transmitter: the *transmitter is sending something* when they turn on the LED and the *transmitter is not sending anything* when they turn off the LED. On the receiver side, the goal is to convert the current I_S generated by the solar cell into a voltage and amplify it, so that we can read the output voltage V_{out} to see if the transmitter was sending something or not. The circuit implements this operation through a series of op-amps. It might look look complicated at first glance, but we can analyze it one section at a time.

(a) Currents i_1, i_2 and i_3 are labeled on the diagram. Assuming the Golden Rules hold, is $I_S = i_1$? $i_1 = i_2$? $i_2 = i_3$? Treat the solar cell as an ideal current source.

(b) Use the Golden Rules to find V_0, V_1, V_2 and V_3 in terms of I_S, R_1, R_2 and R_3.
Hint: Solve for them from left to right, and remember to use the op-amp golden rules.

(c) In the previous part, how could you check your work to gain confidence that you got the right answer?

(d) Now, assume that the transmitter has chosen the values of V_{in} and R to control the intensity of light emitted by LED such that when the *transmitter is sending something*, I_S is equal to 0.1 A and when the *transmitter is not sending anything*, I_S is equal to 0 A. The following figure shows a visual example of how this current I_S might look like as time changes (note that this is just to help you visualize the shape of the current supplied by the solar cell).



For the receiver, suppose $V_{ref} = 2\,V$, $R_1 = 10\,\Omega$, $R_2 = 1000\,\Omega$, and the supply voltages of the op-amps are $V_{DD} = 5\,V$ and $V_{SS} = -5\,V$. Pick a value of R_3 such that V_{out} is V_{DD} when the *transmitter is sending something* and V_{SS} when the *transmitter is not sending anything*?

(e) In the previous part, how could you check your work to gain confidence that you got the right answer?

3. Cool For The Summer

You and a friend want to make a box that helps control an air-conditioning unit. You both have dials that emit a voltage: 0 means you want to leave the temperature as it is. Negative voltages mean that you want to reduce the temperature. (It's hot so we will assume that you never want to increase the temperature — so, we're not talking about a Berkeley summer...)

Your air-conditioning unit, however, responds to positive voltages. The higher the voltage, the more stronger it runs. At zero, it is off. (If it helps, think of this air-conditioning unit as a heat pump. If you run it with negative voltage, it pumps heat in the opposite direction — from outside to inside. If positive voltage, it pumps heat from inside to outside.)

Therefore, you need a box that is an inverting summer — it outputs a weighted sum of two voltages where the weights are both negative. (Weighted because each of you has your own subjective sense of how much to turn the dial down, so you need to compensate for this.)

This problem walks you through this using an op-amp.

(a) As a first step, create a general inverting amplifier and find the voltage gain.

(b) Now add a second input to the amplifier from above. Find the overall voltage gain as a function of the two input voltages.

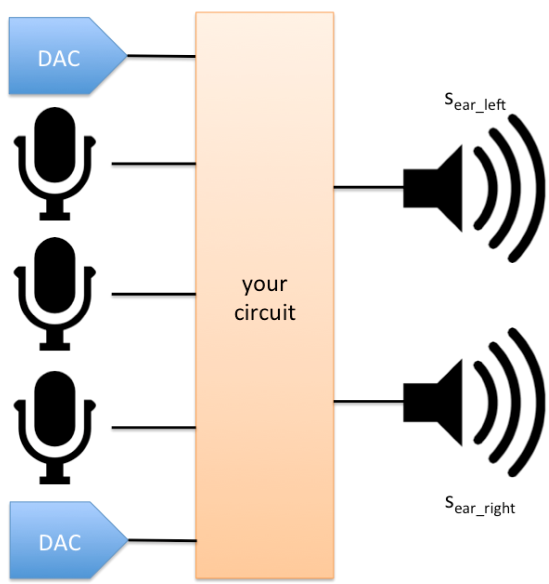
(c) Let's suppose you want to have the overall voltage gain be $V_{out} = -\left(\frac{1}{4}V_{S1} + 2V_{S2}\right)$ where V_{S1} and V_{S2} represent the input voltages from you and your friend. Select resistor values such that this is the overall voltage gain.

(d) Now suppose you have another AC unit that you want to add to the same room. This unit however, functions opposite to the already existing unit; it responds to negative voltages. You want to run both units at the same time. Add another op-amp to this circuit to create an output for the second AC unit.

4. Noise Cancelling Headphones Part 2

Almost everyone has tried "noise cancelling" headphones at some point. The basic goal of noise cancelling headphones is for the user to hear only the desired audio signal and not any other sounds from external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources, and then feeds a signal in to your speakers that cancels (subtracts out) that externally-generated sound.

(a) In discussion, we had just one speaker and one microphone, but almost all headphones today have two speakers (one for each ear). Adding an extra speaker that can be driven by a separate audio stream typically makes things sound more real to us. For similar reasons, having multiple microphones to pick up ambient sounds from multiple different locations can help us do a better job of cancellation, if we can use that information in the right way.



Let's now assume that our system has 3 microphones and 2 speakers, and that the source of our audio is stereo - i.e., we have two different audio streams s_{left} and s_{right} (produced by two different DACs) that represent the ideal sounds we would like the user to hear in their left and right ear. We have three microphone audio signals s_{mic1}, s_{mic2}, and s_{mic3}, and let's assume that without any active noise cancellation, some fraction of the signal picked up by each microphone would be heard by the user in each of their ears. For example, a_{1left} would represent the fraction of the signal picked up by microphone 1 that will be heard in the user's left ear, a_{2right} would represent the fraction of the signal picked up by microphone 2 that will be in the user's right ear, etc.

Let the vector $\vec{s}_{noise}$ represent the noise heard in each ear and $\vec{s}_{mic}$ represent the sound in each mic. Find a matrix $\mathbf{A}$ such that $\vec{s}_{noise} = \mathbf{A}\vec{s}_{mic}$.

(b) Assume no noise canceling, find an equation for $\vec{s}_{ear}$, the sound heard in each ear in terms of the two audio streams and $\vec{s}_{noise}$.

(c) In order to cancel the noise, we want to create a signal that is the inverse of $\vec{s}_{noise}$. Let $\vec{s}_{cancel}$ be the vector representing the cancel signal in each headphone. Find a matrix $\mathbf{B}$ in terms of the matrix $\mathbf{A}$ such that $\vec{s}_{cancel} = \mathbf{B}\vec{s}_{mic}$.

(d) Assume that the microphones can be modeled as voltage sources, whose value v_{micn} is proportional to s_{micn}, design and sketch a circuit that would implement the cancellation matrix $\mathbf{B}$. You should assume that this circuit has three voltage inputs v_{mic1}, v_{mic2}, and v_{mic3} and two voltage outputs v_{cancel_left} and v_{cancel_right} (corresponding to the voltages that will be subtracted from the desired audio streams in order to cancel the externally-produced sounds). In order to simplify the problem, you can assume that all of the v_{mic} voltages are already centered at 0 V (relative to the DAC ground). Furthermore, assume all entries of the $\mathbf{A}$ matrix are positive. You may use op-amps and resistors to implement your circuit. You do not have to pick specific resistor values, but write expressions for each resistor value.

(e) *Practice:* Building upon your solutions to all previous parts, and otherwise making the same assumptions about the relative voltage ranges of v_{mic1}, v_{mic2}, and v_{mic3} and available supply voltages, sketch the complete circuit you would use to create the stereo audio on the two speakers while cancelling the noise picked up by the three microphones.

5. Homework process and study group

Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?
Working in groups of 3-5 will earn credit for your participation grade.