Discussion 3

(Super Quick)
Lin Alg Review:

1. Transpose:
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

So, what is?

$$(XYZ)^T = Z^T Y^T X^T$$

2. Orthogonal Matrix $U \Rightarrow U^H = U^{-1}$

3. Determinant

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

(The hard one)

4. Eigenvalues & Eigenvectors

Given some matrix A

$$A\eta = \lambda\eta$$

What does this mean?

→ In special cases of λ & η, it is possible for matrix multiplication to look like multiplication by a constant.

→ Kind of decomposition of the matrix into vectors η & scalars λ

Eigen value example:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

What is the all impt eigen equation?

$$(A - \lambda I)\eta = 0$$

$$(A - \lambda I) = \begin{bmatrix} 2-\lambda & 3 \\ 4 & 1-\lambda \end{bmatrix}$$

To solve for λ we find the determinant & set it to 0

$$(1-\lambda)(2-\lambda) - 12 = 0$$

$$\lambda^2 - 3\lambda + 2 - 12 = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$(\lambda - 5)(\lambda + 2) = 0 \quad \Rightarrow \quad \lambda = -2, 5$$

Plug in to get values η_1 & η_2

$\lambda = -2$

$$\begin{bmatrix} 4 & 3 \\ 4 & 3 \end{bmatrix} \eta = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 3 & 0 \\ 4 & 3 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow 4\eta_1 + 3\eta_2 = 0 \quad \Rightarrow \eta = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

$$\eta_1 = -\frac{3}{4}\eta_2$$

$\lambda = 5$

$$\begin{bmatrix} -3 & 3 \\ 4 & -4 \end{bmatrix} \eta = 0 \Rightarrow -3\eta_1 = -3\eta_2 \Rightarrow \eta = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We can now see

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \end{bmatrix} = -2 \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

$-6 + 12 = 6$

$-12 + 4 = -8$

And

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$2 + 3 = 5$

$4 + 1 = 5$

Why did we go over this? How many people understand svd that you went over in class?

