EE 16B Designing Information Devices and Systems II
Fall 2015 Section 13B

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1. Phase Response

Sketch the phase (in radians) vs. log ω for the filter specified below with $\omega_1 = 10^3$ rad/s and $\omega_2 = 10^5$ rad/s.

$$H(\omega) = \frac{-5}{1 + j\omega/\omega_1} \frac{1}{1 + j\omega/\omega_2}$$

Hint: You may want to figure out the phase responses of each component of $H(\omega)$ individually and then combine them together.

Solutions:

$\measuredangle H = \measuredangle \frac{-5}{1+\frac{j\omega}{\omega_1}} + \measuredangle \frac{1}{1+\frac{j\omega}{\omega_2}}$

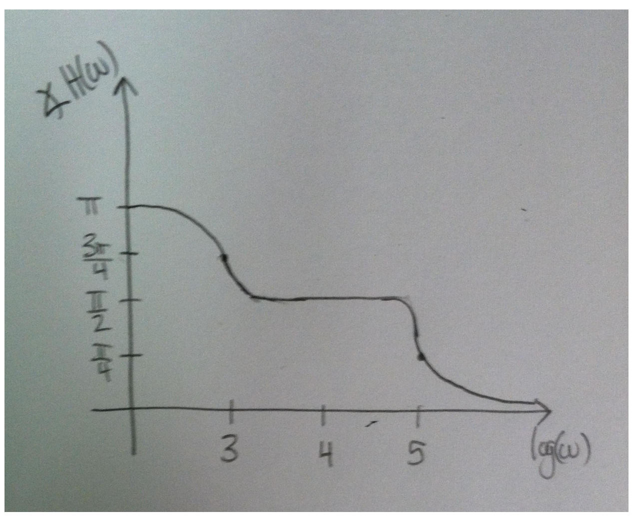
$= \pi - arctan(\frac{\omega}{\omega_1}) - arctan(\frac{\omega}{\omega_2})$

As $\omega \to 0, \measuredangle H \to \pi$

As $\omega \to \infty, \measuredangle H \to 0$

At $\omega = \omega_1, \measuredangle H \approx \frac{3\pi}{4}$

At $\omega = \omega_2, \measuredangle H \approx \frac{\pi}{4}$



2. Filter Design

Consider a system input that has a signal at 100Hz and unwanted noise at 10kHz. For the filter described below, find the value of ω_c that attenuates the unwanted noise by a factor of 20. What is the gain of the desired signal from this filter?

$$H(\omega) = \frac{2}{1 + j\omega/\omega_c}$$

Solutions: $\frac{2}{\sqrt{1^2 + (\frac{10000*2\pi}{\omega_c})^2}} = \frac{1}{20}$

$1^2 + (\frac{10000*2\pi}{\omega_c})^2 = 40^2$

$(\frac{10000-2\pi}{\omega_c})^2 = 40^2 - 1$

$\omega_c = \frac{10000}{\sqrt{1599}} * 2\pi$

$H(\omega) = \frac{2}{1+\frac{j\omega}{\omega_c}}$

$= \frac{2}{\sqrt{1^2 + (\frac{100}{\sqrt{1599}*10000})^2}}$

$= \frac{2}{\sqrt{1^2 + (\frac{1}{\sqrt{1599}*100})^2}}$

$= 1.86$

3. **Dynamic Voltage and Frequency Scaling**

A low-power microcontroller operates in three modes. In high-performance mode, the processor operates at a supply voltage of $V_{DD} = 1.2\text{V}$ and a frequency of 1MHz and consumes $100\mu\text{W}$ of power. Assume the resistance of the gates in the microcontroller is proportional to $1/(V_{DD} - 0.4\text{V})$.

(a) In low-power mode, the processor operates at $V_{DD} = 0.48\text{V}$. What is the highest possible operating frequency in this mode? How much power is consumed?

Solutions: $\frac{f_2}{f_1} = \frac{V_{dd_2}-0.4}{V_{dd_1}-0.4} = \frac{0.8}{0.08} = \frac{1}{10}$, so $f_2 = \frac{1}{10} f_1 = 100kHz$.

$\frac{P_2}{P_1} = \frac{f_2}{f_1} (\frac{V_{dd_2}}{V_{dd_1}})^2 = \frac{1}{10} \frac{1}{2.5^2} = \frac{1}{62.5}$

$P_2 = P_1 * \frac{1}{62.5} = \frac{100\mu W}{62.5} = \frac{8}{5}\mu W = 1.6\mu W$

(b) In sleep mode, the process operates at a frequency of just 1kHz. What is the lowest possible operating voltage that would allow operation at this frequency? How much power is consumed?

Solutions: $\frac{f_3}{f_1} = \frac{V_{dd_3}-0.4}{V_{dd_1}-0.4}$

$\frac{1}{1000} = \frac{V_{dd_3}-0.4}{0.8}$

$V_{dd_3} = 0.4 + 8 * 10^{-4}$

$= 0.4008V$

$\frac{P_3}{P_1} = \frac{f_3}{f_1} (\frac{V_{dd_3}}{V_{dd_1}})^2$

$\frac{V_{dd_3}}{V_{dd_1}} = \frac{0.4008V}{1.2V} \approx \frac{1}{3}$

$= \frac{1}{1000} * (\frac{1}{3}^2) = \frac{1}{9000}$

$P_3 = \frac{1}{9000} * 100\mu W = \frac{1}{90}\mu W = 11.1nW$