EE16B Section 11B

Warmup

Consider a system that updates as $x(k+1) = Ax(k)$. If $A = \begin{bmatrix} 0 & h/4 \\ 1 & 1 \end{bmatrix}$, find a value of h for which the system is:

- Stable
- Unstable
- Marginally stable
- Stable, but "spiralling"

Solution: The e-vals of A are

$$\lambda(\lambda - 1) - h/4 = 0$$

$$\lambda^2 - \lambda - h/4 = 0$$

$$\lambda = \frac{1 \pm \sqrt{1+h}}{2} = 1/2 \pm 1/2\sqrt{1+h}$$

So if:

$h = -1$, then $\lambda_1 = \lambda_2 = 1/2 \Rightarrow$ stable

$h = 3$, then $\lambda = 1.5, 0.5 \Rightarrow$ unstable

$h = 0$, then $\lambda = 1, 0 \Rightarrow$ marginally stable

$h = -2$, then $\lambda = 1/2 \pm 1/2 j \Rightarrow$ stable but spiralling

Questions from Lecture

Demo: Linear Phase Portraits

(Continuous-time, not discrete, so the details are a bit different.)

-0.5 -0.2 1 -2 : Attractor

-2 1 -1 1 : Saddle point

0 1 -1 1 : Spiral repellor

0 1 -1 0 : orbit

1 0 0 1 : star node (source)

-1 0 0 -1 : star node (sink)

Homework 9 Revisited

$$A_{CL} = \begin{bmatrix} 1 & T_s - hT_s^2/2m \\ 0 & 1 - hT_s/m \end{bmatrix}$$

The eigenvalues are 1, $1 - hT_s/m = 1 - 0.002h$.

So try:

$h = 10, 500, 800, 990, 1001$

Midterm Review Exercise