EE16B Section 10B - Controls

Warmup

Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$.

Find the eigenvalues of $\begin{bmatrix} 2 & -5 \\ 1 & 0 \end{bmatrix}$.

Solve $\det(\lambda I - A) = 0$:

$$(\lambda-1)(\lambda-6)+6=0 \qquad\qquad (\lambda-2)\lambda+5=0$$

$$\lambda^2-7\lambda+12=0 \qquad\qquad \lambda^2-2\lambda+5=0$$

$$\lambda=3,4 \qquad\qquad \lambda=\frac{2\pm\sqrt{4-20}}{2}=1\pm 2i$$

Find e-vecs: $(A-\lambda I)v_i=0$

$$\begin{bmatrix} -2 & 3 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=0 \qquad \begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=0$$

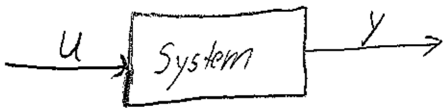
$$2v_1=3v_2 \qquad v_1=v_2$$

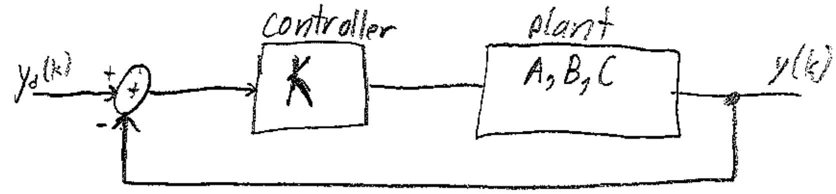
$$\vec{v}_1=\begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad \vec{v}_2=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Questions from Lecture

Today we are discussing both matrix operations and feedback control. We will see that they are closely related!

Open-loop vs. Closed loop





$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k)$$

World/system/physics gives us:

- System behavior (A, B, C)

We control:

- Inputs $u(k)$
- Perhaps an initial condition $x(0)$

$$x(k+1) = A_{CL}x(k) + B_{CL}\, y_d(k)$$
$$y(k) = Cx_k$$

A_{CL}, B_{CL} made up of A, B, C, K

World gives us:

- Plant behavior (A, B, C)

We control:

- Controller parameters (K)
- Desired outputs y_d

Each has advantages!

- Simpler
- No sensor required
- Does not introduce stability problems

- Better performance
- May be required to achieve stability

Super cool aside: the Mandelbrot Set

We have been iterating systems of the form

$$x(k+1) = Ax(k) + B$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} x_1(k) & -x_2(k) \\ x_2(k) & x_1(k) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

What if we make this nonlinear? For which c_1, c_2 will this system be stable if $x_1(0) = x_2(0) = 0$?