SVD is kin to eigenvectors/values

SVD

The all impt SVD eqn

$$M = U \Sigma V^T$$

where

M is a matrix that represents something impt & complex to you

V are orthogonal vectors which are eigenvectors of M $(M v_i = \lambda v_i)$

In other words, V is a current domain

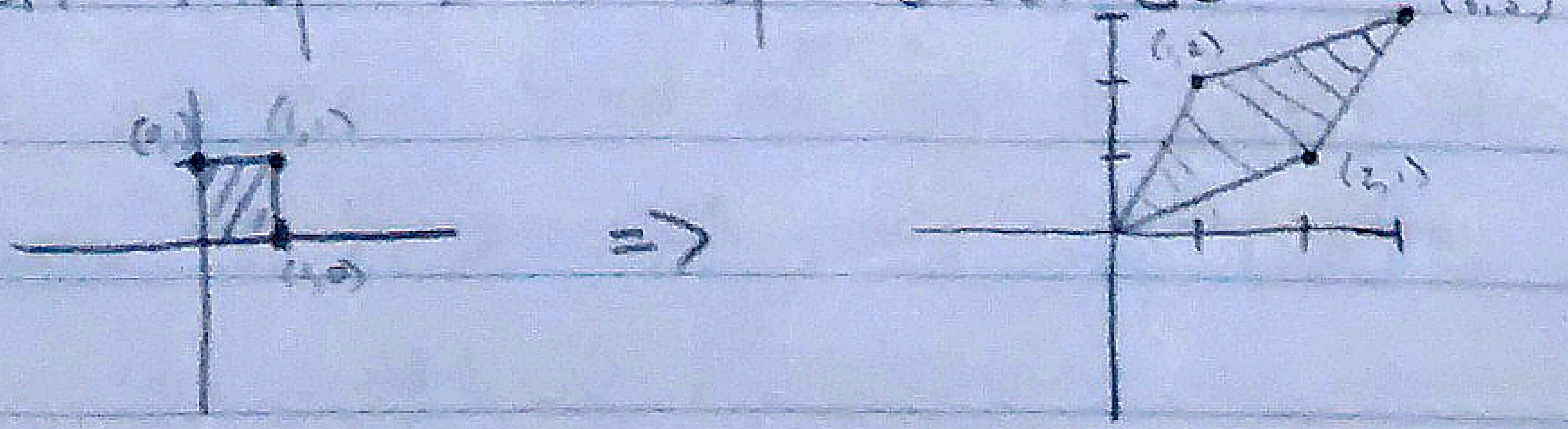
U are orthogonal vectors of a co-domain

Σ are weights that describe M in the co-domain

For example, if I am given some M transform

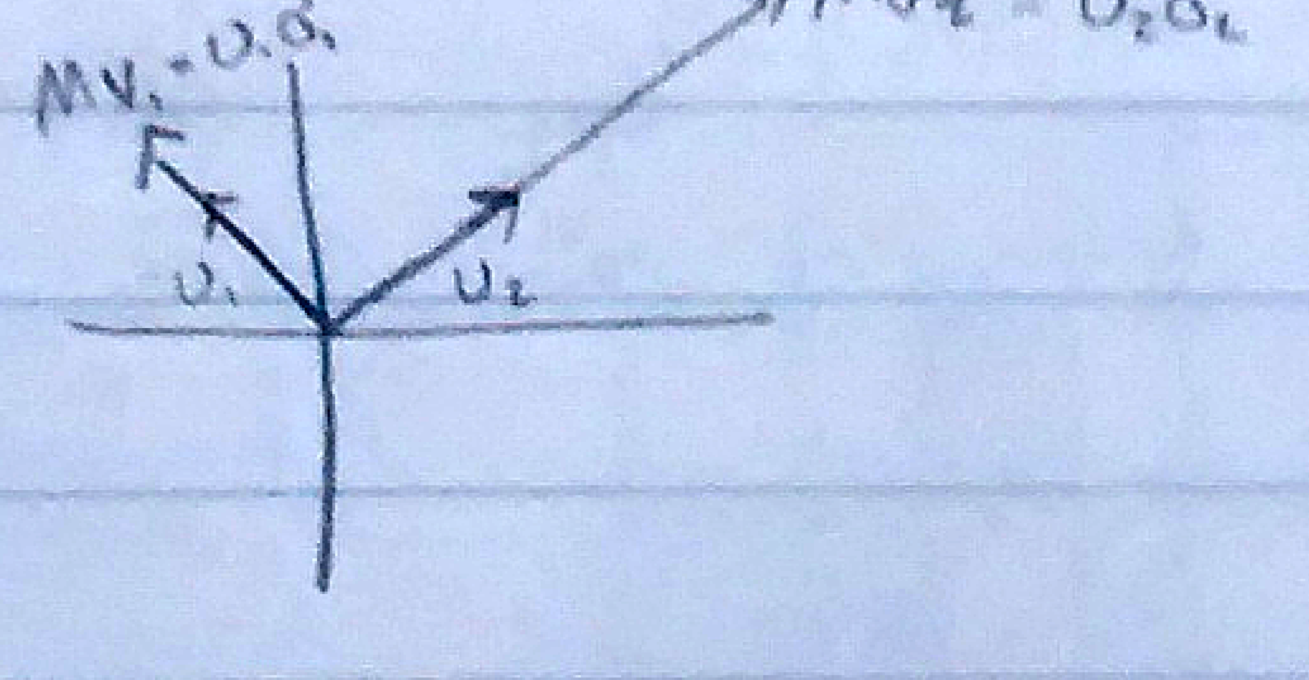
$$M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Then my x & y becomes



I can describe this transformation in parts, first break M into eigen values so that I can describe the transform in an orthogonal domain

Next, turn those eigen vectors into unit vectors & weights



where U is the new co-domain of u_i vectors & σ_i are the weights that translate $Mv_i = u_i\sigma_i$

Thus, we can kind of intuit what it means to break M into 3 parts

$$M = U\Sigma V^T$$

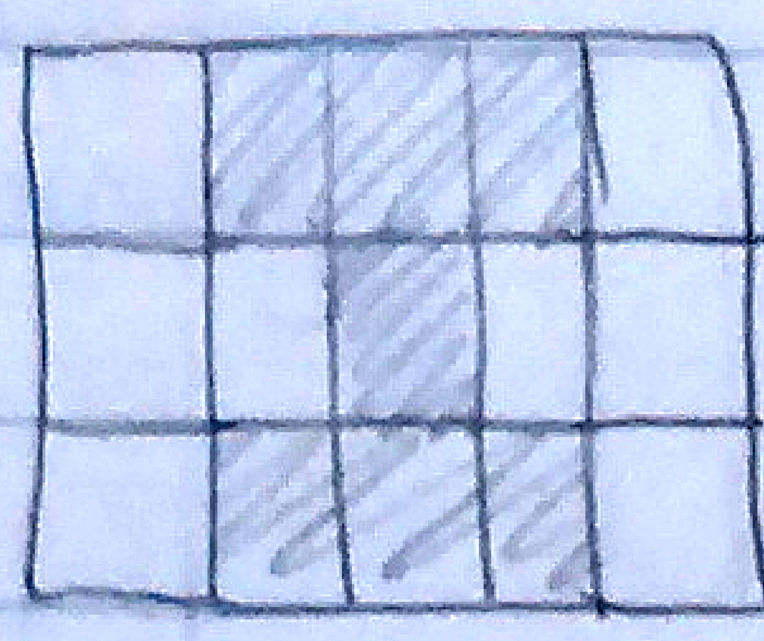
Now, why is this useful? What are the applications of this?

How did we use it in lab?

Noise reduction

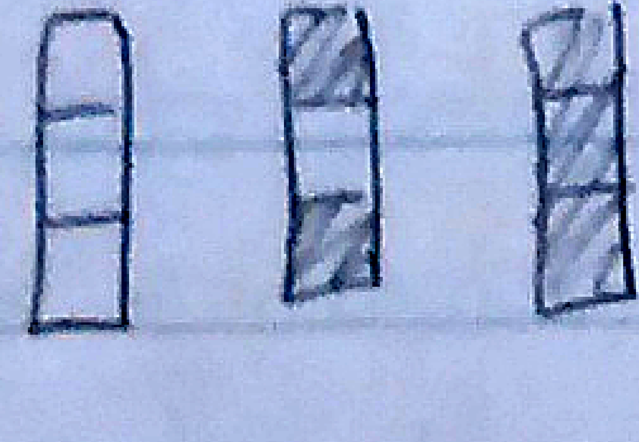
- we can look at the weight (σ_i) of each u_i vector & determine what u_i are very impt in M & which don't change very much, then we keep some number of the largest σ_i.

Example:



Imagine we describe this picture as a 1 for a black box & 0 for white

We can see very easily that there are only 3 vectors in this picture



When we break M into parts we will get back only $\sigma_1, \sigma_2, \sigma_3$ for each of the three independent vectors, & 3 u_i vectors

SVD has given us a way to transform into the three most important vectors

This is what you did in lab, but with some noise mixed in, you got back a lot of vectors, but by only taking the 20 largest σ_i, you picked out the more "important" vectors that could describe the neuron data.

What are other applications?

Netflix recommendations

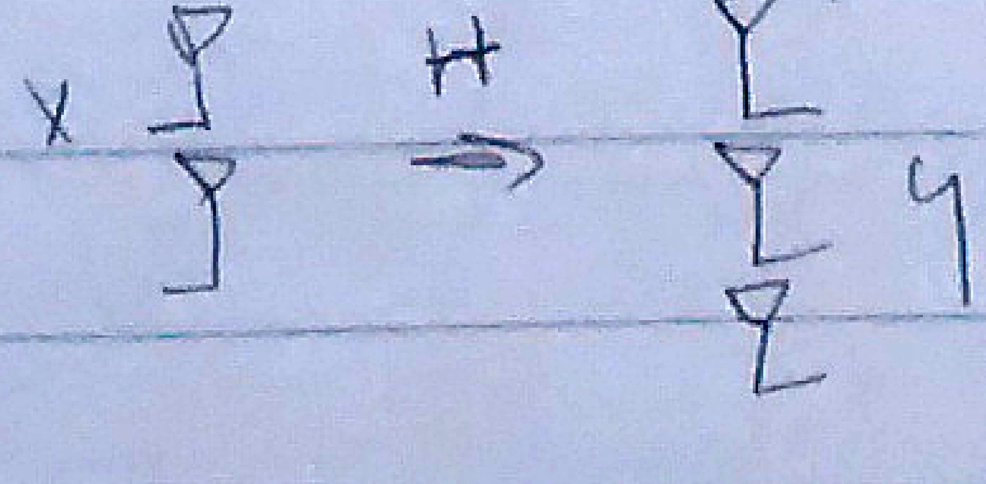
Image compression

Machine learning

Lets look at the SVD problem in your HW.

Another application of SVD is to rewrite a transform into matricies with nicer properties

for example, you are given



What is a formula (w/o SVD) to describe this?

$$y = Hx$$

we can break H into SVD, what is it?

$$H = U \Sigma V^T$$

$$y = \underbrace{U}_{\text{orthogonal matrix}} \underbrace{\Sigma}_{\text{diagonal}} V^T x$$

Now we have an equation in terms of only unitary & a diagonal matrix